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Landfill Leachate Flux Equations: Theoretical Development and Computer Programs

by

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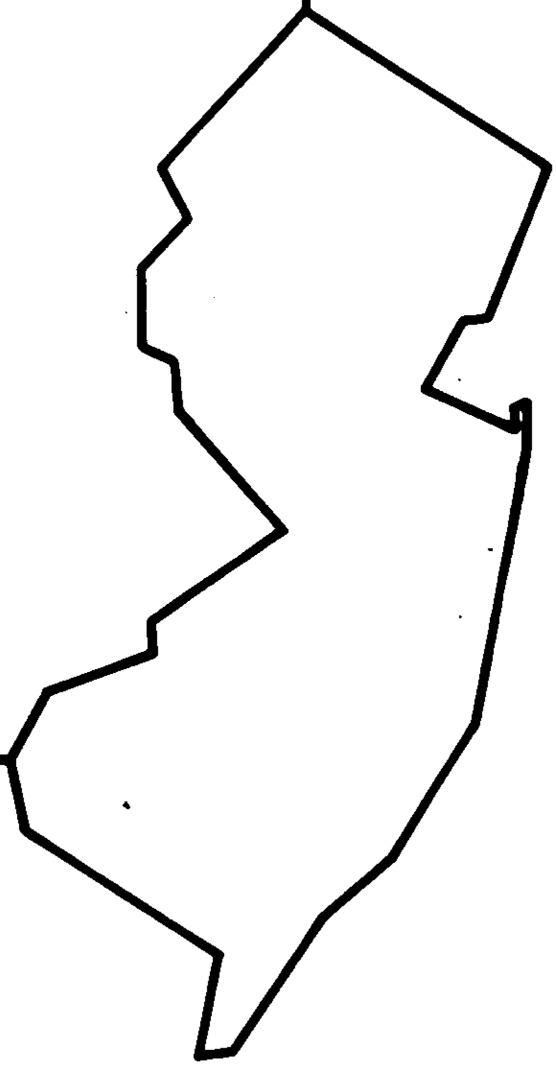

Department of Environmental Protection
Division of Water Resources

1984

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THEORETICAL DEVELOPMENT AND COMPUTER PROGRAMS

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I. Abstract

The volume of leachate which leaks through a landfill liner is an important factor in assessing the environmental impact of the landfill. The analytic equations describing the dissipation of head on a landfill liner and the leakage through the liner are expanded here to describe two additional cases. The first is the case of a time-invariant leakage head (termed the steady state case) which results from a steady recharge. The second is the case of a quasi-steady state head which results from discrete recharge events occuring at a fixed interval.

The effectiveness of a liner can be measured by its efficiency (volume of water which leaks through the liner divided by total volume of water impinging on the liner) or by the average leakage rate. Efficiency varies depending upon the amount and timing of precipitation and can be misleading. Comparing efficiency and the average leakage rate for a typical case shows that the average leakage rate is a better measure of the effectiveness of a landfill liner.

Computer programs to solve the equations for the transient, steady state, and quasi-steady state cases, using the Hewlett-Packard HP-41C programmable calculator are listed, as is a FORTRAN program to model the transient state using irregular precipitation data.

II. INTRODUCTION

The volume of leachate which leaks out of a landfill is an important factor in assessing the environmental impact of the landfill. A currently used approach to minimizing the leachate problem is to line the bottom of the landfill with clay or some other relatively impermeable material. The liner is sloped to a collection drain and covered with a permeable material (e.g. sand or gravel) so that any leachate produced will be intercepted and drained to a central collection point. For any liner which is not totally impermeable some leachate will theoretically escape. Quantifying this amount is important. The purpose of this study is to present (1) the theoretical background to describe leakage through a landfill liner and and (2) computer programs for calculating the leakage through, and effectivness of, a liner.

Equations for the transient response of the system have been previously developed to describe induced leakage and dissipation of a leachate head on a sloped liner (Wong, 1977; and Kmet, Quinn, and Slavik, 1981). These mathematical equations are modified to describe (1) steady state recharge (which produces a steady-state head on the liner) and (2) the case of cyclic, pulsed infiltration. This later case is termed the quasi-steady state case for it gives rise to a cyclic pattern of head growth and dissipation. For each of these two cases, and the original transient case, equation are presented to describe the head on the liner at any time, the efficiency of the liner, and the average leakage rate. The efficiency of the liner and its average leakage rate are two different measures of the effectiveness of the liner in preventing leakage. The utility of each measure is contrasted by examples based on the developed equations.

Under normal precipitation conditions, the leachate head and leakage can be estimated by a numerical technique. Precipitation data from a five-year period are used to estimate leakage from a hypothetical landfill.

Programs for the Hewlett-Packard HP-41C programmable calculator are presented for the transient, steady state, and quasi-steady state cases. A FORTRAN program to model the transient state with rainfall is also presented.

Acknowledgements

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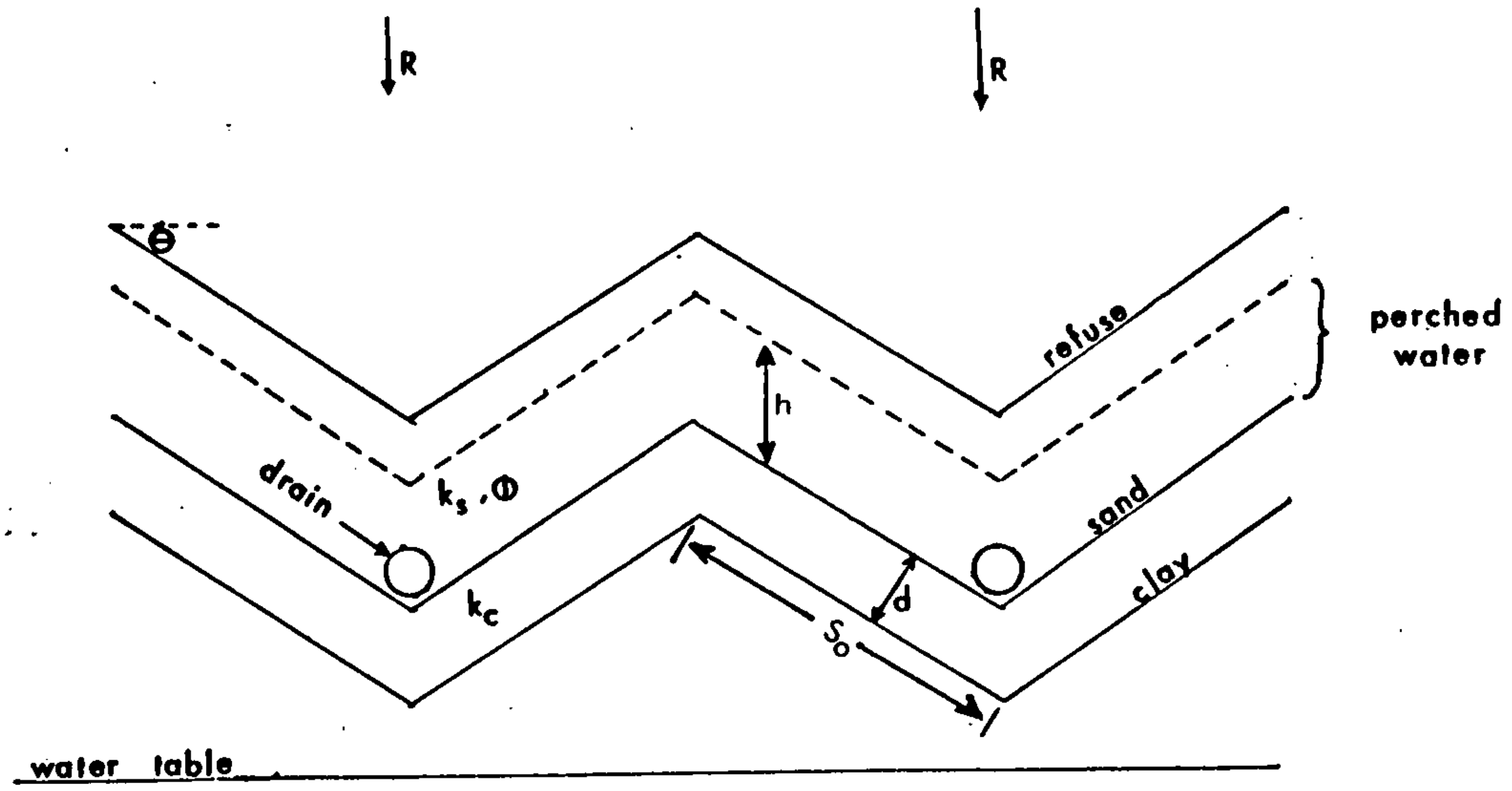
III. Liner Geometry

A cross-section of a landfill is shown in figure 1. The geometry of the clay liner is similar in shape to a piece of corrugated material. The clay liner forms a series of "V"s on the bottom of the landfill. A drain consisting of a porous pipe in a sand blanket is located at the bottom of each "V" and carries off the leachate it receives. The drains are treated as horizontal in this report.

Several variables are needed to describe the landfill geometry. The length of one arm of the "V" from crest to trough, parallel to the slope, is S_o. The liner thickness, perpendicular to the slope, is d. The slope of the liner is Θ. The clay liner has a hydraulic conductivity of k_c while the hydraulic conductivity of the sand is k_s. The porosity of the sand blanket of ϕ. A complete listing of all variables is given in appendix A.

