

## Draft Proposed New Jersey Algebra I Core Content

**Mission:** *Students use mathematics to make sense of the world around them. They use mathematical reasoning to pose and solve problems, communicating their solutions and solution strategies through a variety of representations.*

Algebra is the study of patterns and functions. In Algebra I, students: focus on understanding the big ideas of equivalence and linearity; learn to use a variety of representations, including modeling with variables; begin to build connections between geometric objects and algebraic expressions; and use what they have learned previously about geometry, measurement, data analysis, probability, and discrete mathematics as applications of algebra.

Algebra I builds a strong conceptual foundation for students as they continue the study of mathematics. The core Algebra I content described in this draft follows the outline of the test specifications developed by the Achieve consortium in the production of the Algebra I End of Course Assessment and includes content needed to promote deep understanding of algebraic concepts that builds as students progress through higher levels of mathematics.

Students studying Algebra I should use appropriate tools (e.g., algebra tiles to explore operations with polynomials, including factoring) and technology, such as regular opportunities to use graphing calculators and spreadsheets. Technological tools assist in illustrating the connections between algebra and other areas of mathematics, and demonstrate the power of algebra.

### Big Ideas:

Core content for Algebra I includes a number of discrete skills and concepts, each related to broader mathematical principles. In teaching and learning Algebra I, it is important for teachers and students to comprehend the following big ideas and to connect the individual skills and concepts of Algebra I to these broad principles.

#### **PATTERNS AND FUNCTIONS**

Algebra provides language through which we describe and communicate mathematical patterns that arise in both mathematical and non-mathematical situations, and in particular, when one quantity is a function of a second quantity or where the quantities change in predictable ways. Ways of representing patterns and functions include tables, graphs, symbolic and verbal expressions, sequences, and formulas.

#### **EQUIVALENCE:**

There are many different – but equivalent – forms of a number, expression, function, or equation, and these forms differ in their efficacy and efficiency in interpreting or solving a problem, depending on the context. Algebra extends the properties of numbers to rules involving symbols; when applied properly, these rules allow us to transform an expression, function, or equation into an equivalent form and substitute equivalent forms for each other. Solving problems algebraically typically involves transforming one equation to another equivalent equation until the solution becomes clear.

#### **REPRESENTATION & MODELING WITH VARIABLES**

Quantities can be represented by variables, whether the quantities are unknown (as in  $5x + 3 = 13$ ), changing over time (as in  $h = -16t^2$ ), parameters (the  $m$  and  $b$  in  $y = mx + b$ ), or probabilities (where  $p^2$  represents the probability that an event with probability  $p$  occurs twice). Relationships between quantities can be represented in compact form using expressions, equations, and inequalities. Representing quantities by variables gives us the power to recognize and describe patterns, make generalizations, prove or explain conclusions, and solve problems by converting verbal conditions and constraints into equations that can be solved. Representing quantities with variables also enables us to model situations in all areas of human endeavor and to represent them abstractly.

#### **LINEARITY**

In many situations, the relationship between two quantities is linear so the graphical representation of the relationship is a geometric line. Linear functions can be used to show a relationship between two variables that has a constant rate of change and to represent the relationship between two quantities which vary proportionately. Linear functions can also be used to model, describe, analyze, and compare sets of data. While linearity might be considered to be a subset of the bigger idea of patterns and functions, it is listed here separately as it is so prominent in Algebra I content.

#### **CONNECTIONS BETWEEN ALGEBRA & GEOMETRY**

Geometric objects can be represented algebraically (for example, lines can be described using coordinates), and algebraic expressions can be interpreted geometrically (for example, systems of equations and inequalities can be solved graphically).

#### **CONNECTIONS BETWEEN ALGEBRA & SYSTEMATIC COUNTING, PROBABILITY, AND STATISTICS**

Algebra provides a language and techniques for analyzing situations that involve chance and uncertainty, including the systematic listing and counting of all possible outcomes (as well as informal explorations of Pascal's Triangle), the determination of their probabilities, the calculation of probabilities of various events (e.g. that throwing two dice will yield a total of 7), predictions based on experimental probabilities, and correlations between two variables.

# Draft for Review

## Algebra I Core Content Overview

### **O: Operations on Numbers and Expressions**

#### **O1. Number Sense and Operations**

- O1.a Reasoning with real numbers
- O1.b Using ratios, rates, and proportions
- O1.B1 Using variables in different ways
- O1.B2 Using matrices
- O1.c Using numerical exponential expressions\*
- O2.a Using algebraic exponential expressions \*
- O1.d Using numerical radical expressions\*\*
- O2.d Using algebraic radical expressions\*\*

#### **O2. Algebraic Expressions (a and d listed above)**

- O2.b Operating with polynomial expressions
- O2.c Factoring polynomial expressions

### **L: Linear Relationships**

#### **L1. Linear Functions**

- L1.a Representing linear functions in multiple ways
- L1.b Analyzing linear function
- L1.c Graphing linear functions involving absolute value
- L1.d Using linear models

#### **L2. Linear Equations and Inequalities**

- L2.a Solving linear equations and inequalities
- L2.b Solving equations involving absolute value
- L2.c Graphing linear inequalities
- L2.d Solving systems of linear equations
- L2.e Modeling with single variable linear equations, one-or two-variable inequalities or systems of equations

\* Topics have been combined into one indicator

\*\* Topics have been combined into one indicator

### **N: Non-linear Relationships**

#### **N1. Non-linear Functions**

- N1.a Representing quadratic functions in multiple ways
- N1.b Distinguishing between function types
- N1.c Using quadratic models

#### **N2. Non-linear Equations**

- N2.a Solving literal equations
- N2.b Solving quadratic equations
- N2.B1 Solving simple exponential equations

### **D: Data, Statistics and Probability**

#### **D1. Data and Statistical Analysis**

- D1.a Interpreting linear trends in data
- D1.b Comparing data using summary statistics
- D1.c Evaluating data-based reports in the media

#### **D2. Probability**

- D2.a Using counting principles
- D2.b Determining probability

## O: Operations on Numbers and Expressions

*Successful students will be able to perform operations with real numbers and algebraic expressions, including expressions involving exponents, scientific notation, and square roots, using estimation and an appropriate level of precision. Reasoning skills will be emphasized, including justification of results.*

Essential Questions	Enduring Understandings
<ul style="list-style-type: none"> <li>▪ What are some ways to represent, describe, and analyze patterns (that occur in our world)?</li> <li>▪ When is one representation of a function more useful than another?</li> <li>▪ How can we use algebraic representation to analyze patterns?</li> <li>▪ Why are number and algebraic patterns important as rules?</li> <li>▪ How are arithmetic operations related to functions?</li> <li>▪ How can numeric operations be extended to algebraic objects?</li> <li>▪ Why is it useful to represent real-life situations algebraically?</li> <li>▪ What makes an algebraic algorithm both effective and efficient?</li> </ul>	<ul style="list-style-type: none"> <li>▪ Logical patterns exist and are a regular occurrence in mathematics and the world around us.</li> <li>▪ Algebraic representation can be used to generalize patterns and relationships.</li> <li>▪ The same pattern can be found in many different forms.</li> <li>▪ Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.</li> <li>▪ Functions are a special type of relationship or rule that uniquely associates members of one set with members of another set.</li> <li>▪ Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole.</li> <li>▪ Rules of arithmetic and algebra can be used together with (the concept of) equivalence to transform equations and inequalities so solutions can be found to solve problems.</li> <li>▪ Variables are symbols that take the place of numbers or ranges of numbers; they have different meanings depending on how they are being used.</li> <li>▪ Proportionality involves a relationship in which the ratio of two quantities remains constant as the corresponding values of the quantities change.</li> </ul>

### O1. Number Sense and Operations

### O2. Algebraic Expressions

Content Benchmarks	Comments and Examples
<p>O1.a Use properties of number systems within the set of real numbers to verify or refute conjectures or justify reasoning.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Defining, giving examples of, distinguishing between, and using numbers, and their properties, from each of the following number sets:               <ul style="list-style-type: none"> <li>○ Whole numbers,</li> <li>○ Integers,</li> <li>○ Rationals, and</li> <li>○ Reals.</li> </ul> </li> <li>• Determining whether the square roots of whole numbers are rational or irrational.</li> <li>• Comparing and ordering real numbers, including determining between which two consecutive whole numbers the value of a square root lies.</li> <li>• Showing that a given interval on the real number line, no matter how small, contains both rational and irrational numbers.</li> <li>• Establishing simple facts about rational and irrational numbers using logical arguments and examples.</li> <li>• Providing counterexamples to refute a false conjecture.</li> <li>• <b>State Assessment Limitation:</b> Items involving radicals will be limited to square roots. Students will not be expected to produce formal proofs.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>Extended Constructed Response (ECR):</b> Which of the following numbers are rational and which are irrational? Explain.  <math>\sqrt{(10)(40)}</math>, <math>\sqrt{4^3}</math>, <math>2\sqrt{2}</math>, <math>\sqrt{49}</math></li> <li>• <b>SCR (Non-Calculator) :</b> Which of the following numbers comes closest to the value of <math>\pi</math> without exceeding it? Explain your reasoning.  <math>\sqrt{10}</math>, 3.14, and <math>\frac{22}{7}</math>  <i>(Answer: 3.14 -- The other choices are both too big. The student would be expected to explain how that was determined.)</i></li> <li>• <b>ECR:</b> Give an example to illustrate that if <math>r</math> and <math>s</math> are rational, then both <math>r + s</math> and <math>(r)(s)</math> are rational.</li> </ul>

