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REVISION OF A FLAWED ACCEPTANCE STANDARD

First Interim Report

Project 7788

Development of Statistical Procedures

Prepared By

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16. Abstract

A major revision of AASHTO Standard R9-84, Acceptance Sampling Plans for Highway Construction, has just been completed. The primary goals were to correct a major conceptual error and to reduce the level of complexity. This report discusses the flaws in the original version, describes the basic changes that were made, and presents a significant addition to the new standard in the form of operating characteristic tables that enable the user to quickly and easily select acceptance plans that will provide the desired degree of quality assurance. Computer simulation is used to demonstrate that single-limit variables operating characteristic curves are sufficiently accurate for most double-limit applications. Two examples are included to illustrate the use of the revised standard.

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TABLES

FIGURES

CONTENTS

BACKGROUND

In the early 1960s, the AASHTO Road Test produced a wealth of statistical data that could be used to relate pavement quality to performance. Highway engineers began to recognize that various desirable quality characteristics could be described statistically and, toward the end of that decade, several highway agencies had begun to develop acceptance procedures based on statistical concepts. Today, many highway agencies routinely use statistical acceptance procedures in one form or another.

The first statistical acceptance procedures were often far from optimal. Highway engineers were relatively unfamiliar with statistical terms and procedures, especially in regard to the construction of operating characteristic curves and the analysis of risks. Consequently, the early development of statistical specifications consisted largely of a trial-and-error process and several revisions were often required to obtain a workable specification.

More recently, there has been a significant improvement in the manner in which these specifications are developed. Highway engineers have acquired a better understanding of statistical methods (1 - 3) and the computer has emerged as a valuable aid (4, 5) in performing much of the development and analysis work. The state-of-the-art has now progressed to the extent that statistical specification writing must be regarded as a thoroughly scientific activity.

AASHTO Standard R9 - 84, Acceptance Sampling Plans for Highway Construction (6), was adopted in 1984 to document and standardize practices that had evolved over the previous two decades. It covers both attributes

sampling for defects that are counted and variables sampling for characteristics that are measured on a continuous scale. Primary source documents for these two approaches are Military Standard 105 for attributes sampling (7) and Military Standard 414 for variables sampling (8), both published by the Department of Defense. The theory underlying attributes sampling is relatively simple and is covered in connection with the hypergeometric distribution in many texts on statistics and quality assurance (9 - 11). The theoretical basis for variables sampling is considerably more complex, involving both the beta and noncentral t distributions, and is not as well known (11 - 13).

Unfortunately, the current version of Standard R9 is seriously flawed, both by what it includes as well as by what it omits. It is based on an early method containing both technical and conceptual errors and it fails to cover the analysis of operating characteristic curves, one of the most important steps in the development of any acceptance procedure. A higher level of technical competence must be demanded of a work that is to serve as a procedural guide for the highway quality assurance profession.

BASIC PROBLEMS AND CORRECTIVE MEASURES

The original developers of the methodology used in AASHTO Standard R9 undoubtedly had nothing but the best of intentions. At a time when statistical procedures were new and unfamiliar, and considerable resistance to the new methods was often encountered, it was understandably tempting to make various seemingly harmless modifications to make these procedures more palatable. Obviously, the arbitrary modification of any highly technical procedure by practitioners unfamiliar with the underlying theory is a

dangerous business and, not surprisingly, the validity of some of these methods was seriously compromised. This is essentially what happened in the development of the procedures used in Standard R9. Specific shortcomings and the necessary corrective measures are as follows:

1. Both the attributes and variables plans described in Standard R9 are designed to control percent defective, the percentage of the lot falling outside a lower or upper specification limit, or outside both lower and upper specification limits, as illustrated in Figure 1. As presently written, however, Standard R9 is oriented partly toward percent defective and partly toward population means which leads to considerable confusion. For example, it is stated in the current standard that, for a variables plan with the standard deviation unknown, only one risk (buyer's or seller's) can be controlled. In fact, when quality is measured in terms of percent defective, both the buyer's risk and the seller's risk can be controlled by either variables or attributes plans. This basic contradiction has been corrected by basing the revised standard entirely on the percent defective parameter.