Figure 1. Geometry of Landfill Leachate Collection System



IV. ASSUMPTIONS

Leakage from a landfill is a complex function of infiltration and vertical and horizontal water movement, which is dependant on liner and drain properties, design, and head at bottom of the landfill. In order to make the problem tractable many simplifying assumptions are required. The major assumptions, listed below, are more fully discussed by Kmet, Quinn, and Slavik (1981).

Assumptions:

1. The water table is below the clay liner;
2. The drains are always in a free-draining condition;
3. All materials (clay, sand and refuse) are fully saturated;
4. Any head on the liner becomes effective instantaneously and immediately forces leachate to move to the drain pipes and also through the liner;
5. The leachate slug is rectangular in shape and retains this shape as it dissipates. There is, at all times, a uniform head on the portion of the liner covered by the leachate;
6. The porosity and hydraulic conductivity of the refuse are identical to that of the sand blanket;
7. All flow is governed by Darcy's Law;
8. The landfill geometry is as shown in figure 1;
9. When recharge is added to a partially dissipated leachate slug lying on the liner the entire volume of liquid is redistributed resulting in a new uniform head on the liner, with the saturated length again equal to S_U, and;
10. The clay and sand layers are homogeneous, of constant areal thickness, and of uniform slope.

V. TRANSIENT STATE EQUATIONS

Equations for dissipation of a transient head on a liner are based on the assumption that an initial head h_O instantaneously appears on the liner. This head gradually dissipates as the leachate moves in part through the sand to the drain and in part passes through the clay liner to the underlying soil.

The equations for calculating the head at any subsequent time are derived by equating Darcy's Law with the principal of continuity, the head h and the saturated length s (Kmet, Quinn, and Slavik, 1981).

Darcy's Law is written as

$$Q = k\ I\ A \tag{1}$$

where Q is the volumetric flux rate, k is the hydraulic conductivity, and A is the cross-sectional area for flow. For flow to the drain, parallel to the clay-sand interface, the flow rate at time t is:

$$Q_D = k_s\ \sin\theta\ h\cos\theta\ w \tag{2}$$

where Q_D is the flow rate, h is the head at time t measured perpendicular to the earth's surface (thus h is not parallel to d), and w is the width of the study area.

The continuity equation describing the rate at which the leachate is flowing to the drain at time t is

$$Q_D = \frac{-ds}{dt}\ \phi h\cos\theta w \tag{3}$$

where ds/dt is the time rate of change of the saturated length parallel to the slope.

Combining equations 2 and 3 and eliminating common terms yields

$$ds = \frac{-1}{\phi} k_s \sin\theta dt \qquad (4)$$

Solving differential equation 4 for the initial conditions at t=0, $s=S_o$, the saturated length as a function of time is

$$s = S_o \left(1 - \frac{k_s \sin\theta}{\phi S_o} t\right) \qquad (5)$$

By defining t_1 as:

$$t_1 = \frac{S_o \phi}{k_s \sin\theta} \qquad (6)$$

equation 5 is rewritten as:

$$s = S_o \left(1 - \frac{t}{t_1}\right) \qquad (7)$$

It is easily seen that when the saturated length is equal to 0, the time t is equal to t_1. Thus t_1 is the time necessary for all the leachate to drain off the liner.

The same procedure is used to arrive at an equation for calculating the head at any time. The expression of Darcy's Law for flow through the clay is

$$Q_L = k_c \left(1 + \frac{h\cos\theta}{d}\right) s \cos\theta\, w \qquad (8)$$

where Q_L is the volumetric flux rate through the clay.

The continuity equation describing the rate of head dissipation is

$$Q_L = \frac{-dh}{dt} \phi s \cos\theta w \qquad (9)$$

Combining equations 8 and 9 yields

$$dt = \frac{-\phi}{k_c(1+h\cos\theta/d)} \; dh \tag{10}$$

Solving differential equation 10 for the initial conditions at $t=0$, $h=h_o$, yields

$$h = h_o \left[e^{-at} \left(\frac{d}{h_o\cos\theta} + 1\right) - \frac{d}{h_o\cos\theta}\right] \tag{11}$$

where

$$a = \frac{k_c\cos\theta}{d\phi} \tag{12}$$

Let t_2 be the time when $h=0$ (the time when the leachate head has entirely dissipated). Equation 11 becomes

$$0 = h_o \left[e^{(-at_2)} \left(\frac{d}{h_o\cos\theta} + 1\right) - \frac{d}{h_o\cos\theta}\right] \tag{13}$$

Solving for t_2 results in

$$t_2 = \frac{1}{a} \ln\left(1 + \frac{h_o\cos\theta}{d}\right) \tag{14}$$

If t_m is defined as the lesser of t_1 and t_2, at time t_m no leachate remains lying on the liner. The volume of leachate which leaks through the clay is calculated by integrating from time $t=0$ to $t=t_m$ the time rate of change of the leachate head (dh/dt) times the area over which the leakage occurs. If V_L is the volume which leaks through the liner, the leakage volume integral is written as

$$V_L = \int_0^{t_m} \phi w \cos\theta \left(\frac{-dh}{dt}\right) s \, dt \tag{15}$$

Substituting for dh/dt and s and then integrating results in

$$V_L = V_o\left(\frac{d}{h_o\cos\theta} + 1\right)\left\{\left[1-\frac{1}{k}\right]\left[1-e^{(-at_m)}\right] + e^{(-at_m)}\frac{t_m}{t_1}\right\} \tag{16}$$

where V_o is the original volume of leachate above the clay liner, expressed as

$$V_o = \phi w S_o h_o \cos\theta \tag{17}$$

and k is defined as

$$k = \frac{S_o k_c}{d k_s \tan\theta} \tag{18}$$

By a similar procedure the volume of water which moves through the sand blanket to the drain, V_D, can be shown to be

$$V_D = V_o\left\{\left(\frac{d}{h_o\cos\theta} + 1\right)\left(\frac{1}{k}\right)\left(1-e^{(-at_m)}\right) - \frac{d}{h_o\cos\theta}\,\frac{t_m}{t_1}\right\} \tag{19}$$

Adding equation 16 (V_L) to 19 (V_D) does result in V_o, the original volume liner, for both $t=t_1$ and $t=t_2$, thus providing a continuity check.

The efficiency of the liner under transient conditions (E_t) is defined as the volume of leachate which moves to the drain divided by the original volume of leachate (V_D/V_o). From equation 19 this is shown to be

$$E_t = \left(\frac{d}{h_o\cos\theta} + 1\right)\left(\frac{1}{k}\right)\left(1-e^{(-at_m)}\right) - \frac{d}{h_o\cos\theta}\,\frac{t_m}{t_1} \tag{20}$$

The efficiency of a liner is not a good way to measure its performance as is detailed in later sections. A more useful measure is provided by the average leakage rate (L_t), which is defined as the rate which would produce the observed volume of leakage through the clay liner if the leakage were steady. Mathematically, the total leakage volume can be expressed

$$V_L = L_t t_m S_o w \cos\theta \tag{21}$$

Solving for L_t (using equation 16 for V_L) results in

$$L_t = \frac{\phi h_o}{t_m} \left(\frac{d}{h_o \cos\theta} + 1\right)\left\{\left(1-\frac{1}{k}\right)\left(1-e^{(-at_m)}\right) + e^{(-at_m)}\frac{t_m}{t_1}\right\} \tag{22}$$

For a landfill lined with an efficient liner, the value of t_1 will be much less than t_2 and thus t_m will equal t_1. That is, the leachate will tend to move to the drains instead of leaking through the clay. When this is the case the expressions for V_L, V_D, E_t, and L_t simplify to:

$$V_L = V_o\left(\frac{d}{h_o \cos\theta}+1\right)\left\{\left(1-\frac{1}{k}\right)\left(1-e^{-k}\right) + e^{-k}\right\} \tag{23}$$

$$V_D = V_o\left\{\left(\frac{d}{h_o \cos\theta}+1\right)\frac{1}{k}\left(1-e^{-k}\right) - \frac{d}{h_o \cos\theta}\right\} \tag{24}$$

$$E_t = \left(\frac{d}{h_o \cos\theta}+1\right)\frac{1}{k}\left(1-e^{-k}\right) - \frac{d}{h_o \cos\theta} \tag{25}$$

$$L_t = \frac{\phi h_o}{t_1} \left(\frac{d}{h_o \cos\phi}+1\right)\left\{\left(1-\frac{1}{k}\right)\left(1-e^{-k}\right) + e^{-k}\right\} \tag{26}$$

VI. STEADY STATE EQUATIONS

In an uncapped landfill the recharge due to rainfall may be approximated as a constant, steady movement of water downward through the refuse. This approximation results in a constant head on the liner and a constant leakage rate.

Let h_s be the steady head which is on the liner and R the steady recharge rate (units of length per time). Using Darcy's law (equation 1) the leachate flow rate to the drain, Q_D, and through the liner, Q_L, are expressed as

$$Q_D = k_s \sin\theta h_s \cos\theta w \tag{27}$$

$$Q_L = k_c (1 + \frac{h_s \cos\theta}{d}) S_o \cos\theta w \tag{28}$$

The flow rate of recharge water which moves down through the landfill, Q_R, is expressed as

$$Q_R = R S_o \cos\theta w \tag{29}$$

Continuity requires that Q_R be equal to Q_D plus Q_L. Setting these equal and solving for h_s results in

$$h_s = \frac{S_o (R - k_c)}{k_s \sin\theta (1 + k)} \tag{30}$$

This equation only holds for those values of R greater than k_c. If R is less than k_c then all of the recharge will pass through the liner and h_s will equal 0.