	<p style="text-align: center;"><i>Sample Solution:</i> Both <math>\frac{3}{4}</math> and 2.3 are rational;</p> $\frac{3}{4} + 2.3 = \frac{3}{4} + \frac{23}{10} = \frac{15}{20} + \frac{46}{20} = \frac{61}{20}$ <p style="text-align: center;">which is the ratio of two integers, hence rational. Likewise, <math>\frac{3}{4} \times 2.3 = \frac{3}{4} \times \frac{23}{10} = \frac{69}{400}</math> which is also the ratio of two integers and hence rational.</p> <p><b>Instructional Strategies:</b></p> <ul style="list-style-type: none"> <li>• <b>Core Mathematical Process - Representations:</b> Compare different representations for an irrational number you would expect to encounter in every day life, including a physical representation, common decimal approximations, common fraction approximations, and the value produced by a calculator. Discuss the relative accuracies of the approximations and suggest appropriate circumstances for the use of each. For example, students may identify and compare every day encounters with the ratio of the circumference of a circle to its diameter (<math>\pi</math>).</li> <li>• Watch for <b>Common Misconceptions:</b> Students will frequently confuse <math>2^5</math> (meaning 2·2·2·2·2) with 2·5 (meaning 5 + 5).</li> </ul>
<p>01.b Use rates, ratios and proportions to solve problems, including measurement problems.</p> <ul style="list-style-type: none"> <li>• <b>Scope of Content:</b> For applications, this includes using and interpreting appropriate units of measurement, estimation, and the appropriate level of precision.</li> <li>• <b>Content Clarification:</b> Derived measures are those achieved through calculations with measurements that can be taken directly.</li> </ul>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Using dimensional analysis for unit conversion.</li> <li>• Solving problems using derived measures (e.g. percent change and density).</li> <li>• Solving problems involving scale factor (e.g. similar figures, scale drawings, map scales).</li> <li>• Solving applications related to proportional representation.</li> <li>• Devising and using strategies for making fair decisions.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>ECR (Calculator Permitted):</b> There are 223 students in the freshman class, 168 in the sophomore class, 173 in the junior class, and 138 in the senior class. The student council has 30 members, with these seats allocated based on the number of students in each class. How many student council members should each class have? Show or explain your work. <i>(Possible Answer: With 702 students in all, that's one representative for every 23.4 or 10 (9.5) 7 (7.2) 7 (7.4) 6 (5.9) )</i></li> <li>• <b>Performance Assessment Task:</b> Suppose that a drug company has established that a patient must have 40 mg of a certain prescription drug in the body for the drug to be effective. Moreover, the company's studies indicate that anything in excess of 600 mg is toxic, and its research has shown that the body eliminates 10 percent of the drug every four hours. Imagine you are a doctor prescribing this drug for a patient. How often would you want your patient to take the drug, and in what quantity, to ensure effectiveness while avoiding toxicity? (NCTM <i>Navigating through Mathematical Connections in Grades 9-12</i>)</li> </ul> <p><b>Instructional Strategies:</b></p> <p><b>Core Mathematical Process - Problem Solving:</b> Try the ECR problem above, but with these numbers: "There are 248 students in the freshman class, 199 in the sophomore class, 158 in the junior class, and 97 in the senior class. The student council has 30 members, with these seats allocated based on the number of students in each class. How many student council members should each class have? Show or explain your work." <i>With traditional rounding rules yielding more than 30 representatives, some additional thinking and explaining are necessary. One possible solution might be 11 (10.60) 8 (8.50) 7 (6.75) 4 (4.14), recognizing that if the traditional rounding rule were applied to 8.50, it would result in one too many representatives.</i></p>

	<ul style="list-style-type: none"> <li>• <b>Core Mathematical Process - Connections</b> A new teen recreation center is being proposed. In order to meet one of the requirements by the county council, determine the population density of several surrounding towns and propose a location for the center. Explain your reasoning to the council.</li> <li>• <b>Interdisciplinary Connections:</b> Find a linear relationship between the ratio of the length of a person's thigh bone and his height, and use this to estimate the height of a person whose thigh bone has been found in an archeological dig.</li> <li>• <b>Interdisciplinary Connections:</b> Have students consider the advantages and disadvantages of different voting methods, including weighted voting (ranking 1<sup>st</sup>, 2<sup>nd</sup>, and 3<sup>rd</sup> choices), runoff elections, plurality voting (whoever gets the most votes wins), and majority voting (winner must receive more than half of the votes). Link this discussion to social studies and the study of government.</li> <li>• <b>Core Mathematical Process - Problem Solving:</b> Many students will have encountered the parental solution to splitting the last brownie between two children: one cuts, the other chooses. Have students consider a way to extend this algorithm to three or more children.</li> </ul>
<p>O1.B1 Describe and distinguish among the various uses of variables, including:</p> <ul style="list-style-type: none"> <li>• Symbols for varying quantities (such as <math>3x</math>)</li> <li>• Symbols for fixed unknown values (such as <math>3x - 2 = 7</math>)</li> <li>• Symbols for all numbers in properties (such as <math>x + 0 = x</math>)</li> <li>• Symbols for formulas (such as <math>A = l * w</math>)</li> <li>• Symbols for parameters (such as <math>m</math> and <math>b</math> for slope in <math>y = mx + b</math>)</li> </ul>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Identifying constant and variable terms in algebraic expressions, equations, and inequalities.</li> <li>• Discriminating between <math>x^2</math> and <math>2x</math> (using appropriate applications and algebraic manipulatives).</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>Performance Assessment Task:</b> Every Saturday you play basketball in the local community youth club. At the end of a season after a club tournament, the players in the club meet at a fast-food restaurant for a party. If hamburgers cost 59 cents each, find a way to determine the total cost of hamburgers when various numbers of players in the club each have a hamburger (NCTM <i>Illuminations</i>).</li> </ul> <p><b>Instructional Strategies</b></p> <ul style="list-style-type: none"> <li>• <b>Technology Integration:</b> Using spreadsheet software, examine variables as a set of objects and find the image of a set of objects using a function to gain an output (NCTM <i>Navigating through Algebra in Grades 9-12</i>). For example, the corresponding values of <math>f(n) = 3n</math> are examined by using a single number substituted for <math>n</math> in the function, next by using the set of natural numbers less than or equal to 50 under this function, and finally to considering the variable <math>n</math> as the set of real numbers. Students identify a real world situation where a continuous function using the set of real numbers versus a single number may occur (e.g. the gravitational force on an object of a particular mass as it moves to higher altitudes which might include either mountain climbing or a space shuttle trip; the height of a candle as it burns over time).</li> </ul>
<p>O1.B2 Use matrices to represent and solve problems.</p> <ul style="list-style-type: none"> <li>• Adding and subtracting matrices.</li> <li>• Multiplying a matrix by a scalar.</li> </ul>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Identifying and explaining equality of matrices.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>SCR:</b> Solve for <math>a</math>, <math>b</math> and <math>c</math>: <math>[1 \ 2 \ 3] + 2[a \ b \ c] = [7, 8, 9]</math> (Answer: <math>a = 3, b = 3, c = 3</math>)</li> <li>• <b>Performance Assessment Task:</b> Assume that you are helping your school guidance counselor with the scheduling of courses for the coming year. Use two matrices to represent the enrollment numbers of students, one for males and another for females, in three elective courses (e.g. music, technology, and art) for the past five years in your school. Create a matrix to show the total enrollment in these electives in your school during each of the five years. If you were scheduling for next year, how many students would you project to be in each elective? Explain your reasoning.</li> </ul> <p><b>Instructional Strategies:</b></p>