2. A major omission in the current standard is a convenient method to construct the operating characteristic (OC) curves for the acceptance plans that are developed. OC curves give the probability of acceptance associated with various levels of submitted quality and provide a graphical representation of an acceptance plan's ability to discriminate between acceptable and unacceptable work. A typical example is shown in Figure 2.

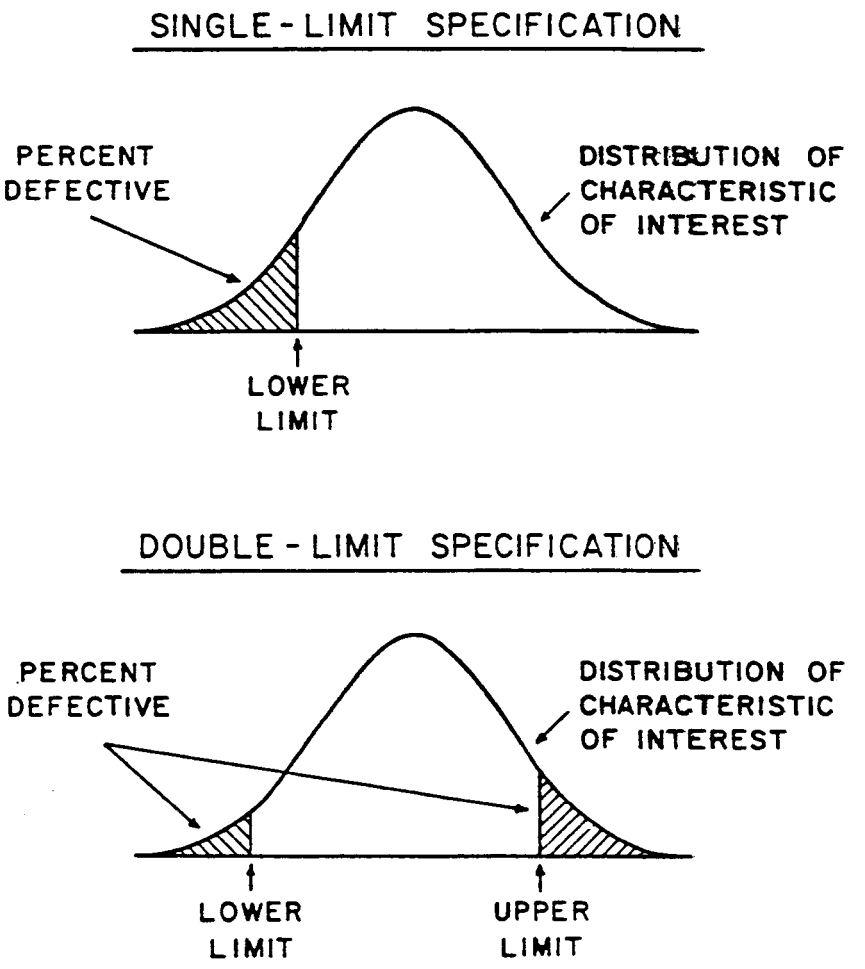

Figure 1. Illustration of the concept of percent defective.