Substituting the equation for h_s into the equations for Q_D and Q_L results in

$$Q_D = wS_o\cos\theta\,\frac{R-k}{1+k}c \tag{31}$$

$$Q_L = (R-\frac{R-k}{1+k}c)S_o\cos\theta w \tag{32}$$

The expression for Q_L can be manipulated to yield a steady-state leachate flow rate, L_s. This rate is

$$L_s = R - \frac{R-k}{1+k}c \tag{33}$$

The steady-state efficiency (E_s) is Q_D divided by Q_R:

$$E_s = \frac{1-k}{1+k}c/R \tag{34}$$

VII. QUASI-STEADY STATE EQUATIONS

Infiltration through a landfill probably does not occur at a steady rate (as was assumed in Section VI) but more likely in discrete events. Thus recharge to the leachate head will be followed by an interval of no recharge, during which the head will dissipate.

Equations describing uniform periodic recharge can be easily established. Let a recharge event of magnitude R^* (units of length) occur every t_R days (e.g., 1 inch every 10 days). t_R is the return period between rainfall events. If either t_1 or t_2 is less than t_R all of the leachate will either drain off or leak through the liner before the next recharge event. If this is the case then h_q, the head immediately following a rainfall event, is

$$h_q = R^*/\phi \tag{35}$$

For the remainder of this section, it is assumed that t_R is less than either t_1 or t_2.

The calculation of h_q is straightforward. The volume of leachate on the liner immediately after a recharge event must be equal to the volume just before the event plus the recharge volume. Or:

$$\phi h_q S_o \cos\theta w = \phi h \, s_R \cos\theta w + R^* S_o \cos\theta w \tag{36}$$

where h_R and s_R are the saturated head and length respectively at time t_R. From equation 11 and 7 it is known that h_R and s_R are expressed as

$$h_R = h_q \left\{e^{(-at_R)}\left(\frac{d}{h_q \cos\theta}+1\right) - \frac{d}{h_q \cos\theta}\right\} \tag{37}$$

$$s_R = S_o \left(1-\frac{t_R}{t_1}\right) \tag{38}$$

Solving for h_q in equation 36 using equations 37 and 38 results in

$$h_q = \frac{(R^*/\phi)-(d/\cos\theta)(1-e^{(-at_R)})(1-t_R/t_1)}{1 - e^{(-at_R)}(1-t_R/t_1)} \tag{39}$$

The volume of water which leaks through the liner (V_L) is the integral over time of the saturated length s multiplied by the time rate of change of the head dh/dt:

$$V_L = \int_0^{t_R} \phi w \cos\theta \left(\frac{-dh}{dt}\right) s\,dt \tag{40}$$

Substituting for h and s and integrating yields

$$V_L = V_o\left(\frac{d}{h_q\cos\theta}+1\right)\{1 - e^{(-at_R)} + \frac{1}{k}[e^{(-at_R)}(at_R+1)-1]\} \tag{41}$$

The average leakage rate between recharge events, L_q, is derived by dividing V_L by the time t_R and by the cross sectional area infiltration occurs through, swcosθ. This results in

$$L_q = \frac{\phi h_q}{t_R}\left(\frac{d}{h_q\cos\theta}+1\right)\{1 - e^{(-at_R)} + \frac{1}{k}[e^{(-at_R)}(at_R+1)-1]\} \tag{42}$$

The volume of water which moves to the drain between recharge events is defined as V_D and is mathematically defined as

$$V_D = V_o\{\left(\frac{d}{h_q\cos\theta}+1\right)\frac{1}{k}(1-e^{(-at_R)}) - \frac{d}{h_q\cos\theta}\frac{t_R}{t_1}\} \tag{43}$$

The efficiency of the liner (E_q) is calculated by dividing the volume of water which moves to the drain (V_D) by the volume of recharge (V_o):

$$E_q = \frac{\Phi h}{R^*} q \{ (\frac{d}{h_q \cos\theta} + 1) \frac{1}{k} (1 - e^{(-at_R)}) - \frac{d}{h_q \cos\theta} \frac{t_R}{t_1} \} \tag{44}$$

A check on continuity, made by setting the volume of leachate on the liner just after the recharge event equal to V_D plus V_L plus the volume on the liner just before the recharge, is satisfied.

VIII. RELATIONSHIP BETWEEN QUASI-STEADY STATE AND STEADY STATE

If the total rainfall per year is kept constant while the recharge return period is shortened, then the quasi-steady state case approaches the steady state case. Table 1 shows a particular situation in which 52 inches (1.32 meters) per year is applied to a liner. (The yearly average rainfall for New Jersey is approxmiately 42 inches. However during wet years 52 inches can be measures at a station. Also, since waste often either contains water or generates it as it decomposes, a higher infiltration rate may be justified. The cases presented here are clearly worst case scenarios.) If all 52 inches is applied at one time then the average leakage rate through the liner, L_q, is 21,138 gallons per year per acre (gal/yr/acre), the initial head buildup, h_q, is 14.4 feet, and the liner efficiency, E_q, is 98.5%. If the frequency is increased to two recharge events per year (with each contributing 26 inches of recharge) then L_q becomes 24,775 gal/yr/acre, h_q becomes 7.2 feet, and E_q becomes 98.2%. Increasing the frequency to 3650 times per year (e.g., it rains 10 times per day with each rainfall event creating 0.014 inches of recharge) L_q becomes 67,303 gal/yr/acre, h_q is 3.0 feet, and E_q is 95.2%. The steady state recharge case of 52 inches per year (or 1.37×10^{-7} feet per second) results in a leakage rate L_s of 67,326 gal/yr/acre, a steady state leachate head h_s of 3.0 feet, and efficiency, E_s, of 95.2%.

The greater efficiency of, and lesser leakage through, the example liner under a single recharge event per year as compared to more frequent events is initially puzzling. One would expect that a higher head on the liner should produce more leakage through the clay. This discrepancy is resolved by calculating t_1, the time needed for all leachate to slide down the liner to the drain. t_1 is 79.4 days for this case. (For this example, and all reasonably designed liner systems t_2 - the length of time necessary for the leachate slug to completely leak through the clay - is much greater than t_1. Thus t_2 need not practically be considered.) After a recharge event the liner will have a leachate head on it for 79.4 days if no additional recharge events occur. For the first case in table 1 (1 event per year) this means there will be 79.4 days of leakage and 285.6 days of during which there will be no head on the liner. The leakage rate will change during the time the leachate is on the liner because the head and saturated length will be decreasing. But during the course of a year the liner will lose 21,138 gallons per acre to the underlying soil. Because this leakage actually occurs only over 79.4 days the instantaneous recharge rate will always be much higher than the yearly average.

For the second case (2 recharge events per year) again there exists a leachate head on the liner for 79.4 days after each event. Thus a head exists on the liner for 158.8 days per year. Since the head is always lower than that of the first case, the instantaneous leakage rate at any time will also be less. But because the leachate is on the liner for a greater amount of time more total leakage per year is observed.

If there are 8 or more recharge events per year (a return period of 46 days or less) there will always be a leachate head on the liner. When this is the case the more frequent the recharge the greater the leakage per year.

Table 1. Quasi-steady state vs. Steady State

(Total yearly recharge volume held constant)

# of rainfall events per year	L_q (gal/yr/acre)	E_q (%)	h_q (feet)
1	21,138	98.5	14.4
2	24,775	98.3	7.2
4	32,047	97.7	3.6
8	48,360	96.6	3.1
16	57,872	95.9	3.0
32	62,689	95.6	3.0
64	64,883	95.4	3.0
365	66,320	95.2	2.9
730	66,523	95.2	2.9
3650	67,303	95.2	3.0
Steady State:	L_s=67,326	E_s=95.2	h_s=3.0

<u>Landfill parameters</u>

k_s = 1 X 10^{-2} cm/sec = 1 X 10^{-4} meters/sec
k_c = 1 X 10^{-7} cm/sec = 1 X 10^{-9} meters/sec
d = 3 feet = 0.9144 meters
S_o = 150 feet = 45.92 meters
θ = 2% = 1.14°
ϕ = 0.3
R = 52 inches/year = 1.32 m/year

IX. EFFICIENCY AND LEAKAGE AS MEASURES OF EFFECTIVENESS

It is very appealing to discuss the effectiveness of a liner in preventing leachate leakage in terms of its efficiency. However, this can be misleading. Equation 34 shows that for steady state the greater the rainfall R the greater the efficiency. This is because as the head on the liner increases the amount of water moving to the drain increases at a greater rate than the volume of water leaking through the clay. Table 2 shows for one steady state case how the efficiency, leakage, and head change as the recharge is varied. As recharge increases from 1 to 100 inches per year the liner's efficiency increases from 0% to 96.4%. However the leakage also increases from 27,152 to 99,114 gallons per year per acre.

The quasi-steady state case is different. Table 3 shows that the efficiency and the leakage amounts increase as the rainfall volume increases but the return period (t_r) remains constant.

To say that the liner is more efficient under greater recharge volumes is correct but is misleading in that the main purpose of a liner is to prevent leachate from entering the ground water. Thus the average yearly leakage rate is a better number by which to compare the effectiveness of two liners, or of one liner under differing recharge conditions.