	<ul style="list-style-type: none"> <li>• <b>Interdisciplinary Connections:</b> Determine the political composition of the United States Congress during the 20<sup>th</sup> Century by decade, first researching the types of party affiliations (e.g. Republican) and the associated numbers (of party representatives) in the House of Representatives and the Senate. Use matrices to represent tabular information for each, with a third matrix that displays the numbers of the U.S. Congress members by party affiliation for each decade. Using an historical timeline, identify significant laws that were enacted during each decade in the activity above.</li> <li>• <b>Technology Integration:</b> Use the internet to research and present the data for the task above, as well as a graphing calculator or a spreadsheet to accomplish the task.</li> </ul>
<p>O1.c &amp; O2.a</p> <p>Apply the laws of exponents to numerical and algebraic expressions with integral exponents to rewrite them in different but equivalent forms or to solve problems.</p> <ul style="list-style-type: none"> <li>• <b>Scope of Content:</b> For applications, this includes using and interpreting appropriate units of measurement, estimation, and the appropriate level of precision.</li> </ul>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Representing, computing, and solving problems using numbers in scientific notation.</li> <li>• Translating to expressions with only positive exponents.</li> </ul> <p><b>Examples:</b> <math>\frac{7 \cdot 3^{-3}}{2^{-4} \cdot 3^5} = \frac{7 \cdot 2^4}{3^3 \cdot 3^5}</math>      <math>\frac{3x^{-2}y^3}{2x^{-5}y^{-3}} = \frac{3}{2}x^3y^6</math></p> <ul style="list-style-type: none"> <li>• Translating to expressions with variables appearing only in the numerator.</li> </ul> <p><b>Example:</b> <math>\frac{3s^3}{2r^5} = \frac{3}{2}s^3r^{-5}</math></p> <ul style="list-style-type: none"> <li>• <b>State Assessment Assumption:</b> All algebraic expressions are defined.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>SCR:</b> Multiply, giving the answer without exponents.</li> </ul> $\frac{2}{5^{-4}} \cdot \frac{3^{-3}}{5^2} \cdot \frac{3^5}{2^5} \cdot \frac{1}{4^0}$ <p style="text-align: center;"><i>Sample Solution:</i></p> $\frac{2}{5^{-4}} \cdot \frac{3^{-3}}{5^2} \cdot \frac{3^5}{2^5} \cdot \frac{1}{4^0} = \frac{2 \cdot 3^5 \cdot 5^4}{3^3 \cdot 5^2 \cdot 2^5} = \frac{3^2 \cdot 5^2}{2^4} = \frac{225}{16}$ <ul style="list-style-type: none"> <li>• <b>SCR:</b> Write the expression in simplest form.</li> </ul> $(2a^2b^3)^5 \quad (\text{Answer: } (2a^2b^3)^5 = 32a^{10}b^{15})$ <ul style="list-style-type: none"> <li>• <b>SCR:</b> Write the expression in simplest form.</li> </ul> $\frac{3a^2 + 6ab}{3a} \quad (\text{Answer: } \frac{3a^2 + 6ab}{3a} = a + 2b)$
<p>O1.d &amp; O2.d</p> <p>Use the properties of radicals to convert numerical or algebraic expressions containing square roots in different but equivalent forms or to solve problems.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Adding, subtracting, multiplying, dividing, and manipulating numerical or algebraic expressions with square roots. Results may be required to be given in exact form.</li> <li>• Using the distance formula, based on the Pythagorean Theorem, to solve problems.</li> <li>• When taking square roots of variable expressions, absolute values must be included when appropriate.</li> </ul> <p><b>Examples:</b> <math>\sqrt{x^3} = x\sqrt{x}</math> because <math>\sqrt{x^3}</math> was assumed to be real.</p> $\sqrt{x^2} =  x , \quad \sqrt{x^4} = x^2, \quad \sqrt{x^6} =  x^3  \text{ or }  x ^3,$

$$\sqrt{x^8} = x^4, \quad \sqrt{x^{10}} = |x^5| \text{ or } |x|^5$$

- For applications, this includes using and interpreting appropriate units of measurement, and the appropriate level of precision.
- **State Assessment Assumption:** All radical expressions represent real numbers.
- **State Assessment Limitation:** Expressions under radicals will be limited to monomials. When rationalization of a denominator is required, the radical in the denominator will contain no variables.

**Sample Assessments:**

- **ECR:** Show or explain how  $(2\sqrt{2})^2$  is equal to 8.

$$\text{Sample Solution: } (2\sqrt{2})^2 = 4\sqrt{4} = 4(2) = 8$$

- **ECR:** Show or explain how  $5\sqrt{2}$  is equal to  $\sqrt{50}$ .

*Sample Solutions: Showing that the number on the left equals that on the right:*

$$5\sqrt{2} = \sqrt{25} \cdot \sqrt{2} = \sqrt{50}$$

OR

*Squaring both numbers to get 50:*

$$\begin{aligned} (5\sqrt{2})^2 &= 25 \cdot 2 = 50 \\ (\sqrt{50})^2 &= 50 \end{aligned}$$

- **ECR:** Rewrite the radicals to determine the sum of  $\sqrt{8} + \sqrt{18}$ .

*Sample solution:*

$$\sqrt{8} + \sqrt{18} = \sqrt{4 \cdot 2} + \sqrt{9 \cdot 2} = 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$$

- **Example:**  $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

- **Example:**  $\frac{\sqrt{6} + \sqrt{9}}{\sqrt{3}} = \sqrt{2} + \sqrt{3}$

- **SCR (Non-Calculator):** Determine the exact perimeter of a quadrilateral with vertices (1, 1), (-1, 2), (2, 4), and (4, 3). Show or explain your work.

$$(\text{Answer: } 2\sqrt{13} + 2\sqrt{5})$$

- **MC (Calculator Permitted):** Determine the perimeter of a quadrilateral with vertices (4, 4), (1, 2), (-1, 3), and (2, 5). Which of the following comes closest to your answer?

\*A. 11.7      B. 12.9      C. 14.1      D. 15.3

- **SCR (Non-Calculator):** If the legs of a right triangle measure  $\sqrt{5}$  and  $\sqrt{7}$ , determine the exact measure of the hypotenuse in simplest form. Show or explain your work.

*Sample Solution:*

$$(\sqrt{5})^2 + (\sqrt{7})^2 = 5 + 7 = 12$$

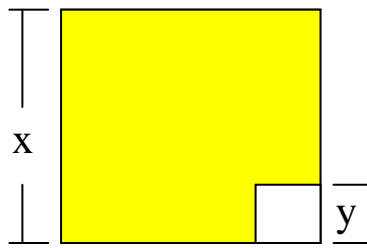
$$\text{The length of the hypotenuse} = \sqrt{12} = 2\sqrt{3}.$$

- **Example:** Explain how  $\sqrt{25x^6}$  is equal to  $5|x^3|$  and to  $5|x|^3$

$$\text{Sample Solution: } \sqrt{25x^6} = \sqrt{5^2 \cdot x^2 \cdot x^2 \cdot x^2}$$