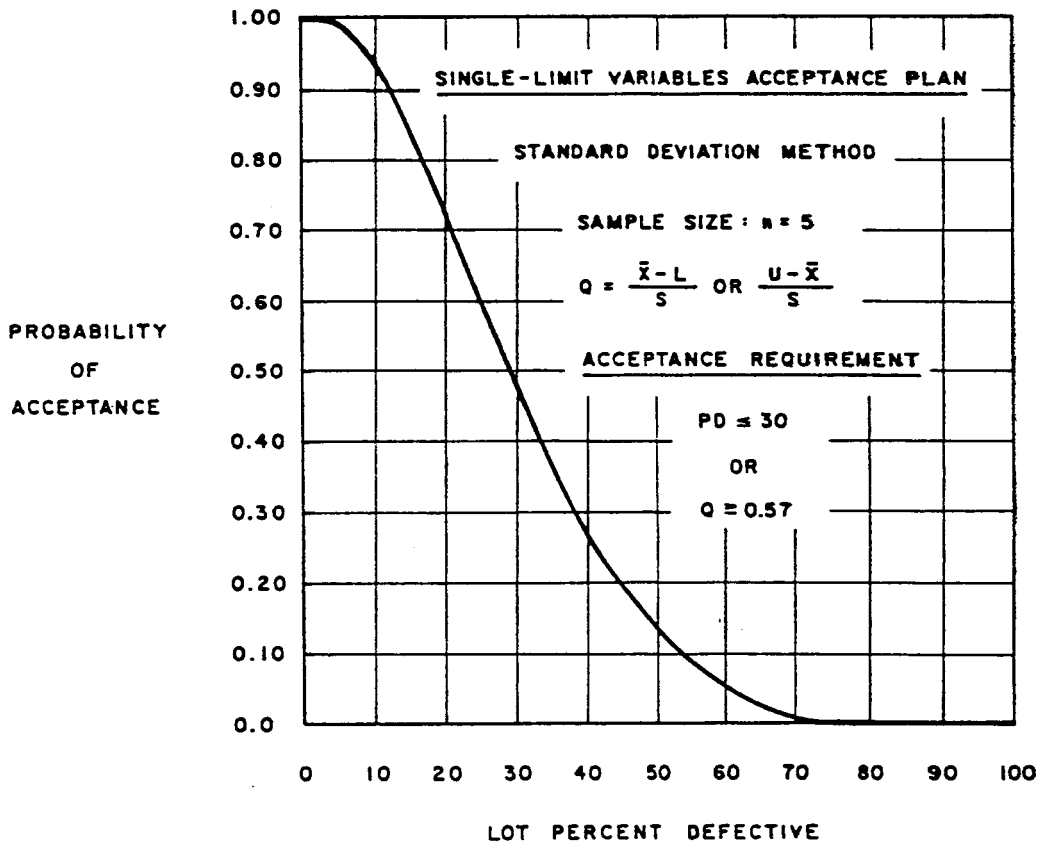

Figure 2. Typical operating characteristic curve for a variables acceptance plan.

The importance of examining OC curves cannot be overemphasized. In this manner, the risks to both the specifying agency and the contractor can be determined in advance and modifications of the acceptance plan can be made, if necessary, before embarrassing and troublesome situations arise in the field. This shortcoming of the current standard has been corrected by the development of several new operating characteristic tables for both attributes and variables plans.

3. When constructing an OC curve for a variables plan, the problem cited in Item 1 becomes much more apparent. Because the variables approach was derived to control percent defective, there is a unique probability of acceptance associated with any particular level of lot percent defective, as can be seen in Figure 2. (This is precisely correct for single-limit plans and is approximately correct for double-limit plans.) However, if the acceptance procedure were oriented around population means, as it is in the current version of Standard R9, there would no longer be a unique OC curve since each level of population mean could correspond to a wide range of percent defective, depending upon the value of the population standard deviation. Rewriting the standard entirely around the percent defective parameter has corrected this problem.

4. The table for the estimation of percent defective in the current version of the standard is not in the most logical or useful form and it omits several potentially useful sample sizes. The new table includes several additional sample sizes, it is accurate to a greater number of decimal places, and two revised formats are provided.

5. The current table for attributes sampling was taken from Military Standard 105 (7). It gives the recommended sample size and acceptance number (maximum allowable number of defective items in a sample) based on lot size and the user's definition of acceptable quality level (AQL). In its present form, it does not allow the user to know or control the risks that are involved and, like the percent defective estimation table, several useful sample sizes have been omitted. This table has been completely revised to be more suitable for highway construction applications.

6. The current version of the standard emphasizes the range method for variables acceptance plans. The standard deviation method is included, but the user is required to estimate the standard deviation from the range, thereby failing to capitalize on the standard deviation's superior mathematical efficiency. In order to realize the cost savings associated with the smaller sample sizes required with the standard deviation method, this is made the primary procedure in the revised version of the standard. The range method has been retained, and some new tables have been provided, but this procedure has now been relegated to an appendix.

7. If an acceptance specification developed by the method outlined in the current version of Standard R9 were to be challenged in court, it is possible that the weaknesses in the standard could be used to attack the validity of the specification. While it is true that the acceptance plan could be perfectly satisfactory even though the methodology used to develop it was flawed, the highway agency might still be cast in an unfavorable light. This potential vulnerability can be avoided by using valid

statistical procedures in a rigorous fashion. It is believed that the revised version of Standard R9 will encourage the proper use of these methods.

8. Finally, a major drawback of the present version of Standard R9 is its technical complexity. At best, it will fail to promote a wider use and acceptance of statistical quality assurance and, at worst, it could even be a deterrent. A primary goal in rewriting the standard was to make it considerably easier to understand and use.