Table 2. Steady State Case: Efficiency vs. Leakage (Increasing volume of recharge)

R (inches/year)	E_s (%)	L_s (gal/yr/acre)	h_s (feet)
1	0.0	27,152	0.0
5	73.3	36,200	0.2
10	85.4	39,512	0.5
25	92.7	49,445	1.4
50	95.1	66,002	2.9
100	96.4	99,114	5.8

Landfill Parameters

k_s = 1 X 10^{-2} cm/sec = 1 X 10^{-4} m/sec
k_c = 1 X 10^{-7} cm/sec = 1 X 10^{-9} m/sec
d = 3 feet = 0.9144 meters
S_o = 150 feet = 45.72 meters
θ = 2% = 1.14°
ϕ = 0.3

Table 3. Quasi-steady State: Efficiency vs. Leakage (Increasing volume of recharge)

R^* (inches)	E_q (%)	L_q (gal/yr/acre)	h_q (feet)
0.096	74.5	34,646	0.23
0.192	86.1	37,819	0.52
0.479	93.0	47,302	1.41
0.960	93.35	63,198	2.89
1.918	96.51	94,855	5.84

Landfill parameters

t_r = 7 days
k_s = 1 X 10^{-2} cm/sec = 1 X 10^{-4} m/sec
k_c = 1 X 10^{-7} cm/sec = 1 X 10^{-9} m/sec
d = 3 feet = 0.9144 meters
S_o = 150 feet = 45.72 feet
θ = 2% = 1.14°
ϕ = 0.3

X. USE OF PRECIPITATION DATA WITH TRANSIENT STATE EQUATIONS

The use of rainfall data is harder to analytically describe than either the steady or quasi-steady state cases. The irregular amounts and timings of actual rainfall will create irregular recharge patterns which do not fit into one simple, analytic formula. A numerical technique is necessary.

The numerical technique used here is very simple. At an initial time an initial head is assumed to exist on the liner. This head is allowed to dissipate, sending leachate to the drain and through the liner. If the leachate head disappears before the next recharge event the leakage rate falls to zero. If a recharge event occurs before the head dissipates then the recharge volume is added to the remaining leachate lying on the liner and the total amount uniformally redistributed over the liner.

The total leakage is the integral of the leakage rate over the time that a head is actually on the liner. The equations governing the transient dissipation of a head on a liner (equations 7 and 11) are used to calculate the head and saturated length at any time after the last previous recharge event.

An example of the iterative transient state method is illustrated using five years of daily rainfall data from the Trenton, New Jersey airport. Calculations were performed using the FORTRAN program listed in Appendix C. Figure 2 displays monthly summaries of the rainfall data. The transient state numerical method was used to predict leakage through a landfill assumed to have an initial head of zero in January, 1968. The assumed physical parameters of the landfill are included on Table 4.

Leakage between each rainfall event was calculated using equation 16. The results were then summed by month for display purposes. Figure 3 shows the monthly leakage. Table 4 lists the computer output from the program. It is evident from the graphs and the summary that the leakage lags the rainfall. A rainfall peak or trough is followed the next month by a leakage peak or trough. Note that for figure 3 a leakage value of 0.1 inches is equivalent to 43 gallons per acre.