$$= 5|x \cdot x \cdot x| = 5|x^3| = 5|x||x||x| = 5|x|^3$$

	<p><b>Instructional Strategies:</b></p> <ul style="list-style-type: none"> <li>• <b>Core Mathematical Processes - Connections:</b> In civil engineering, surveyors are often asked to determine the approximate distance across a body of water. For example, find the distance across a lake that is bounded by four streets, each adjacent to the northern, western, southern, and eastern shore, respectively, and meeting at perpendicular intersections. Each street is one mile long. Find the approximate distance from the southwestern corner to the northeastern corner of the lake.</li> </ul>
<p>O2.b Add, subtract and multiply polynomial expressions.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Performing operations on polynomials within and without a context.</li> <li>• <b>State Assessment Limitation:</b> Multiplication is limited to a monomial multiplied by a polynomial or a binomial multiplied by a binomial.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>ECR:</b> Subtract: <math>3x^5(x-2) - 2x^4(x^2+2)</math>  <i>Sample Solution:</i>  <math>3x^5(x-2) - 2x^4(x^2+2) = 3x^6 - 6x^5 - 2x^6 - 4x^4 = x^6 - 6x^5 - 4x^4</math></li> <li>• <b>ECR:</b> Multiply: <math>(x+a)(x+b)</math>  <i>Sample Solution:</i>  <math>(x+a)(x+b) = x^2 + ax + bx + ab = x^2 + (a+b)x + ab</math></li> </ul> <p><b>Instructional Strategies:</b></p> <ul style="list-style-type: none"> <li>• <b>Core Mathematical Processes - Connections:</b> Represent the surface area of a rectangular shipping carton (with dimensions l, w, and h) as a polynomial expression. Use this expression to create a table of possible dimensions which would be associated with a particular surface area. For example, with only 10 square feet of cardboard, find the dimensions of some boxes (rectangular prisms) that could be constructed from the cardboard.</li> <li>• <b>Interdisciplinary Connections:</b> Shirley is interested in estimating how much she will be earning in the future if she stays with the same company. Shirley's salary is \$35,000 per year. Let <math>x</math> represent her annual percent increase expressed as a decimal. Express Shirley's next year's salary as a polynomial. <b>Extension:</b> What polynomial expression would represent Shirley's salary in two years?</li> </ul>
<p>O2.c Factor simple polynomial expressions.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• <b>Scope of State Assessment Content:</b> <ul style="list-style-type: none"> <li>○ Factoring out common monomial factors</li> <li>○ Factoring perfect-square trinomials</li> <li>○ Factoring differences of squares</li> <li>○ Factoring out quadratics of the form <math>ax^2 + bx + c</math> that factor over the set of integers</li> <li>○ The factoring process may require more than one step.</li> </ul> </li> <li>• Extending the use of the distributive property to division of a polynomial by a monomial.</li> <li>• Factoring expressions within and without a context.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>ECR:</b> Factor completely: <math>6u^5 - 15u^3</math>  <i>Sample Solution:</i> <math>6u^5 - 15u^3 = 3u^3(2u^2 - 5)</math></li> <li>• <b>ECR:</b> Factor completely: <math>3x^3 + 21x^2 + 30x</math>  <i>Sample Solution:</i>  <math>3x^3 + 21x^2 + 30x \Rightarrow 3x(x^2 + 7x + 10) \Rightarrow 3x(x+2)(x+5)</math></li> <li>• <b>SCR (Non-Calculator):</b> A small square of plastic is to be cut from the plastic cover of a cubical storage container. Express the area of the remaining (shaded) portion of the cover in factored form.</li> </ul>

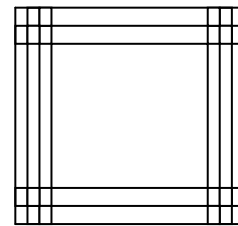


( Answer:  $x^2 - y^2 = (x + y)(x - y)$  )

**Instructional Strategies:**

- **Core Mathematical Process - Representations:** Use algebra tiles to model expressions and develop an understanding of factoring.

- **Interdisciplinary Connections:**  
Use algebra tiles to represent polynomial expressions in multiple ways, comparing them from an artistic viewpoint. For example, while  $(2 + x + 2)(3 + x + 3)$ ,  $(x + 4)(x + 6)$ , and  $x^2 + 10x + 24$  are all algebraically equal, the algebra-tile representations of the first two algebraic expressions would vary greatly in artistic symmetry.



- **Core Mathematical Process - Representations:** Abu Kamil used geometric models many centuries ago to solve algebraic problems. Use a sheet of paper to model  $(a + b)^2$  and remove the  $b^2$  section from the corner of the paper. After cutting, rearrange the pieces and explain how this represents the factorization of  $a^2 + 2ab + b^2$ .

**L: Linear Relationships**

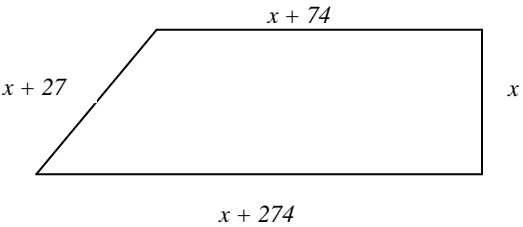
Successful students will be able to solve and graph the solution sets of linear equations, inequalities and systems of linear equations and to use words, tables, graphs, and symbols to represent, analyze, and model with linear functions. In contextual problems students graph and interpret their solutions in terms of the context. They apply such problem solving heuristics as: identifying missing or irrelevant information; testing ideas; considering analogous or special cases; making appropriate estimates; using inductive or deductive reasoning; analyzing situations using symbols, tables, graphs, or diagrams; evaluating progress regularly; checking for reasonableness of results; using technology appropriately; deriving independent methods to verify results; and using the symbols and terms of mathematics correctly and precisely. Function notation should be introduced and used regularly but not exclusively.

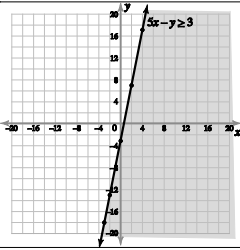
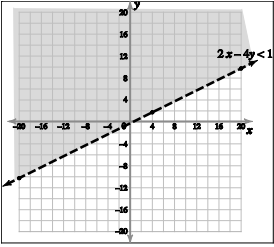
Essential Questions	Enduring Understandings
<ul style="list-style-type: none"> <li>▪ How can change be best represented mathematically?</li> <li>▪ How can we use mathematical language to describe change?</li> <li>▪ How can we use mathematical models to describe change or change over time?</li> <li>▪ How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?</li> <li>▪ How are patterns of change related to the behavior of functions?</li> <li>▪ How are functions and their graphs related?</li> <li>▪ How can technology be used to investigate properties of linear functions and their graphs?</li> <li>▪ How can systems of equations be used to solve real-life situations?</li> </ul>	<ul style="list-style-type: none"> <li>▪ Graphs and equations are alternative (and often equivalent) ways for depicting and analyzing patterns of change.</li> <li>▪ Functional relationships can be expressed in real contexts, graphs, algebraic equations, tables, and words; each representation of a given function is simply a different way of expressing the same idea.</li> <li>▪ The value of a particular representation depends on its purpose.</li> <li>▪ A variety of families of functions can be used to model and solve real world situations.</li> </ul>

**L1. Linear Functions**

Content Benchmarks	Comments and Examples
<p>L1.a Recognize, describe and represent linear relationships using words, tables, numerical patterns, graphs and equations.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Translating among representations of linear relationships.</li> <li>• Using correct terminology and notation for functions (e.g. <math>f(x)</math>, independent and dependent variables, etc.).</li> <li>• Analyzing, interpreting, and describing linear relationships represented recursively, rather than by an explicit formula, including those produced using a spreadsheet.</li> <li>• Generating and describing <i>arithmetic</i> sequences recursively (e.g., NEXT = NOW + A); identifying <i>arithmetic</i> sequences expressed recursively.</li> <li>• <b>Scope of State Assessment Content:</b> Where a student is required to graph the equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g. dollars, seconds, etc.).</li> <li>• <b>State Assessment Limitation:</b> Subscript notation will not be used or required for items involving sequences.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>SCR:</b> Explain how the relationship between length of the side of a square and its perimeter can be represented by a direct proportion. <i>Sample Solution:</i> <math>P</math> and <math>s</math> vary directly: <math>\frac{P}{s} = 4</math></li> <li>• <b>SCR:</b> Given the sequence: 5, 7, 9, 11, ... If 5 is considered the first term, what linear expression could generate this pattern? <i>(Sample Solution: NEXT = NOW + 2)</i></li> <li>• <b>ECR:</b> Sketch the graph of the sequence in which the first term is 8 and the following equation holds: NEXT = NOW – 3</li> <li>• <b>SCR:</b> If cell B8 in a spreadsheet is supposed to be 5 more than whatever number is in cell B7, what equation could you use to calculate that entry? <i>(Sample Solution: B8 =B7 + 5)</i></li> <li>• <b>ECR:</b> Express the following sentence in equation form: two times the quantity of a number increased by eight is equivalent to five less than the same number. <i>(Sample solution: <math>2(x + 8) = x - 5</math>)</i></li> <li>• <b>ECR:</b> As you ride home from a football game, the number of kilometers you are away from home depends (largely) on the number of minutes you have been riding. Suppose that you are 13 km from home when you have been riding for 10 minutes, and 8 km from home when you have been riding for 15 minutes. (Assume that the distance varies linearly with time.) Make a graph with the vertical axis representing distance home and the horizontal axis representing time. Label your graph. Plot the data given as two points on your graph. About how long did it take (on average) to travel 1 km? About how far was the football game from your home? Explain your answer.</li> </ul>
<p>L1.b Describe, analyze and use key characteristics of linear functions and their graphs.</p> <ul style="list-style-type: none"> <li>• <b>Content Clarification:</b> Key characteristics include constant slope and <math>x</math>- and <math>y</math>-intercepts.</li> </ul>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Interpreting slopes of given lines to determine whether lines are parallel, perpendicular, intersecting or coincident.</li> <li>• Identifying and distinguishing among parameters and the independent and dependent variables in a linear relationship.</li> <li>• Describing the effects of varying the parameters <math>m</math> and <math>b</math> in linear functions of the form <math>f(x) = mx + b</math>.</li> <li>• Applying direct proportions, as special linear relationships, and analyzing their graphs in a context.</li> </ul>