DEVELOPMENT OF NEW TABLES

In order to correct the deficiencies of the current standard, it was first necessary to develop several new tables. These form the core around which the rest of the standard has been constructed and are discussed in the order in which they appear in the appendices of the revised standard.

In Appendix A of the new standard, the previous table for attributes sampling has been replaced with operating characteristic tables that give probability of acceptance for selected levels of population (lot) percent defective for many different combinations of sample size and acceptance number. Those plans having relatively undesirable OC curves have been omitted and not all plans in these tables will be suitable for all situations. The primary benefit of the new tables is that it is possible to tell at a glance how different plans will perform over a wide range of submitted quality.

The new attributes tables appear in four sections, one each for lot sizes of 20, 100, 500, and infinity. Two of these tables, for lot sizes of

100 and infinity, are presented as Tables 1 and 2. The tables are constructed so that it will never be necessary to interpolate between acceptance numbers or between sample sizes up to a sample size of n = 10. Some interpolation may be necessary for larger sample sizes or for specific lot sizes, although the OC curves are relatively insensitive to lot size. For plans with variable lot sizes, it will be necessary to plot bounding OC curves.

Appendix B of the new standard contains the corresponding operating characteristic tables for variables acceptance plans (standard deviation method), one of which is presented as Table 3. The acceptance plans in these tables are specified by sample size and either the maximum allowable estimated percent defective (M) or the minimum allowable value (k) of the quality index (Q). The quality index is computed by Equation 1 or 2, as appropriate.

$$Q_L = \frac{\bar{X} - L}{S} \quad (1)$$

$$Q_U = \frac{U - \bar{X}}{S} \quad (2)$$

in which

Q = quality index

$\bar{X}$ = sample mean

S = sample standard deviation

L, U = lower and upper specification limits outside of which the material or work is defined to be defective