Figure 2. Monthly Rainfall at Trenton Airport

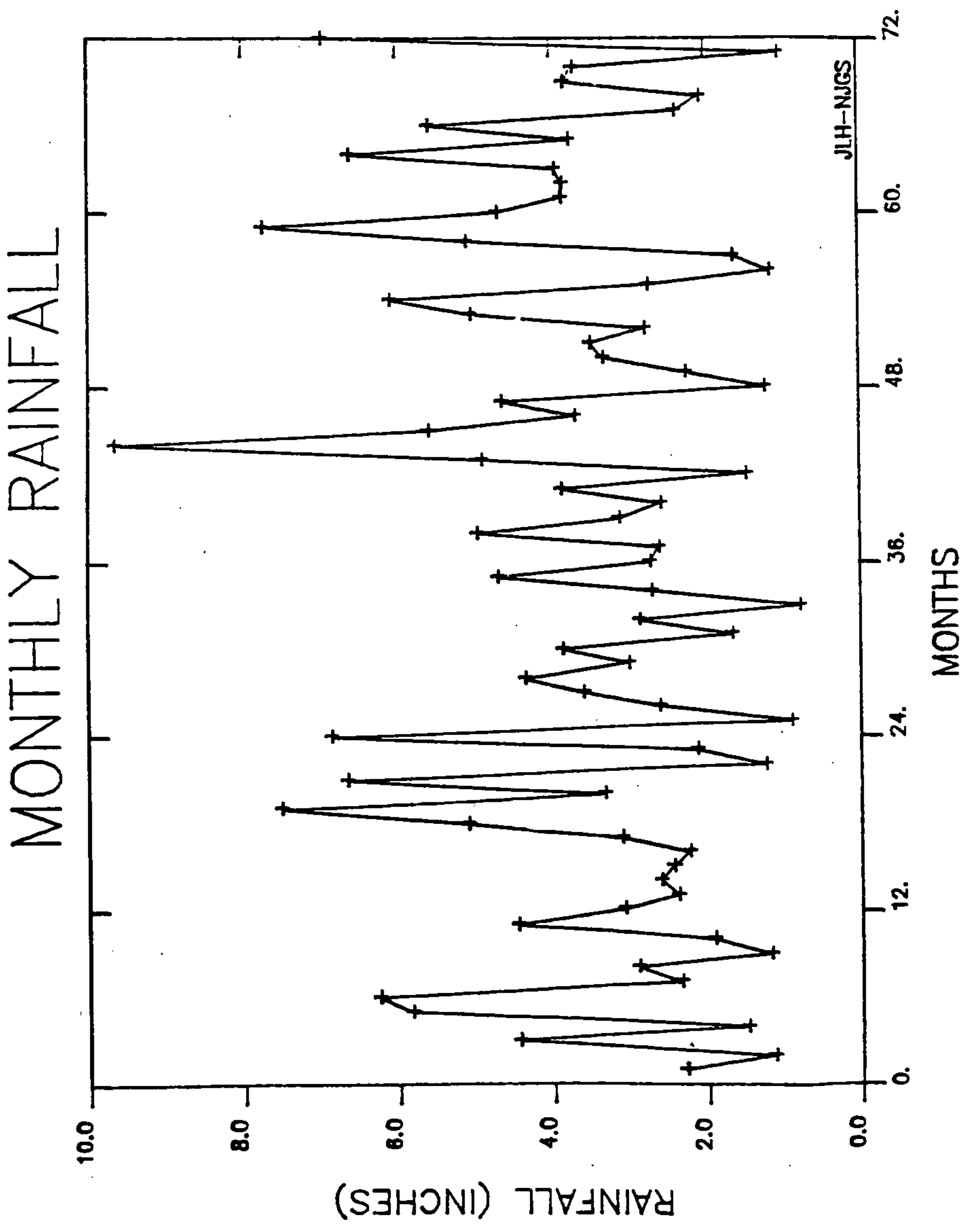


Figure 3. Calculated Monthly Leakage

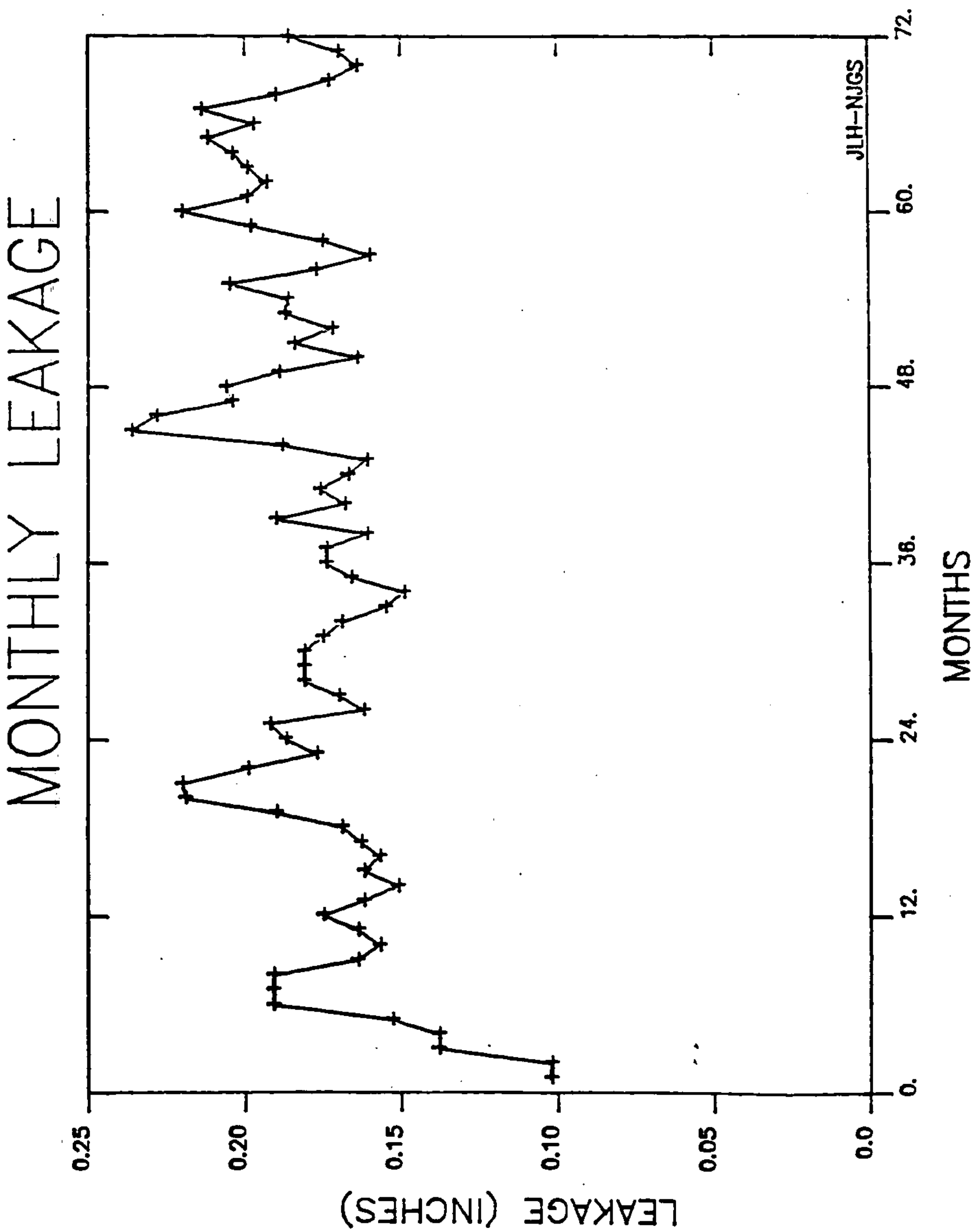


Table 4. Transient State Numerical Example: Program Output

```
FILE: RAINLEAK OUTPUT   A   NEW JERSEY DEPARTMENT OF TRANSPORTATION - CMS


  DATA INPUT:

     KS     =     0.10E-01 CM/SEC
     KC     =     0.10E-06 CM/SEC
     SO     =         150. FEET
     D      =          3.0 FEET
     PORO   =         0.30
     SLOPE  =         2.00 %


  CALCULATED PARAMETERS

     THETA   =   0.200E-01
     K       =   0.250E-01
     T1      =        79.38 DAYS
     A       =     0.32E-03




     NUMBER OF DAYS              =        2189.
     NUMBER OF RAINFALL EVENTS =           724

     LEFT ON LINER:
          HT  =        2.790 FEET
          ST  =      148.110 FEET



     MASS BALANCE:

          TOTAL RAIN                    =     258.774 INCHES
                                        =    3234.030 CU. FT.

          LEACHATE THROUGH LINER        =      12.863 INCHES
                                        =     160.758 CU. FT.

          LEACHATE TO DRAIN             =     235.813 INCHES
                                        =    2947.071 CU. FT.

          LEACHATE LEFT ON LINER        =       9.918 INCHES
                                        =     123.950 CU. FT.

          ERROR                         =       0.001 INCHES
                                        =       2.251 CU. FT.


  MONTH      RAIN        DRAIN     LEAKAGE
    1       2.290       0.419      0.102
    2       1.150       0.715      0.102
    3       4.440       1.490      0.138
    4       1.490       1.724      0.138
    5       5.840       1.980      0.153
    6       6.260       3.742      0.191
    7       2.350       3.866      0.191
```

```
  8      2.900      3.712      0.191
  9      1.190      2.875      0.164
 10      1.920      2.349      0.157
 11      4.460      2.603      0.164
 12      3.070      2.964      0.175
 13      2.380      2.539      0.162
 14      2.600      2.474      0.151
 15      2.440      2.627      0.162
 16      2.230      2.342      0.157
 17      3.110      2.555      0.163
 18      5.090      2.895      0.169
 19      7.510      3.550      0.190
 20      3.330      4.817      0.219
 21      6.660      5.154      0.220
 22      1.250      4.298      0.199
 23      2.130      3.244      0.177
 24      6.860      3.393      0.187
 25      0.910      3.633      0.192
 26      2.610      2.882      0.162
 27      3.600      2.780      0.170
 28      4.350      3.399      0.181
 29      3.010      3.218      0.181
 30      3.870      3.343      0.181
 31      1.670      2.984      0.175
 32      2.870      2.855      0.169
 33      0.790      2.361      0.155
 34      2.710      2.063      0.149
 35      4.700      2.762      0.166
 36      2.730      2.874      0.174
 37      2.610      2.980      0.174
 38      4.960      2.726      0.161
 39      3.130      3.597      0.190
 40      2.590      3.375      0.168
 41      3.880      2.983      0.176
 42      1.480      2.814      0.167
 43      4.900      2.432      0.161
 44      9.660      3.607      0.188
 45      5.590      5.837      0.236
 46      3.700      5.245      0.228
 47      4.640      4.331      0.204
 48      1.240      4.270      0.206
 49      2.260      3.503      0.189
 50      3.330      2.901      0.164
 51      3.500      3.318      0.184
 52      2.790      2.946      0.172
 53      5.040      3.513      0.187
 54      6.090      3.433      0.186
 55      2.740      4.266      0.205
 56      1.170      3.316      0.177
 57      1.640      2.523      0.160
 58      5.090      2.991      0.175
 59      7.740      3.971      0.198
 60      4.690      4.712      0.220
 61      3.860      4.225      0.199
 62      3.850      4.352      0.193
```

```
   63       3.950       4.001       0.199
   64       6.610       4.451       0.204
   65       3.760       4.433       0.212
   66       5.580       4.010       0.197
   67       2.380       4.575       0.214
   68       2.070       3.839       0.190
   69       3.830       3.183       0.173
   70       3.710       3.305       0.164
   71       1.050       2.881       0.170
   72       6.970       3.367       0.186
```

XI. CONCLUSIONS

There are several ways to simplify the analysis of the quantity of leachate leaking through a landfill liner. The assumption that the flow is either in a steady, quasi-steady, or transient state allows the development of easily programmed equations. Analysis of these equations shows that as a means of comparative evaluation a landfill's liner efficiency is not as desirable a quantity as is its average leakage rate.

The analytical equations are simplifications of the real world case and a more accurate treatment can be made by using measured rainfall data and a numerical scheme. These numerical results provide an approximation to the volume of leachate which leaks from a landfill to the underlying soil.

The methods presented here allow a landfill designer to compare different liner parameters to determine which is more important, for example, comparing the effect on leakage volume of increasing slope vs. increasing the liner thickness vs. increasing the sand permeability. By doing this the designer can chose the most cost effective way to control leakage volume and meet any performance standards placed on the liner.

XII. BIBLIOGRAPHY

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Kmet, Peter, Quinn, Kenneth, and Slavik, Cynthia (1981). "Analysis of Design Parameters Affecting the Collection Efficiency of Clay Lined Landfills." In:Fourth Annual Madison Conference of Applied Research and Practice on Municipal and Industrial Waste, Department of Engineering and Applied Science, University of Wisconsin-Extension, Madison, WI.

APPENDIX A. VARIABLES

variable	description/definition	units
a	k/t_1	[1/T]
d	clay liner thickness perpendicular to slope	[L]
E_q	quasi-steady state liner efficiency	[%]
E_s	steady state liner efficiency	[%]
E_t	transient state liner efficiency	[%]
h	leachate head	[L]
h_q	maximum quasi-steady state head	[L]
h_s	steady state head	[L]
h_o	initial transient state head	[L]
k	$S_o k_c/(d k_s \tan\theta)$	[-]
k_c	clay liner hydraulic conductivity	[L/T]
k_s	sand blanket hydraulic conductivity	[L/T]
L_q	quasi-steady state leakage	[L/T]
L_s	steady state leakage	[L/T]
L_t	transient state leakage	[L/T]
Q_D	volumetric flux rate to drain	[L^3/T]
Q_L	volumetric flux rate through liner	[L^3/T]
R_*	steady state recharge	[L]
R^*	quasi-steady state recharge	[L]
s	saturated length parallel to slope	[L]
S_o	liner peak to trough distance	[L]
t_m	minimum of t_1 and t_2	[T]
t_1	$S_o\phi/(k_s \sin\theta)$	[T]
t_2	$(1/a)\ln(1+h_o\cos\theta/d)$	[T]
V_D	volume of leachate intercepted by liner and transmitted to drains	[L^3]
V_L	volume of leachate which leaks through liner	[L^3]
V_o	original volume of leachate on liner	[L^3]
w	unit width	[-]
θ	slope of clay liner and sand blanket	[-]
ϕ	porosity of sand blanket	[-]