	<p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>SCR:</b> Write an equation for a line parallel to the line through (1, -2) and (-3, 5). (Possible solution: <math>y = \frac{-7}{4}x + \frac{-1}{4}</math>)</li> <li>• <b>ECR:</b> Compare and contrast the positions of the graphs for the following three functions and explain how the positions are related to the equations:  <math>f(x) = 5x</math>, <math>g(x) = 5x + 2</math>, and <math>h(x) = 5x - 2</math>.</li> <li>• <b>Performance Assessment Task:</b> The Math Club needs to raise money for its annual neighborhood park beautification project. The club members decide to have a one-day car wash to raise money for this project. After estimating the cost of the activities, determine the total cost of sponges, rags, soap, buckets, and other materials, and investigate the average local charge for washing one car. Write a general rule to determine how much money can be raised for any number of cars. Realistically, can the car wash raise enough money to support this activity?</li> </ul> <p><b>Instructional Strategies:</b></p> <ul style="list-style-type: none"> <li>• <b>Interdisciplinary Connection:</b> Investigate the relationship between stopping distance and speed of travel in a car. Gather data from the driver's education manual or online through the Motor Vehicle Commission (<b>Technology Integration</b>), graph the values found, note that the relationship is linear, and look for an equation that fits the data.</li> </ul>
<p>L1.c Graph the absolute value of a linear function and determine and analyze its key characteristics.</p> <ul style="list-style-type: none"> <li>• <b>Scope of Content:</b> Key characteristics include vertex, slope of each branch, intercepts, domain and range, maximum, minimum, transformations, and opening direction.</li> </ul>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• <b>Scope of Assessment Content:</b> Students are expected to label axes and scales when required to graph an equation or function.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>ECR:</b> Graph each of the following absolute value equations and compare and contrast the graphs with the graph of <math>p(x) =  x </math>:  <math>q(x) = - x </math>, <math>r(x) =  2x </math>, <math>s(x) =  x + 2 </math>, and <math>t(x) =  x  + 2</math></li> </ul>
<p>L1.d Recognize, express and solve problems that can be modeled using linear functions. Interpret solutions in terms of the context of the problem.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Interpreting slope and y-intercept in the context of a problem.</li> <li>• Using and interpreting appropriate units of measurement, estimation and the appropriate level of precision for applications.</li> <li>• <b>Scope of Assessment Content:</b> Students are expected to label axes and scales when required to graph an equation or function.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>SCR:</b> The linear function <math>40t = d</math> can be used to describe the motion of a certain car, where t represents the time in hours and d represents distance traveled, in miles. What does the coefficient, 40, represent in the equation? Include units with the answer.  <i>(Answer: 40 represents the rate of speed)</i></li> </ul> <p><b>Instructional Strategies:</b></p> <ul style="list-style-type: none"> <li>• <b>Interdisciplinary Connections:</b> Use Hooke's Law, <math>F = dk</math>, to determine the constant k when force is being applied to a spring and the distance of the stretch of the spring is known.</li> </ul>
<b>L2. Linear Equations and Inequalities</b>	
<b>Content Benchmarks</b>	<b>Comments and Examples</b>
<p>L2.a Solve single-variable linear equations and inequalities with rational coefficients.</p> <ul style="list-style-type: none"> <li>• <b>Content Clarification:</b> Linear equations may have no solution (empty set), an</li> </ul>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Describing and distinguishing among the types of equations that can be constructed by equating linear expressions: <ul style="list-style-type: none"> <li>○ Identities (such as <math>x + 0 = x</math>)</li> <li>○ Equations for which there is no solution (such as <math>x + 3 = x</math>)</li> <li>○ Formulas</li> </ul> </li> </ul>

<p>infinite number of solutions (identity) or a unique solution.</p>	<ul style="list-style-type: none"> <li>○ Equations where the solution is unique</li> <li>○ Equations relating two variables.</li> <li>• Solving multi-step equations and inequalities.</li> <li>• Representing solution sets for inequalities symbolically as intervals or graphically on a number line.</li> <li>• <b>State Assessment Limitation:</b> Limited to single variable, first degree for both equations and inequalities.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>MC:</b> Which of the following equations has no solution? <ul style="list-style-type: none"> <li>A. <math>x + 0 = x</math></li> <li>B. <math>y + 1 = 2x</math></li> <li>* C. <math>x + 4 = x</math></li> <li>D. <math>3x = 9</math></li> </ul> </li> <li>• <b>MC (Calculator Permitted):</b> <div style="text-align: center;">  </div> <p>Sandra's property has the shape of a trapezoid with the dimensions shown. If the perimeter of the property is 3,279 feet, what is the value of <math>x</math>?</p> <ul style="list-style-type: none"> <li>* A. 726 ft</li> <li>B. 781.25 ft</li> <li>C. 913.5 ft</li> <li>D. 1452 ft</li> </ul> <p style="text-align: right;"><i>Solution:</i> <math>375 + 4x = 3279</math> <math>x = 726</math></p> </li> <li>• <b>SCR:</b> Solve the equation <math>\frac{x}{2} - \frac{x+1}{3} = 2</math>. Show or explain your work. <div style="text-align: center;"> <math display="block">\frac{x}{2} - \frac{x+1}{3} = 2</math> <math display="block">6\left[\frac{x}{2} - \frac{x+1}{3}\right] = 6[2]</math> <p><i>Sample Solution:</i> <math>3x - 2(x+1) = 12</math> <math>3x - 2x - 2 = 12</math> <math>x = 14</math></p> </div> </li> <li>• <b>SCR:</b> Solve <math>3 - x &lt; 5</math> <div style="text-align: center;"> <math display="block">3 - x &lt; 5</math> <p><i>Sample Solution:</i> <math>-x &lt; 2</math> <math>x &gt; -2</math></p> </div> </li> <li>• <b>ECR:</b> Determine and explain the solutions for each of the following three equations:  A) <math>x + 0 = x + 2</math>      B) <math>x + 0 = x</math>      C) <math>x + 0 = 2x</math> </li> </ul>
<p>L2.b Solve equations involving the absolute value of a linear expression.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Determining all possible values in the solution.</li> <li>• <b>State Assessment Limitation:</b> Equations will include only one absolute value expression and will be one of the following forms:</li> </ul>

	<p><math> ax + b  = c, a x + b  = c,  ax  + b = c,  ax + b_1  + b_2 = c.</math></p> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li><b>Example:</b> Solve: <math> x + 3  = 7.</math>  <i>Sample Solutions:</i>  <math> x + 3  = 7</math>  <math>x + 3 = 7</math> or <math>x + 3 = -7</math>  <math>x = 4</math> or <math>-10</math></li> </ul> <p style="text-align: center;"><i>OR</i></p> <p>Since <math> x - b </math> can be interpreted as the distance from <math>x</math> to <math>b</math>, the solutions of the above absolute value equation may be interpreted as the numbers, <math>x</math>, that are 7 units from <math>-3</math>. (i.e., <math>x = -3 + 7 = 4</math> or <math>x = -3 - 7 = -10</math>).</p>
<p>L2.c Graph and analyze the graph of the solution set of a two-variable linear inequality.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>Representing algebraic solutions graphically on the coordinate plane.</li> <li>Using a shaded half-plane with solid or open boundary for graphs of two-variable inequalities.</li> <li>Providing examples of ordered pairs that are included in the solution set of a two-variable linear inequality.</li> <li><b>Scope of State Assessment Content:</b> Students will be expected to provide examples of ordered pairs that are included in the solution set of a two-variable linear inequality.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li><b>SCR:</b> Graph <math>5x - y \geq 3.</math>  <i>Solution:</i>  </li> <li><b>SCR:</b> Graph <math>2x - 4y &lt; 1.</math>  <i>Solution:</i>  </li> <li><b>ECR:</b> Determine a point in the solution set for <math>3x + 2y &lt; 6</math> by graphing.</li> </ul> <p><b>Instructional Strategies:</b></p> <ul style="list-style-type: none"> <li><b>Technology Integration:</b> The Cape May-Lewes Ferry has space for cars and buses. Using the internet, investigate how many of each can be transported on a single trip. Use variables to represent the unknowns (e.g. <math>x</math> for cars and <math>y</math> for buses) and develop the graph of the inequality, using either paper-and-pencil or a graphing calculator. Recognizing that the solutions have to be whole numbers, students should identify the</li> </ul>