ATTRIBUTES ACCEPTANCE PLANS — LOT SIZE = 100

SAMPLE SIZE (n)	ACCEPTANCE NUMBER (c)	PROBABILITY OF ACCEPTANCE FOR SELECTED LEVELS OF LOT PERCENT DEFECTIVE													
		5	10	15	20	25	30	35	40	45	50	55	60	65	70
1	0	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30
2	0	0.90	0.81	0.72	0.64	0.56	0.49	0.42	0.36	0.30	0.25	0.20	0.16	0.12	0.09
3	0	0.86	0.73	0.61	0.51	0.42	0.34	0.27	0.21	0.16	0.12	0.09	0.06	0.04	0.03
3	1	0.99	0.97	0.94	0.90	0.85	0.79	0.72	0.65	0.58	0.50	0.42	0.35	0.28	0.21
4	0	0.81	0.65	0.52	0.40	0.31	0.23	0.17	0.12	0.09	0.06	0.04	0.02	0.01	0.01
4	1	0.99	0.95	0.89	0.82	0.74	0.65	0.56	0.47	0.39	0.31	0.24	0.17	0.12	0.08
5	1	0.98	0.92	0.84	0.74	0.63	0.53	0.42	0.33	0.25	0.18	0.13	0.08	0.05	0.03
5	2	1.00	0.99	0.98	0.95	0.90	0.84	0.77	0.69	0.60	0.50	0.40	0.31	0.23	0.16
6	1	0.97	0.89	0.78	0.66	0.53	0.41	0.31	0.23	0.16	0.10	0.06	0.04	0.02	0.01
6	2	1.00	0.99	0.96	0.91	0.84	0.75	0.65	0.54	0.44	0.34	0.25	0.17	0.11	0.06
7	1	0.96	0.86	0.72	0.57	0.44	0.32	0.22	0.15	0.09	0.06	0.03	0.02	0.01	0.00
7	2	1.00	0.98	0.93	0.86	0.76	0.65	0.53	0.42	0.31	0.22	0.14	0.09	0.05	0.02
8	1	0.95	0.82	0.66	0.50	0.36	0.24	0.16	0.10	0.06	0.03	0.01	0.01	0.00	0.00
8	2	1.00	0.97	0.90	0.80	0.68	0.55	0.42	0.31	0.21	0.13	0.08	0.04	0.02	0.01
8	3	1.00	1.00	0.98	0.95	0.90	0.81	0.71	0.60	0.48	0.36	0.25	0.16	0.10	0.05
9	1	0.94	0.78	0.60	0.43	0.29	0.18	0.11	0.06	0.03	0.02	0.01	0.00	0.00	0.00
9	2	1.00	0.96	0.87	0.74	0.60	0.46	0.33	0.22	0.14	0.08	0.04	0.02	0.01	0.00
9	3	1.00	0.99	0.97	0.92	0.84	0.74	0.61	0.48	0.35	0.24	0.15	0.09	0.05	0.02
10	1	0.92	0.74	0.54	0.36	0.23	0.14	0.07	0.04	0.02	0.01	0.00	0.00	0.00	0.00
10	2	0.99	0.94	0.83	0.68	0.52	0.37	0.25	0.15	0.09	0.05	0.02	0.01	0.00	0.00
10	3	1.00	0.99	0.96	0.89	0.79	0.65	0.51	0.37	0.25	0.16	0.09	0.05	0.02	0.01
15	2	0.98	0.83	0.60	0.38	0.21	0.11	0.05	0.02	0.01	0.00	0.00	0.00	0.00	0.00
15	3	1.00	0.96	0.84	0.65	0.45	0.28	0.15	0.07	0.03	0.01	0.00	0.00	0.00	0.00
15	4	1.00	0.99	0.95	0.85	0.70	0.51	0.34	0.20	0.10	0.05	0.02	0.01	0.00	0.00
15	5	1.00	1.00	0.99	0.95	0.87	0.73	0.57	0.39	0.24	0.13	0.06	0.02	0.01	0.00
20	2	0.95	0.68	0.38	0.18	0.07	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	3	0.99	0.89	0.65	0.39	0.20	0.08	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00
20	4	1.00	0.97	0.85	0.64	0.40	0.21	0.09	0.03	0.01	0.00	0.00	0.00	0.00	0.00
20	5	1.00	1.00	0.95	0.83	0.62	0.40	0.22	0.10	0.04	0.01	0.00	0.00	0.00	0.00
20	6	1.00	1.00	0.99	0.94	0.81	0.62	0.40	0.22	0.10	0.04	0.01	0.00	0.00	0.00
30	3	0.97	0.65	0.28	0.08	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	4	1.00	0.86	0.51	0.21	0.06	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	5	1.00	0.96	0.73	0.40	0.16	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	6	1.00	0.99	0.89	0.62	0.31	0.12	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00
30	7	1.00	1.00	0.96	0.80	0.51	0.24	0.08	0.02	0.00	0.00	0.00	0.00	0.00	0.00
30	8	1.00	1.00	0.99	0.91	0.70	0.41	0.18	0.06	0.01	0.00	0.00	0.00	0.00	0.00
50	5	1.00	0.63	0.13	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	6	1.00	0.84	0.29	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	7	1.00	0.95	0.50	0.11	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	8	1.00	0.99	0.71	0.23	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	9	1.00	1.00	0.87	0.40	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	10	1.00	1.00	0.95	0.60	0.18	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	11	1.00	1.00	0.99	0.77	0.32	0.06	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	9	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	9	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	10	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	11	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	12	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	13	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	14	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	15	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	16	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	17	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	18	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

PROBABILITY OF ACCEPTANCE IS A FUNCTION OF LOT SIZE FOR ATTRIBUTES PLANS USING DISCRETE DATA. FOR VARIABLE LOT SIZES, IT WILL BE NECESSARY TO PLOT BOUNDING OPERATING CHARACTERISTIC CURVES. THE VALUES IN THIS TABLE ARE APPROPRIATE FOR BOTH SINGLE-LIMIT AND DOUBLE-LIMIT APPLICATIONS AND ARE UNAFFECTED BY THE DISTRIBUTIONAL FORM OF THE POPULATION.

Table 1. Operating characteristics of attributes acceptance plans with a lot size of N = 100.