Appendix B. HP-41C Programs for Transient, Steady and Quasi-steady States

```
01♦LBL "TS"
02 XEQ "INIT"
03 "POROSITY?"
04 PROMPT
05 STO 06
06 "H0? FEET"
07 PROMPT
08 STO 07
09♦LBL "TSC"
10 ADV
11 ADV
12 SF 12
13 " * TS *"
14 PRA
15 ADV
16 CF 12
17 XEQ "CONV"
18 RCL 03
19 RCL 07
20 /
21 RCL 15
22 COS
23 /
24 STO 33
25 RCL 30
26 CHS
27 E↑X
28 STO 34
29 1
30 ENTER↑
31 RCL 30
32 1/X
33 -
34 1
35 ENTER↑
36 RCL 34
37 -
38 *
39 RCL 34
40 +
41 RCL 33
42 1
43 +
44 *
45 RCL 06
46 *
47 RCL 07
48 *
49 STO 21
50 328828.6
51 *
52 STO 22
53 RCL 33
54 1
55 +
56 RCL 30
57 /
58 RCL 34
59 CHS
60 1
61 +
62 *
63 RCL 33
64 -
65 STO 23
66♦LBL 20
67 XEQ "ANS"
68 15
69 ACCHR
70 " ="
71 ACA
72 RCL 06
73 ACX
74 PRBUF
75 "H0 ="
76 ACA
77 RCL 07
78 ACX
79 " FEET"
80 ACA
81 PRBUF
82 ADV
83 "E"
84 ACA
85 116
86 ACCHR
87 " ="
88 ACA
89 RCL 23
90 130
91 *
92 ACX
93 37
94 ACCHR
95 PRBUF
96 "L"
97 ACA
98 116
99 ACCHR
100 " ="
101 ACA
102 RCL 22
103 ACX
104 " GAL/ACRE"
105 ACA
106 PRBUF
107 "T1 ="
108 ACA
109 RCL 21
110 365
111 /
112 ACX
113 " YEARS"
114 ACA
115 PRBUF
116 END
```

```
01+LBL "SS"
02 XEQ "INIT"
03 "R? IN/YEAR"
04 PROMPT
05 STO 10
06 1
07 STO 06
08+LBL "SSC"
09 RCL 10
10 4380
11 /
12 STO 18
13 ADV
14 ADV
15 SF 12
16 " * SS *"
17 PRA
18 CF 12
19 ADV
20 XEQ "CONV"
21 RCL 15
22 RCL 18
23 X>Y?
24 GTO 15
25 0
26 STO 23
27 STO 20
28 RCL 10
29 STO 24
30 271524
31 *
32 STO 25
33 GTO 20
34+LBL 15
35 RCL 18
36 RCL 15
37 -
38 RCL 04
39 *
40 RCL 30
41 1
42 +
43 RCL 16
44 SIN
45 *
46 RCL 14
47 *
48 /
49 STO 20
50 RCL 18
51 RCL 15
52 -
53 RCL 30
54 1
55 +
56 /
57 CHS
58 RCL 10
59 +
60 STO 24
61 119927512
62 *
63 STO 25
64 RCL 15
65 RCL 18
66 /
67 CHS
68 1
69 +
70 RCL 30
71 1
72 +
73 /
74 STO 23
75+LBL 20
76 XEQ "ANS"
77 "R ="
78 ACA
79 RCL 10
80 ACX
81 " IN/YEAR"
82 ACA
83 PRBUF
84 "R ="
85 ACA
86 RCL 10
87 27152.4
88 *
89 ACX
90 PRBUF
91 "      "
92 ACA
93 "GAL/YR/ACRE"
94 ACA
95 PRBUF
96 ADV
97 "E"
98 ACA
99 115
100 ACCHR
101 " ="
102 ACA
103 RCL 23
104 100
105 *
106 ACX
107 37
108 ACCHR
109 PRBUF
110 "L"
111 ACA
112 115
113 ACCHR
114 " ="
115 ACA
116 RCL 25
117 ACX
118 PRBUF
119 "      "
120 ACA
121 "GAL/YR/ACRE"
122 ACA
123 PRBUF
124 "H"
125 ACA
126 115
127 ACCHR
128 " ="
129 ACA
130 RCL 20
131 ACX
132 " FEET"
133 ACA
134 PRBUF
135 ADV
136 END
```

```
01*LBL "OSS"
02 XEQ "INIT"
03 "POROSITY?"
04 PROMPT
05 STO 06
06 "R? INCHES"
07 PROMPT
08 STO 03
09 "T-R? DAYS"
10 PROMPT
11 STO 09
12*LBL "OSSC"
13 SF 12
14 ADV
15 ADV
16 " * OSS *"
17 PRA
18 CF 12
19 ADV
20*LBL 99
21 XEQ "CONV"
22 RCL 17
23 RCL 09
24 /
25 STO 18
26 RCL 31
27 RCL 09
28 X<=Y?
29 GTO 10
30 RCL 17
31 RCL 06
32 /
33 STO 20
34 1/X
35 RCL 03
36 *
37 RCL 16
38 COS
39 /
40 STO 33
41 RCL 30
42 CHS
43 E↑X
44 STO 34
45 1
46 ENTER↑
47 RCL 30
48 1/X
49 -
50 1
51 RCL 34
52 -
53 *
54 RCL 34
55 +
56 RCL 33
57 1
58 +
59 *
60 RCL 06
61 *
62 RCL 20
63 *
64 RCL 09
65 /
66 STO 24
67 118927512
68 *
69 STO 25
70 RCL 33
71 1
72 +
73 RCL 32
74 /
75 1
76 RCL 34
77 -
78 *
79 RCL 33
80 -
81 STO 23
82 GTO 05
83*LBL 10
84 1
85 ENTER↑
86 RCL 09
87 RCL 31
88 /
89 -
90 STO 37
91 RCL 32
92 RCL 09
93 *
94 CHS
95 E↑X
96 STO 34
97 1
98 ENTER↑
99 RCL 34
100 -
101 RCL 37
102 *
103 RCL 03
104 *
105 RCL 16
106 COS
107 /
108 CHS
109 RCL 17
110 RCL 06
111 /
112 +
113 1
114 ENTER↑
115 RCL 34
116 RCL 37
117 *
118 -
119 /
120 STO 20
121 1/X
122 RCL 03
123 *
124 RCL 16
125 COS
126 /
127 STO 33
128 RCL 32
129 RCL 09
130 *
131 1
132 +
133 RCL 34
134 *
135 1
136 -
137 RCL 30
138 /
139 RCL 34
140 -
141 1
142 +
143 RCL 33
144 1
145 +
146 *
147 RCL 06
148 *
149 RCL 20
150 *
151 RCL 09
152 /
153 STO 24
154 118927512
155 *
156 STO 25
157 RCL 33
158 1
159 +
160 RCL 30
161 /
162 1
163 ENTER↑
164 RCL 34
165 -
166 *
167 RCL 33
168 RCL 09
169 *
170 RCL 31
171 /
172 -
173 RCL 06
174 *
175 RCL 20
176 *
177 RCL 17
178 /
179 STO 23
180*LBL 05
181 XEQ "ANS"
182 15
183 ACCHR
184 " ="
185 ACA
186 RCL 06
187 ACX
188 PRBUF
189 "R* ="
190 ACA
191 RCL 08
192 ACX
193 " INCHES
194 ACA
195 PRBUF
196 "T"
197 ACA
198 114
199 ACCHR
200 " ="
201 ACA
202 RCL 09
203 ACX
204 " DAYS"
205 ACA
206 PRBUF
207 "R ="
208 ACA
209 RCL 18
210 118927512
211 *
212 ACX
213 PRBUF
214 "       "
215 ACA
216 "GAL/YR/ACRE"
217 ACA
218 PRBUF
219 ADV
220 "E"
221 ACA
222 113
223 ACCHR
224 " ="
225 ACA
226 RCL 23
227 100
228 *
229 ACX
230 " "
231 ACA
232 37
233 ACCHR
234 PRBUF
235 "L"
236 ACA
237 113
238 ACCHR
239 " ="
240 ACA
241 RCL 25
242 ACX
243 PRBUF
244 "        "
245 ACA
246 "GAL/YR/ACRE"
247 ACA
248 PRBUF
249 "H"
250 ACA
251 113
252 ACCHR
253 " ="
254 ACA
255 RCL 20
256 ACX
257 " FEET
258 ACA
259 PRBUF
260 ADV
261 ADV
262 RCL 25
263 END
```

```
01♦LBL "AHS"
02♦LBL 20
03 SCI 2
04 "KS ="
05 ACA
06 RCL 01
07 ACX
08 " CM/SEC"
09 ACA
10 PRBUF
11 "KC ="
12 ACA
13 RCL 02
14 ACX
15 " CM/SEC"
16 ACA
17 PRBUF
18 FIX 2
19 "DC ="
20 ACA
21 RCL 03
22 ACX
23 " FEET"
24 ACA
25 PRBUF
26 "S0 ="
27 ACA
28 RCL 04
29 ACX
30 " FEET"
31 ACA
32 PRBUF
33 16
34 ACCHR
35 " = "
36 ACA
37 RCL 05
38 ACX
39 " "
40 ACA
41 37
42 ACCHR
43 PRBUF
44♦LBL 99
45 RTN
46 END
```

```
01♦LBL "INIT"
02 "KS CM/SEC"
03 PROMPT
04 STO 01
05 "KC CM/SEC"
06 PROMPT
07 STO 02
08 "DC? FEET"
09 PROMPT
10 STO 03
11 "S0? FEET"
12 PROMPT
13 STO 04
14 "SLOPE? %"
15 PROMPT
16 STO 05
17 RTN
18 END
```

```
01♦LBL "CONV"
02 RCL 01
03 2834.65
04 *
05 STO 14
06 RCL 02
07 2834.6
08 *
09 STO 15
10 RCL 05
11 100
12 /
13 ATAN
14 STO 16
15 RCL 08
16 12
17 /
18 STO 17
19 RCL 04
20 RCL 03
21 /
22 RCL 02
23 *
24 RCL 01
25 /
26 RCL 16
27 TAN
28 /
29 STO 30
30 RCL 04
31 RCL 14
32 /
33 RCL 06
34 *
35 RCL 16
36 SIN
37 /
38 STO 31
39 1/X
40 RCL 30
41 *
42 STO 32
43 RTN
44 END
```

```
  *  TS  *

KS = 1.00-02 CM/SEC
KC = 1.00-07 CM/SEC
DC = 3.00 FEET
S0 = 150.00 FEET
θ =  2.00 %
φ = 0.30
H0 = 5.00 FEET

Et = 99.02%
Lt = 9,723.90 GAL/ACRE
T1 = 6.22 YEARS




  *  SS  *

KS = 1.00-02 CM/SEC
KC = 1.00-07 CM/SEC
DC = 3.00 FEET
S0 = 150.00 FEET
θ =  2.00 %
R = 52.00 IN/YEAR
R = 1,411,924.90
      GAL/YR/ACRE

Es = 95.23%
Ls = 67,326.74
       GAL/YR/ACRE
Hs = 2.99 FEET




   *  QSS  *

 KS = 1.00-02 CM/SEC
 KC = 1.00-07 CM/SEC
 DC = 3.00 FEET
 S0 = 150.00 FEET
 θ =  2.00 %
 φ = 0.30
 R* = 52.00 INCHES
 Tr = 365.00 DAYS
 R = 1,411,924.90
      GAL/YR/ACRE

 Eq = 93.50 %
 Lq = 21,138.84
        GAL/YR/ACRE
 Hq = 14.44 FEET
```