	<p>points whose coefficients are non-negative integers and in the first quadrant on or below the line.</p>
<p>L2.d Solve systems of linear equations in two variables using algebraic and graphic procedures.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• <b>Scope of State Assessment Content:</b> Systems of equations may include intersecting, parallel, or coincident lines, some of which may be equations of horizontal or vertical lines.</li> <li>• <b>Scope of State Assessment Content:</b> Students are expected to label axes and scales when required to graph an equation or function. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g. dollars, seconds, etc).</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>ECR:</b> Solve the linear system by the method that you think is best. Show or explain your work. Explain why you chose that method.  <math display="block">7x - 8y = 6</math> <math display="block">4x + y = 9</math> </li> <li>• <b>Non-Example</b> (Not good material for a multiple choice format):  <math display="block">x + y = 4</math> <math display="block">x - y = 2</math> <p>Which is the solution (x, y) to the system of equations shown?  A. (1, 3)    B. (2, 2)    C. (3, 1)    D. (4, 0)</p> <p><i>(This content could be more appropriately tested where students are not given four choices and therefore actually required to “solve” the system rather than just plugging in the response choices.)</i></p></li> </ul>
<p>L2.e Recognize, express and solve problems that can be modeled using single-variable linear equations; one- or two-variable inequalities; or two-variable systems of linear equations.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Interpreting solutions in terms of the context of the problem.</li> <li>• Using and interpreting appropriate units of measurement, estimation and the appropriate level of precision for applications.</li> <li>• <b>Scope of State Assessment Content:</b> Students are expected to label axes and scales when required to graph an equation or function.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>SCR (Calculator Permitted):</b> Jim spent \$200 on gifts for his family. He spent the money on toys, clothes, and a \$15 DVD. He spent 4 times as much on clothes as he did on toys. Write an equation in one variable that can be used to determine how much money Jim spent on toys. Solve the equation to determine how much Jim spent on toys.  <i>(Sample Solution:</i>  <math display="block">\text{Let } t = \text{money spent on toys} \rightarrow 4t + t + 15 = 200</math> <math display="block">5t = 185</math> <math display="block">t = 37 \text{ )}</math> </li> <li>• <b>MC (Non-Calculator):</b> A triangle is formed by the intersections of the x-axis, the y-axis, and the line <math>2x + 3y = 6</math>. What is the area of the triangle?  A. <math>\frac{2}{3}</math>    B. 2    * C. 3    D. 6</li> <li>• <b>SCR (Calculator Permitted):</b> The measure of one angle of an acute triangle is twice the measure of the first angle while the third is <math>30^\circ</math> more than the first angle. Determine the measures of the three angles.  <i>(Sample Solution:    <math>m\angle 1 = a = 37.5^\circ</math>, <math>m\angle 2 = 2a = 75^\circ</math>, <math>m\angle 3 = a + 30 = 67.5^\circ</math>. )</i></li> <li>• <b>ECR (Calculator Permitted):</b> Cell phone plan A charges a fixed cost of \$45.00 per month, which includes 200 minutes. Each additional minute, or part of a minute, for Plan A costs \$0.30. Cell phone Plan B charges a fixed cost of \$65.00 per month, which includes 300 minutes.</li> </ul>

	<p>Each additional minute, or part of a minute, for Plan B costs \$0.15. How many minutes need to be used for the plans to have the same cost? Show or explain your work.</p> <p><b>Instructional Strategies:</b></p> <ul style="list-style-type: none"> <li>• <b>Interdisciplinary Connections:</b> Make a model of the relationship between Celsius and Fahrenheit temperatures. Represent the relationship as an equation, and check the equation against two known data points – 0 degrees C = 32 degrees F and 100 degrees C = 212 degrees F. Use the equation to convert between Celsius and Fahrenheit temperatures.</li> <li>• <b>Core Mathematical Processes - Connections:</b> A landscaping contractor uses a combination of two brands of fertilizers, each containing a different amount of phosphates and nitrates. In a package, brand A has 4 lb. of phosphates and 2 lb. of nitrates. Brand B contains 6 lb. of phosphates and 5 lb. of nitrates. On her current job, the lawn requires at least 24 lb. of phosphates and at least 16 lb. of nitrates. How much of each fertilizer does the contractor need? (<i>Students represent the given conditions as inequalities and use the intersection of their regions as the set of feasible answers.</i>)</li> </ul>
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N: Non-linear Relationships	
<p>Successful students will be able to recognize, represent, analyze, graph, solve equations and apply some non-linear functions, including quadratic and exponential. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and, where appropriate, solve contextual problems. In contextual problems students will be required to graph and interpret their solutions in terms of the context. They should be able to apply such problem solving heuristics as: identifying missing or irrelevant information; testing ideas; considering analogous or special cases; making appropriate estimates; using inductive or deductive reasoning; analyzing situations using symbols, tables, graphs, or diagrams; evaluating progress regularly; checking for reasonableness of results; using technology appropriately; deriving independent methods to verify results; and using the symbols and terms of mathematics correctly and precisely. Function notation should be introduced and used regularly but not exclusively.</p>	
Essential Questions	Enduring Understandings
<ul style="list-style-type: none"> <li>▪ How can we use mathematical language to describe non-linear change?</li> <li>▪ How can we model situations using quadratics?</li> <li>▪ How can we model situations using exponents?</li> </ul>	<ul style="list-style-type: none"> <li>▪ Graphs and equations are alternative (and often equivalent) ways for depicting and analyzing patterns of non-linear change.</li> <li>▪ Mathematical models can be used to describe physical relationships; these relationships are often non-linear.</li> <li>▪ Real world situations, involving quadratic or exponential relationships, can be solved using multiple representations.</li> </ul>
N1. Non-linear Functions	
Content Benchmarks	Comments and Examples
<p>N1.a Recognize, describe, represent and analyze a quadratic function using words, tables, graphs or equations.</p> <ul style="list-style-type: none"> <li>• <b>Content Clarification</b> Key characteristics include vertex, zeros, y-intercept, domain and range, maximum, minimum and opening direction.</li> </ul>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Determining and analyzing key characteristics of quadratic functions and their graphs.</li> <li>• Using correct function notation and terminology (e.g. <math>f(x)</math>, independent and dependent variables, etc.).</li> <li>• Sketching a graph of a quadratic equation using the zeros and vertex when given the equation.</li> </ul> <p><b>Scope of State Assessment Content:</b></p> <ul style="list-style-type: none"> <li>○ Students are expected to label axes and scales when required to graph an equation or function. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g. dollars, seconds, etc).</li> <li>○ Quadratic functions may be represented in the following forms:</li> </ul>

	<p>polynomial: <math>f(x) = ax^2 + bx + c</math>  factored: <math>f(x) = a(x - r)(x - s)</math></p> <ul style="list-style-type: none"> <li>• <b>State Assessment Limitations:</b> <ul style="list-style-type: none"> <li>○ In this section (Non-Linear Functions), all coefficients will be integers. Quadratic functions will have integral coefficient and vertices and rational zeros.</li> <li>○ In constructed response items, students will not be required to derive quadratic equations from tables, graphs or words.</li> <li>○ Completing the square will not be required.</li> </ul> </li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>SCR:</b> Determine the vertex of the function <math>f(x) = 4x^2 - 8x - 5</math>  <i>Sample Solutions:</i>  <math display="block">f(x) = 4x^2 - 8x - 5</math> <math display="block">f(x) = (2x - 5)(2x + 1)</math> <math display="block">0 = (2x - 5)(2x + 1)</math> <math display="block">x = \left\{ -\frac{1}{2}, \frac{5}{2} \right\}</math> <p>To find the x-value of the vertex, average the zeros:</p> <math display="block">x = \frac{\left( -\frac{1}{2} + \frac{5}{2} \right)}{2} = 1</math> <math display="block">f(1) = 4(1)^2 - 8(1) - 5 = -9</math> <math display="block">(1, -9)</math> <p style="text-align: center;">OR</p> <p>Substitute 4 and -8 into <math>x = \frac{-b}{2a}</math>, and then solve for <math>f(x)</math>.</p> <math display="block">x = \frac{-(-8)}{2(4)} = 1</math> <math display="block">f(1) = 4(1)^2 - 8(1) - 5 = -9</math> <math display="block">(1, -9)</math> </li> </ul> <p><b>Instructional Strategies:</b></p> <ul style="list-style-type: none"> <li>• <b>Interdisciplinary Connections:</b> Using an electronic spreadsheet, demonstrate algebraic equivalence by demonstrating that the functions <math>m(x)</math> and <math>a(x)</math> form the identity <math>m(x) = a(x)</math> in an authentic problem. For example, investigate the following request before a local planning board: A subdivision is being placed on a piece of land 1000 m by 1500 m. A boulevard of trees and an access road of uniform width form the border of the subdivision. The area of the inner rectangle of houses and parks is to be at least 1.35 million <math>m^2</math> to accommodate the planned homes and parks. What is the largest width that can be set aside inside the perimeter for the border composed of the boulevard and road? (NCTM <i>Navigations through Algebra In Grades 9-12</i>)</li> </ul>
<p>N1.b Analyze a table, numerical pattern, graph, equation or context to determine whether a linear, quadratic or exponential relationship could be represented.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Distinguishing between linear and non-linear functions, including quadratic, exponential and other non-linear relationships.</li> <li>• Generating and describing <i>geometric</i> sequences recursively; identifying geometric sequences expressed recursively.</li> <li>• Recognizing when an exponential model is appropriate (growth or decay).</li> <li>• Determining if an exponential function is increasing or decreasing.</li> <li>• Demonstrating the effect of compound interest, decay, or growth using</li> </ul>