ATTRIBUTES ACCEPTANCE PLANS — LOT SIZE = INFINITE

SAMPLE SIZE (n)	ACCEPTANCE NUMBER (c)	PROBABILITY OF ACCEPTANCE FOR SELECTED LEVELS OF LOT PERCENT DEFECTIVE													
		5	10	15	20	25	30	35	40	45	50	55	60	65	70
1	0	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30
2	0	0.90	0.81	0.72	0.64	0.56	0.49	0.42	0.36	0.30	0.25	0.20	0.16	0.12	0.09
3	0	0.86	0.73	0.61	0.51	0.42	0.34	0.27	0.22	0.17	0.13	0.09	0.06	0.04	0.03
3	1	0.99	0.97	0.94	0.90	0.84	0.78	0.72	0.65	0.57	0.50	0.43	0.35	0.28	0.22
4	0	0.81	0.66	0.52	0.41	0.32	0.24	0.18	0.13	0.09	0.06	0.04	0.03	0.02	0.01
4	1	0.99	0.95	0.89	0.82	0.74	0.65	0.56	0.48	0.39	0.31	0.24	0.18	0.13	0.08
5	1	0.98	0.92	0.84	0.74	0.63	0.53	0.43	0.34	0.26	0.19	0.13	0.09	0.05	0.03
5	2	1.00	0.99	0.97	0.94	0.90	0.84	0.76	0.68	0.59	0.50	0.41	0.32	0.24	0.16
6	1	0.97	0.89	0.78	0.66	0.53	0.42	0.32	0.23	0.16	0.11	0.07	0.04	0.02	0.01
6	2	1.00	0.98	0.95	0.90	0.83	0.74	0.65	0.54	0.44	0.34	0.26	0.18	0.12	0.07
7	1	0.96	0.85	0.72	0.58	0.44	0.33	0.23	0.16	0.10	0.06	0.04	0.02	0.01	0.00
7	2	1.00	0.97	0.93	0.85	0.76	0.65	0.53	0.42	0.32	0.23	0.15	0.10	0.06	0.03
8	1	0.94	0.81	0.66	0.50	0.37	0.26	0.17	0.11	0.06	0.04	0.02	0.01	0.00	0.00
8	2	0.99	0.96	0.89	0.80	0.68	0.55	0.43	0.32	0.22	0.14	0.09	0.05	0.03	0.01
8	3	1.00	0.99	0.98	0.94	0.89	0.81	0.71	0.59	0.48	0.36	0.26	0.17	0.11	0.06
9	1	0.93	0.77	0.60	0.44	0.30	0.20	0.12	0.07	0.04	0.02	0.01	0.00	0.00	0.00
9	2	0.99	0.95	0.86	0.74	0.60	0.46	0.34	0.23	0.15	0.09	0.05	0.03	0.01	0.00
9	3	1.00	0.99	0.97	0.91	0.83	0.73	0.61	0.48	0.36	0.25	0.17	0.10	0.05	0.03
10	1	0.91	0.74	0.54	0.38	0.24	0.15	0.09	0.05	0.02	0.01	0.00	0.00	0.00	0.00
10	2	0.99	0.93	0.82	0.68	0.53	0.38	0.26	0.17	0.10	0.05	0.03	0.01	0.00	0.00
10	3	1.00	0.99	0.95	0.88	0.78	0.65	0.51	0.38	0.27	0.17	0.10	0.05	0.03	0.01
15	2	0.96	0.82	0.60	0.40	0.24	0.13	0.06	0.03	0.01	0.00	0.00	0.00	0.00	0.00
15	3	0.99	0.94	0.82	0.65	0.46	0.30	0.17	0.09	0.04	0.02	0.01	0.00	0.00	0.00
15	4	1.00	0.99	0.94	0.84	0.68	0.52	0.35	0.22	0.12	0.06	0.03	0.01	0.00	0.00
15	5	1.00	1.00	0.98	0.94	0.85	0.72	0.56	0.40	0.26	0.15	0.08	0.03	0.01	0.00
20	2	0.92	0.68	0.40	0.21	0.09	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	3	0.98	0.87	0.65	0.41	0.23	0.11	0.04	0.02	0.00	0.00	0.00	0.00	0.00	0.00
20	4	1.00	0.96	0.83	0.63	0.41	0.24	0.12	0.05	0.02	0.01	0.00	0.00	0.00	0.00
20	5	1.00	0.99	0.93	0.80	0.62	0.42	0.25	0.13	0.06	0.02	0.01	0.00	0.00	0.00
20	6	1.00	1.00	0.98	0.91	0.79	0.61	0.42	0.25	0.13	0.06	0.02	0.01	0.00	0.00
30	3	0.94	0.65	0.32	0.12	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	4	0.98	0.82	0.52	0.26	0.10	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	5	1.00	0.93	0.71	0.43	0.20	0.08	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00
30	6	1.00	0.97	0.85	0.61	0.35	0.16	0.06	0.02	0.00	0.00	0.00	0.00	0.00	0.00
30	7	1.00	0.99	0.93	0.76	0.51	0.28	0.12	0.04	0.01	0.00	0.00	0.00	0.00	0.00
30	8	1.00	1.00	0.97	0.87	0.67	0.43	0.22	0.09	0.03	0.01	0.00	0.00	0.00	0.00
50	5	0.96	0.62	0.22	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	6	0.99	0.77	0.36	0.10	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	7	1.00	0.88	0.52	0.19	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	8	1.00	0.94	0.67	0.31	0.09	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	9	1.00	0.98	0.79	0.44	0.16	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	10	1.00	0.99	0.88	0.58	0.26	0.08	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	11	1.00	1.00	0.94	0.71	0.38	0.14	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00
100	8	0.94	0.32	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	9	0.97	0.45	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	10	0.99	0.58	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	11	1.00	0.70	0.16	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	12	1.00	0.80	0.25	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	13	1.00	0.88	0.35	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	14	1.00	0.93	0.46	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	15	1.00	0.96	0.57	0.13	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	16	1.00	0.98	0.67	0.19	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	17	1.00	0.99	0.76	0.27	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	18	1.00	1.00	0.84	0.36	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