Appendix C. FORTRAN Program for Iterative Transient State with Sample Data

```
FILE: RAINLEAK FORTRAN  A   NEW JERSEY DEPARTMENT OF TRANSPORTATION - CMS


C
C
C
C
C           RAINLEAK
C
C    THIS PROGRAM COMPUTES THE LEAKAGE THROUGH A LANDFILL LINER
C    BASED ON THE LINER'S PHYSICAL CHARACTERISTICS AND MEASURED
C    RAINFALL DATES AND AMOUNTS.  ALL OF THE RAIN IS ASSUMED TO
C    INFILTRATE INTO THE LANDFILL AND IMPINGE ON THE LINER.
C
C
C   REFERENCE:
C     LANDFILL LEACHAGE FLUX EQUATIONS
C     JEFFREY L. HOFFMAN, SENIOR GEOLOGIST
C     NEW JERSEY GEOLOGICAL SURVEY
C
C
C
C
C
C   VARIABLES     MEANING
C   ---------     -------
C
C     A           = K/T1 (1/DAYS)
C     CO          = COSINE(THETA)
C     D           THICKNESS OF CLAY LAYER PERPENDICULAT TO SLOPE (FEET)
C     DDMMYY      DAY,MONTH,YEAR OF RAINFALL EVENT (FORMAT = 3I2)
C     DNEW        DAY OF RAINFALL OF PREVIOUS LOOP
C     DOLD        DAY OF RAINFALL OF PREVIOUS LOOP
C     DTERM       = 1 + D/(HO*CO)  (DIMENSIONLESS)
C     EX          = EXP(-A*TM)     (DIMENSIONLESS)
C     EXTERM      = 1 - EX         (DIMENSIONLESS)
C     F1          = (MNTH(MOLD) - DOLD)/T  ==>DAYS BETWEEN RAINS IN
C                               OLD MONTH
C     F2          = DNEW/T  ==> DAYS BEWTEEN RAINS IN NEW MONTH
C     HT          SATURATED HEAD PERPENDICULAR TO SLOPE AT END OF DRY
C                               PERIOD (FEET)
C     HO          SATURAGED HEAD PERPENDICULAT TO SLOPE JUST AFTER
C                               RAINFALL EVENT (FEET)
C     I           DO LOOP PARAMETER
C     I4          = 4
C     K           = SO*KC/(D*KS*ATAN(THETA))  (DIMENSIONLESS)
C     KC          CLAY LINER PERMEABILITY (FEET/DAY)
C     KCCMS       CLAY LINER PERMEABILITY (CENTIMETERS/SECOND)
C     KS          SAND BLANKET PERMEABILITY (FEET/DAY)
C     KSCMS       SAND BLANKET PERMEABILITY (CENTIMETERS/SECOND)
C     MKNT        TOTAL NUMBER OF MONTHS SINCE SIMULATION BEGAN
C     MKNTM1      = MKNT -1
C     MNEW        MONTH OF RAINFALL EVENT OF CURRENT LOOP
C     MNTH(12)    NUMBER OF DAYS IN EACH MONTH (IGNORING LEAP YEARS)
C     MOLD        MONTH OF RAINFALL EVENT OF PREVIOUS LOOP
C     NCOUNT      COUNTER FOR NUMBER OF RAINFALL EVENTS
C     PORO        POROSITY OF SAND BLANKET  (DIMENSIONLESS)
C     RAIN        RAIN AMOUNT OF PREVIOUS LOOP (FEET)
```

```
C     RCUM        CUMMULATIVE RAINFALL (INCHES)
C     RINNEW      RAIN AMOUNT OF CURRENT LOOP (INCHES)
C     RINOLD      RAIN AMOUNT OF PREVIOUS LOOP (INCHES)
C     SI          SINE(THETA)
C     SLOPE       SLOPE OF LINER  (%)
C     ST          SATURATED LENGTH PARALLEL TO SLOPE JUST BEFORE NEXT
C                          RAINFALL EVENT (FEET)
C     SO          LINER PEAK TO TROUGH DISTANCE PARALLEL TO SLOPE (FEET)
C     T           TIME BETWEEN PREVIOUS RAINFALL EVENT AND EVENT OF
C                          CURRENT LOOP (DAYS)
C     TCUM        COUNTER FOR TOTAL NUMBER OF DAYS IN SIMULATION
C     THETA
C     TM          MINIMUM OF T1 AND T2 (DAYS)
C     T1          = SO*PORO/(KS*SI)  ==> NUMBER OF DAYS A HEAD CAN STAY
C                          ON LINER BEFORE SLIDING OFF TO DRAIN
C     T2          = (ALOG(1 + HO*CO/D))/A  ==> NUMBER OF DAYS A HEAD CAN
C                             STAY ON LINER BEFORE LEAKING THROUGH
C     VD          LEAKAGE TO DRAIN (FEET)
C     VDINCH      LEAKAGE TO DRAIN (INCHES)
C     VDSUM       CUMMULATIVE LEAKAGE TO DRAIN (FEET)
C     VL          LEAKAGE THROUGH LINER (FEET)
C     VLEFIN      LEACHATE LEFT ON LINER (INCHES)
C     VLEFT       LEACHATE LEFT ON LINER (CUBIC FEET)
C     VLINCH      LEAKAGE THROUGH LINER (INCHES)
C     VLSUM       CUMMULATIVE LEAKAGE THROUGH LINER (FEET)
C     VRES        CUMMULATIVE MASS ERROR (CUBIC FEET)
C     VRESIN      CUMMULATIVE MASS ERROR (INCHES)
C     VRSUM       CUMMULATIVE RAINFALL (CUBIC FEET)
C     VO          VOLUME OF LEACHATE ON LINER JUST BEFORE A RAINFALL
C                              EVENT (CUBIC FEET)
C     YNEW        YEAR OF RAINFALL EVENT OF CURRENT LOOP
C     YOLD        YEAR OF RAINFALL EVENT OF PREVIOUS LOOP
C     ZZ(75,6)    MONTHLY SUMMARIES
C                   COL. 1:  RAIN (INCHES)
C                   COL. 2:  LEACHATE TO DRAIN (INCHES)
C                   COL. 3:  LEAKAGE THROUGH LINER (INCHES)
C
C
C
C
C
C      ---DATA INPUT---
C
C    CARD         VARIABLE
C    ----         --------
C
C      1           KSCMS
C      2           KCCMS
C      3           SO
C      4           D
C      5           PORO
C      6           SLOPE
C      7+          DDMMYY, RAIN
C
C
```