	<p>iteration.</p> <ul style="list-style-type: none"> <li>Extending a table, numerical pattern or graph given the type of relationship (quadratic or exponential).</li> <li>Using first and second differences to determine the type of function represented.</li> <li>Graphing exponential functions.</li> <li><b>Scope of State Assessment Content:</b> Students may be required to explain their reasoning.</li> <li><b>State Assessment Limitation:</b> Exponential functions in the form <math>y=ab^x</math> will include integer exponents and rational non-zero values for both <math>a</math> and <math>b</math>, and <math>b &gt; 0</math>. When exponents are specifically named for exponential functions, the exponents will be integers.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li><b>ECR:</b> Given the following increasing numerical pattern, determine the type of relationship that exists (linear, quadratic or exponential) and justify your conclusion. 3, 6, 12, 24, 48, ...</li> <li><b>ECR:</b> Make a table and graph for the sequence NEXT = 4 * NOW where the first term is 1 and explain why this corresponds to an exponential function.</li> <li><b>Performance Assessment Task:</b> To plan for paying college tuition, you investigate at least three options for a savings plan, using internet resources. Explain how a family might prepare by investing in an annuity in order to have at least \$100,000 in the annuity by the time a child is 18 years old if they start saving immediately after the child is born.</li> </ul> <p><b>Instructional Strategies:</b></p> <ul style="list-style-type: none"> <li><b>Global Perspective:</b> Take data involving two variables in an area of global impact (e.g. population growth and decline, pandemic flu outbreak, global warming, etc.) from an online resource such as the World Almanac. Construct a visual representation and predict the type of equation or function which would best model the data then chose from linear, quadratic, or exponential methods and discuss how well the model fits, as well as the limitations. Extend the model to make predictions on future impact of these variables.</li> <li><b>Technology Integration</b> In the activity above, in addition to internet resources, students use a computerized statistics application or calculator to fit a function to the data. Students use technology to help them discuss or communicate the mathematical representations of the application.</li> </ul>
<p>NI.c Recognize and solve problems that can be modeled using a quadratic function. Interpret the solution in terms of the context of the original problem.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>Using and interpreting appropriate units of measurement, estimation and the appropriate level of precision for applications.</li> <li><b>State Assessment Limitations:</b> <ul style="list-style-type: none"> <li>Quadratic functions will have integral coefficient and vertices and rational zeros.</li> <li>For physics applications, formulas will be provided (e.g. <math>s=-16t^2 + 48t + 64</math>).</li> <li>Contexts will be accessible for students working at this level (e.g. area, Pythagorean relationships or motion). No formal physics notation will be used (e.g. <math>v_0</math>, <math>s_0</math>, etc.). All quadratic equations will have integral coefficients.</li> </ul> </li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li><b>SCR:</b> A hiker accidentally drops a full water bottle off of a bridge. How many seconds will it take to hit the water? Assuming the bottle drops from a height of 300 feet, the model for the height of the bottle at time <math>t</math> is  <math display="block">h = -16t^2 + 300</math> <i>(Sample Solution: <math>0 = -16t^2 + 300 \rightarrow t \approx 4.33</math> seconds)</i></li> </ul>

	<p><b>Instructional Strategies:</b></p> <ul style="list-style-type: none"> <li>• <b>Technology Integration:</b> Students investigate what size square to cut from each corner of a rectangular piece of cardboard in order to make the largest possible open-top box. Students make models, record the size of the square and the volume for each model, plotting the points on a graph using spreadsheet software. They note the relationship is not linear and make a conjecture about maximum volume. Students also generate an algebraic expression and equation describing this situation</li> </ul>
<b>N2. Non-linear Equations</b>	
<b>Content Benchmarks</b>	<b>Comments and Examples</b>
<p>N2.a Solve equations involving several variables for one variable in terms of the others.</p> <ul style="list-style-type: none"> <li>• <b>Content Clarification:</b> Equations and formulas that have several variables (letters) are called “literal equations.” The task will usually be to solve the equation for one of the variables.</li> <li>• <b>Scope of Content:</b> Students should build on previous knowledge, applying it to the solving of non-linear literal equations.</li> </ul>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Understanding that solving a literal equation follows the same rules as solving any other equation.</li> <li>• <b>State Assessment Limitation:</b> <ul style="list-style-type: none"> <li>○ In this section (Non-Linear Equations), all coefficients will be integers.</li> <li>○ Equations may be linear or simple non-linear equations for which students must solve for, at most, a second-degree variable.</li> </ul> </li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>SCR:</b> Solve for <math>r</math>: <math>V = \pi r^2 h</math>. <i>(Sample Solution: <math>\sqrt{\frac{V}{\pi h}} = r</math>)</i></li> <li>• <b>SCR:</b> Solve for <math>y</math>: <math>z = 3x^2y + 4y</math> <i>(Sample Solution: <math>y = z/(3x^2 + 4)</math>)</i></li> </ul>
<p>N2.b Solve single-variable quadratic equations.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• <b>State Assessment Limitation:</b> Quadratic equations will have integral coefficients and rational solutions. Students may use any valid method to determine solutions for a quadratic equation.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>SCR (Non-Calculator):</b> Solve the following for <math>x</math>: <math>x(2x + 5) = 0</math> <i>(Sample Solution: <math>x = 0</math> and <math>x = -5/2</math>)</i></li> <li>• <b>SCR (Non-Calculator):</b> Solve the following for <math>x</math>: <math>3x^2 - x - 10 = -8</math> <i>(Sample Solution: <math>x = 1</math> and <math>x = -2/3</math>)</i></li> <li>• <b>SCR (Non-Calculator):</b> <math>2x^2 - 3x + 1 = 0</math> What is the solution set for the equation above? Show or explain your work. <i>( Sample Solutions: <math>\{ 1/2, 1\}</math> or <math>s = 1/2</math> or <math>1</math> )</i></li> </ul>
<p>N2B1. Provide and describe multiple representations of solutions to simple exponential equations using concrete models, tables, graphs, symbolic expressions, and technology.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• <b>State Assessment Limitation:</b> Exponential equations will be one-step.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>ECR:</b> Using a table, graph, and/or symbolic expressions, solve the following equation. Provide more than one representation of the solutions and explain your work. <math display="block">32 = 2^x</math></li> </ul> <p><b>Instructional Strategies:</b></p> <ul style="list-style-type: none"> <li>• <b>Interdisciplinary Connections:</b> Determine the best choice of a payroll (or allowance) option after one month: a constant rate of \$5.00 per day or 2 cents for the first day of the month, 4 cents on the second day of the month, etc., where every day is double the amount the day before.</li> </ul>

	<ul style="list-style-type: none"> <li>• <b>Interdisciplinary Connections:</b> Find the half-life of a decaying substance by providing and describing solutions to a simple exponential equation.</li> </ul>
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### D: Data, Statistics, and Probability

Successful students will be able to apply algebraic knowledge to the interpretation and analysis of data, statistics and probability. Analysis and interpretation of univariate and bivariate data includes the use of summary statistics for sets of data and estimation of lines of best fit. While some important components in the study of data and statistics, such as misleading uses of data, sampling techniques, bias, question formulation, and experiment design are addressed when possible in this Algebra I End-of-Course Exam, those topics will be expected to be assessed in more depth in the classroom. These benchmarks are intended to support and reinforce algebra concepts. For this reason, several sample algebraic solutions are provided for examples.

Essential Questions	Enduring Understandings
<ul style="list-style-type: none"> <li>▪ How can the collection, organization, interpretation, and display of data be used to answer questions?</li> <li>▪ How can the representation of data influence decisions?</li> <li>▪ When does order matter?</li> <li>▪ How can experimental and theoretical probabilities be used to make predictions or draw conclusions?</li> </ul>	<ul style="list-style-type: none"> <li>▪ The message conveyed by the data depends on the display.</li> <li>▪ The results of a statistical investigation can be used to support or refute an argument.</li> <li>▪ Tables, charts, tree diagrams, and multiplication can be used to determine how many ways an event can occur.</li> <li>▪ Probability is about predictions over the long term rather than predictions of individual events.</li> </ul>

### D1: Data and Statistical Analysis

Content Benchmarks	Comments and Examples
D1.a. Interpret and compare linear models for data that exhibit a linear trend in the context of a problem.	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Creating scatter plots and estimating a line of best fit.</li> <li>• Interpreting the slope and y-intercept of the regression line (line of best fit) in the context of the model.</li> <li>• Using lines of best fit to extrapolate or interpolate within the range of the data and within the context of the problem.</li> <li>• Determining when, within the context of a problem, it may be unreasonable to extrapolate beyond a certain point.</li> <li>• Evaluating the use of data in authentic scenarios with regard to the concepts of correlation versus causation.</li> <li>• <b>Scope of State Assessment Content:</b> Students are expected to label axes and scales when required to graph an equation or function. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g. dollars, seconds, etc).</li> <li>• <b>State Assessment Limitation:</b> Students will not be required to use regression to calculate a line of best fit. In constructed response items, students will not be required to draw a line of best fit.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>ECR:</b> Given the set of points (1, 1), (2, 3), (4, 7), (6, 9), and (7, 13), if a sixth point were included in the set, which of the following would have the greatest impact, as a sixth point in the set, on the slope of a line of best fit? Justify your answer. (3, 4), (6, 10), (8, 23), or (10, 19)</li> <li>• <b>ECR:</b> If a linear trend describes population growth in a small town over 5 years, explain why it would not be best to use the same linear trend to predict population in the town after 100 years.</li> </ul>

<p>D1.b Use measures of center and spread to compare and analyze data sets.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>Analyzing data sets and using summary statistics to compare the data sets and to answer questions regarding the data.</li> <li>Determining the effect outliers have on various measures of center and spread.</li> <li><b>State Assessment Limitation:</b> <ul style="list-style-type: none"> <li>No item will assess only the calculations of mean, median, or mode. Items will require the use of those concepts and/or calculations and will be at an appropriate cognitive level and difficulty for Algebra I.</li> <li>Measures of spread are limited to range.</li> </ul> </li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li><b>ECR:</b> Explain what happens to the mean, median and mode when a value, <math>x</math>, is added to each data point.</li> <li><b>ECR:</b> Given the following data set: 55, 55, 57, 58, 60, and 63. Describe how the measures of center or spread will or will not change if an additional data point of 57.5 is included with the set.</li> <li><b>SCR (Calculator permitted):</b> A student has scores of 78, 82, 91, 84, and 67 on the first five tests in a semester. What score must she earn on the sixth test in order to raise her average to 82? Show or explain your work.</li> </ul> $\frac{78 + 82 + 91 + 84 + 67 + x}{6} = 82$ <p><i>Sample Solution:</i></p> $\Rightarrow 402 + x = 6(82)$ $\Rightarrow x = 492 - 402 = 90$ <ul style="list-style-type: none"> <li><b>Performance Assessment Task:</b> You have been hired as a consultant to the faculty, based on your expertise as a mathematician studying issues of fairness and measures of central tendency. You have been asked to propose and defend a “fair” grading system for use in this school. How should everyone’s grade in classes be calculated? The faculty will want great evidence and argument, presented respectfully but effectively (<i>Grant Wiggins</i>).</li> </ul>
<p>D1.c. Evaluate the reliability of reports based on data published in the media.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>Explaining the impact of bias and the phrasing of questions asked during data collection.</li> <li>Identifying and explaining misleading uses of data and data displays.</li> <li>Analyzing the appropriateness of a data display and the reasonableness of conclusions based on statistical studies.</li> <li>Explaining the difference between randomized experiments and observational data.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li><b>Performance Assessment Task:</b> Find a graph in your local newspaper which provides data you expect to use for a social studies paper. Describe the data presented. Tell whether the graph is misleading or fair and how you know.</li> </ul>

## D2: Systematic Listing & Counting & Probability

Content Benchmarks	Comments and Examples
<p>D2.a Use counting principles to determine the number of ways an event can occur. Interpret and justify solutions.</p> <ul style="list-style-type: none"> <li>• <b>Content Assumption</b> All spinners, number cubes and coins are fair unless otherwise noted.</li> </ul>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Using an understanding of permutations and combinations to solve problems with and without replacement.</li> <li>• <b>State Assessment Limitation:</b> Neither factorial notation nor factorial forms of formulas for combination <math>({}_nC_r)</math> or permutation <math>({}_nP_r)</math> will be used in items or be required to solve items on the end-of-course assessment. However, students may use any valid method to solve the problems. Numbers involved will be manageable without formulas.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>ECR (Non-Calculator):</b> Compare the number of ways the letters of the words FROG and DEER can be arranged to form different four-letter sequences (e.g. to form different passwords). Explain your answer.</li> <li>• <b>SCR:</b> If a person has twice as many shirts as pairs of pants, how many different combinations can be made of a shirt and pair of pants, based on the number of pants? <i>Sample Solution:</i> <math>p</math> = number of pairs of pants <math>2p</math> = number of shirts <math>p(2p) = 2p^2</math> = number of combinations of pants and shirts</li> </ul>
<p>D2.b Apply probability concepts to determine the likelihood an event will occur in practical situations.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Determining, exactly or approximately, the probability that an event will occur based on simple experiments (e.g. tossing number cubes, flipping coins, spinning spinners), counting principles, or data.</li> <li>• Making predictions based on experimental and theoretical probabilities and comparing results.</li> <li>• <b>State Assessment Assumptions:</b> All events are equally likely and samples are assumed to be representative of the population, unless otherwise stated. All spinners, number cubes, and coins are assumed to be fair unless otherwise noted.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>• <b>SCR:</b> If there are 4 brown, 4 black, and 4 blue socks in a drawer, what is the probability that a matched pair will be selected when drawing out first one and then another, without replacing the first sock or being able to see the socks as they are drawn?</li> <li>• <b>SCR:</b> In a sample of 100 randomly selected students, 37 of them could identify the difference in two brands of soft drinks. Based on these data, what is the best estimate of how many of the 2352 students in the school could distinguish between the soft drinks? <i>Sample Solution:</i> <math display="block">\frac{37}{100} = \frac{x}{2352}</math><math display="block">100x = 37(2352)</math><math display="block">x = 870.24</math><math display="block">\therefore 870 \text{ students would be expected.}</math></li> </ul>

<p><b>D2.C1</b> Determine and apply probabilities in complex situations.</p>	<p><b>Instructional Focus:</b></p> <ul style="list-style-type: none"> <li>• Employing multiple representations (Venn diagrams, trees, tables, and area methods) to analyze and summarize information concerning compound events. <ul style="list-style-type: none"> <li>○ Mutually exclusive or not</li> </ul> </li> </ul>
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	<ul style="list-style-type: none"> <li>○ Complementary events</li> <li>○ Dependent or independent</li> <li>○ Multiplication rule</li> <li>● Determining conditional probability.</li> <li>● Solving problems involving probability with simulations (using spinners, dice, calculators, and computers) and theoretical models.</li> <li>● Recognizing that simulation results are likely to differ from one run of the simulation to the next; observe that the results of the simulation tend to converge as the number of trials increases (Law of Large Numbers).</li> <li>● Evaluating medical test results and treatment options, analyzing risk in situations where anecdotal evidence is provided, interpreting media reports and evaluate conclusions.</li> </ul> <p><b>Sample Assessments:</b></p> <ul style="list-style-type: none"> <li>● <b>ECR:</b> Jim has tossed a coin 8 times and gotten heads every time. He thinks that he is more likely to get tails on the next flip. Do you agree or disagree? Explain.</li> <li>● <b>ECR:</b> Ann is considering two different investments. The first investment is a stock which has a 25% chance of returning 10%, a 25% chance of returning 4%, and a 50% chance of losing 2%. The second investment is guaranteed to earn 5%. Which investment should she make? Justify your response.</li> </ul> <p><b>Instructional Strategies:</b></p> <ul style="list-style-type: none"> <li>● Employ Venn diagrams to summarize information concerning compound events.</li> <li>● <b>Interdisciplinary Connections:</b> Use probability to interpret odds and risks of financial investments options and recognize common misconceptions. Students investigate return and risk for various investments, including certificates of deposit, stocks, bonds, and real estate.</li> <li>● <b>Interdisciplinary Connections:</b> Analyze the risks associated with a particular accident, illness, or course of treatment expressed as a probability. Present various incidents that have a similar probability of occurrence.</li> </ul>
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