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C                                                                       RAI01110
C  ---UNIT DEFINITIONS---                                               RAI01120
C                                                                       RAI01130
C     UNIT       DEFINITION                                             RAI01140
C     ----       ----------                                             RAI01150
C                                                                       RAI01160
C      5          DATA INPUT                                            RAI01170
C      6          FULL NUMERIC OUTPUT                                   RAI01180
C      7          DATA FOR GRAPHICAL USE                                RAI01190
C                                                                       RAI01200
C                                                                       RAI01210
C                                                                       RAI01220
C                                                                       RAI01230
C                                                                       RAI01240
C-----------------------------------------------------------------------RAI01250
      INTEGER DOLD, MOLD, YOLD, DNEW, MNEW, YNEW                        RAI01260
      REAL KS, KSCMS, KC, KCCMS, K, THETA, T1, T2, A, HO                RAI01270
      INTEGER MNTH(12)/31,28,31,30,31,30,31,31,30,31,30,31/             RAI01280
      REAL ZZ(75,6)/450*0.0/                                            RAI01290
C-----------------------------------------------------------------------RAI01300
C                                                                       RAI01310
C............READ IN INPUT DATA AND OUTPUT IT                           RAI01320
      READ (5,*) KSCMS                                                  RAI01330
      READ (5,*) KCCMS                                                  RAI01340
      READ (5,*) SO                                                     RAI01350
      READ (5,*) D                                                      RAI01360
      READ (5,*) PORO                                                   RAI01370
      READ (5,*) SLOPE                                                  RAI01380
      WRITE (6,602) KSCMS,KCCMS,SO,D,PORO,SLOPE                         RAI01390
C............COMPUTE PARAMETERS                                         RAI01400
      KS=2835.*KSCMS                                                    RAI01410
      KC=2835.*KCCMS                                                    RAI01420
      THETA = ATAN(SLOPE/100.)                                          RAI01430
      CO = COS(THETA)                                                   RAI01440
      SI = SIN(THETA)                                                   RAI01450
      K = SO*KC/(D*KS*ATAN(THETA))                                      RAI01460
      T1 = SO*PORO/(KS*SI)                                              RAI01470
      A = K/T1                                                          RAI01480
      WRITE (6,603) THETA, K, T1, A                                     RAI01490
C............INITIALIZE SUMMATION VARIABLES                             RAI01500
      NCOUNT = 0                                                        RAI01510
      HT = 0.0                                                          RAI01520
      ST = 0.0                                                          RAI01530
      VDSUM = 0.0                                                       RAI01540
      VLSUM = 0.0                                                       RAI01550
      RCUM = 0.0                                                        RAI01560
      TCUM = 0.0                                                        RAI01570
      MKNT = 1                                                          RAI01580
C............READ FIRST RAINFALL DATE AND MAGNITUDE                     RAI01590
      READ (5,501) DOLD, MOLD, YOLD, RINOLD                             RAI01600
      ZZ(1,1) = RINOLD                                                  RAI01610
C                                                                       RAI01620
C--->RAIN LOOP                                                          RAI01630
C                                                                       RAI01640
C............READ NEW DATE AND RAIN                                     RAI01650
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```
   10 READ (5,501,END=60) DNEW, MNEW, YNEW, RINNEW                      RAI01660
      NCOUNT = NCOUNT + 1                                               RAI01670
C...........COMPUTE INTERVAL BETWEEN RAINFALL EVENTS (T)                RAI01680
      IF (MOLD .NE. MNEW) GO TO 20                                      RAI01690
      T = DNEW - DOLD                                                   RAI01700
      GO TO 30                                                          RAI01710
   20 T = MNTH(MOLD) - DOLD + DNEW                                      RAI01720
      IF ((MOLD .EQ. 2) .AND. (MOD(YNEW,4) .EQ. 0)) T = T+1             RAI01730
      MKNT = MKNT + 1                                                   RAI01740
C...........ACCUMULATE VARIABLES JUST READ IN                           RAI01750
   30 ZZ(MKNT,1) = ZZ(MKNT,1) + RINNEW                                  RAI01760
      TCUM = TCUM + T                                                   RAI01770
      RCUM = RCUM + RINNEW                                              RAI01780
C...........COMPUTE TERMS IN SOLUTION                                   RAI01790
      RAIN = RINOLD/12.                                                 RAI01800
      HO = HT*ST/SO + RAIN/PORC                                         RAI01810
      T2 = (ALOG(1 + HO*CO/D))/A                                        RAI01820
      TM = AMIN1(T1, T2, T)                                             RAI01830
      VO = PORO*HT*ST*CO + RAIN*SO*CO                                   RAI01840
      EX = EXP(-A*TM)                                                   RAI01850
      DTERM = 1 + D/(HO*CO)                                             RAI01860
      EXTERM = 1 - EX                                                   RAI01870
C...........COMPUTE LEAKAGE                                             RAI01880
      VD = VO*(DTERM*EXTERM/K - D*TM/(HO*CO*T1))                        RAI01890
      VL = VO*DTERM * (EXTERM*(1-1/K) + EX*TM/T1)                       RAI01900
      VDINCH = 12.*VD/(SO*CO)                                           RAI01910
      VLINCH = 12.*VL/(SO*CO)                                           RAI01920
      VDSUM = VDSUM + VD                                                RAI01930
      VLSUM = VLSUM + VL                                                RAI01940
      IF (MOLD .NE. MNEW) GO TO 40                                      RAI01950
      ZZ(MKNT,2) = ZZ(MKNT,2) + VDINCH                                  RAI01960
      ZZ(MKNT,3) = ZZ(MKNT,3) + VLINCH                                  RAI01970
      GO TO 50                                                          RAI01980
   40 MKNTM1 = MKNT - 1                                                 RAI01990
      F1 = (MNTH(MOLD) - DOLD)/T                                        RAI02000
      F2 = DNEW/T                                                       RAI02010
      ZZ(MKNTM1,2) = ZZ(MKNTM1,2) + F1*VDINCH                           RAI02020
      ZZ(MKNT,2) = ZZ(MKNT,2) + F2*VDINCH                               RAI02030
      ZZ(MKNTM1,3) = ZZ(MKNTM1,3) + F1*VLINCH                           RAI02040
      ZZ(MKNT,3) = ZZ(MKNT,3) + F2*VLINCH                               RAI02050
   50 CONTINUE                                                          RAI02060
      HT = HO * (EX*DTERM - D/(HO*CO))                                  RAI02070
      ST = SO * (1 - TM/T1)                                             RAI02080
C...........REASSIGN HOLDING VALUES                                     RAI02090
      DOLD = DNEW                                                       RAI02100
      MOLD = MNEW                                                       RAI02110
      YOLD = YNEW                                                       RAI02120
      RINOLD = RINNEW                                                   RAI02130
C...........READ IN NEXT EVENT                                          RAI02140
      GO TO 10                                                          RAI02150
   60 CONTINUE                                                          RAI02160
C                                                                       RAI02170
C--->END OF RAIN LOOP                                                   RAI02180
C                                                                       RAI02190
C...........MASS BALANCE                                                RAI02200
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```
      VRSUM = (RCUM/12.)*SO*CO
      VLINCH = 12.*VLSUM/(SO*CO)
      VDINCH = 12.*VDSUM/(SO*CO)
      VLEFT = PORO*HT*ST*CO
      VLEFIN = 12.*VLEFT/(SO*CO)
      VRES = VRSUM - (VDSUM + VLSUM + VLEFT)
      VRESIN = VRES /(12.*SO*CO)
C...........OUTPUT RESULTS
      WRITE (6,609) TCUM, NCOUNT
      WRITE (6,610) HT, ST
      WRITE (6,611) RCUM, VRSUM, VLINCH, VLSUM, VDINCH, VDSUM,
     &              VLEFIN, VLEFT, VRESIN, VRES
      I4 = 4
      WRITE (6,612)
      WRITE (7,*) MKNT, I4
      DO 70 I = 1,MKNT
        WRITE (7,702) I, ZZ(I,1), ZZ(I,2), ZZ(I,3)
        WRITE (6,702) I, ZZ(I,1), ZZ(I,2), ZZ(I,3)
   70 CONTINUE
C...........FORMATS
  501 FORMAT (3I2,2X,F10.3)
  602 FORMAT (2X,'DATA INPUT:',/
     &  /5X,'KS    =    ',E9.2,' CM/SEC',
     &  /5X,'KC    =    ',E9.2,' CM/SEC',
     &  /5X,'SO    =    ',F9.0,' FEET',
     &  /5X,'D     =    ',F9.1,' FEET',
     &  /5X,'PORO  =    ',F9.2,
     &  /5X,'SLOPE =    ',F9.2,' %')
  603 FORMAT (//2X,'CALCULATED PARAMETERS',/
     &/5X,'THETA  =  ',E9.3,
     &/5X,'K      =  ',E9.3,
     &/5X,'T1     =  ',F9.2,' DAYS',
     &/5X,'A      =  ',E9.2///)
  604 FORMAT (7(2X,F8.3))
  609 FORMAT (/5X 'NUMBER OF DAYS                = ',F10.0,
     &         /5X 'NUMBER OF RAINFALL EVENTS = ',I10)
  610 FORMAT (/5X,'LEFT ON LINER:',
     &/10X,'HT  = ',F10.3,' FEET',
     &/10X,'ST  = ',F10.3,' FEET')
  611 FORMAT (///5X,'MASS BALANCE:',
     &//10X,'TOTAL RAIN                    = ',F10.3,' INCHES',
     & /10X,'                              = ',F10.3,' CU. FT.',
     &//10X,'LEACHATE THROUGH LINER        = ',F10.3,' INCHES',
     & /10X,'                              = ',F10.3,' CU. FT.',
     &//10X,'LEACHATE TO DRAIN             = ',F10.3,' INCHES',
     & /10X,'                              = ',F10.3,' CU. FT.',
     &//10X,'LEACHATE LEFT ON LINER        = ',F10.3,' INCHES',
     & /10X,'                              = ',F10.3,' CU. FT.',
     &//10X,'ERROR                         = ',F10.3,' INCHES',
     & /10X,'                              = ',F10.3,' CU. FT.')
  612 FORMAT (//2X,'MONTH', 4X, 'RAIN', 5X, 'DRAIN', 3X, 'LEAKAGE')
  701 FORMAT (3I2,7(2X,F8.3))
  702 FORMAT (I5, 7(2X,F8.3))
C
  999 STOP
```

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      END                                                               RAI02760
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FILE: RAINLEAK DATA      A    NEW JERSEY DEPARTMENT OF TRANSPORTATION - CMS


0.01
0.0000001
150.
3.
0.3
2.0
 3 168          0.070
 4 168          0.010
 6 168          0.150
14 168          1.740
23 168          0.180
28 168          0.010
29 168          0.040
30 168          0.090
 2 268          0.520
10 268          0.030
29 268          0.600
 1 368          0.100
 9 368          0.010
10 368          0.100
12 368          1.800
13 368          0.450
17 368          0.640
18 368          0.760
23 368          0.560
29 368          0.020
 1 468          0.080
 4 468          0.010
 5 468          0.020
 8 468          0.060
22 468          0.010
24 468          1.210
27 468          0.010
30 468          0.090
 1 568          0.010
 2 568          0.010
 3 568          0.030
 4 568          0.010
 5 568          0.020
 6 568          0.030
11 568          0.780
12 568          0.090
16 568          0.620
18 568          0.040
19 568          0.260
23 568          0.380
24 568          0.190
27 568          0.070
28 568          0.850
29 568          2.320
30 568          0.130
 2 668          0.020
11 668          0.010
12 668          4.420
13 668          0.010
```