PROBABILITY OF ACCEPTANCE IS A FUNCTION OF LOT SIZE FOR ATTRIBUTES PLANS USING DISCRETE DATA. FOR VARIABLE LOT SIZES, IT WILL BE NECESSARY TO PLOT BOUNDING OPERATING CHARACTERISTIC CURVES. THE VALUES IN THIS TABLE ARE APPROPRIATE FOR BOTH SINGLE-LIMIT AND DOUBLE-LIMIT APPLICATIONS AND ARE UNINFLUENCED BY THE DISTRIBUTIONAL FORM OF THE POPULATION.

Table 2. Operating characteristics of attributes acceptance plans with an infinite lot size.

VARIABLES ACCEPTANCE PLANS *** VARIABILITY-UNKNOWN PROCEDURE *** STANDARD DEVIATION METHOD

SAMPLE SIZE (n)	MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M)	MINIMUM ALLOWABLE QUALITY INDEX (k)	PROBABILITY OF ACCEPTANCE FOR SELECTED LEVELS OF LOT PERCENT DEFECTIVE						
			10	20	30	40	50	60	70
3	34	0.556	0.89	0.71	0.52	0.35	0.22	0.12	0.05
3	36	0.492	0.91	0.74	0.56	0.39	0.24	0.13	0.06
3	38	0.425	0.93	0.78	0.60	0.42	0.27	0.15	0.07
3	40	0.357	0.95	0.81	0.64	0.46	0.30	0.17	0.08
3	42	0.287	0.96	0.84	0.68	0.50	0.33	0.20	0.09
3	44	0.216	0.97	0.87	0.71	0.54	0.37	0.22	0.11
3	46	0.145	0.98	0.89	0.75	0.58	0.41	0.26	0.13
3	48	0.073	0.98	0.91	0.79	0.63	0.46	0.29	0.15
4	28	0.660	0.88	0.66	0.44	0.27	0.14	0.06	0.02
4	30	0.600	0.91	0.70	0.48	0.29	0.16	0.07	0.02
4	32	0.540	0.93	0.74	0.52	0.33	0.18	0.08	0.03
4	34	0.480	0.94	0.77	0.56	0.36	0.20	0.10	0.03
4	36	0.420	0.96	0.81	0.60	0.40	0.23	0.11	0.04
4	38	0.360	0.97	0.84	0.64	0.44	0.26	0.13	0.05
4	40	0.300	0.97	0.86	0.69	0.48	0.30	0.15	0.06
4	42	0.240	0.98	0.89	0.72	0.53	0.33	0.18	0.07
4	44	0.180	0.99	0.91	0.76	0.57	0.37	0.20	0.09
4	46	0.120	0.99	0.93	0.79	0.61	0.41	0.23	0.10
5	26	0.692	0.89	0.65	0.40	0.22	0.10	0.04	0.01
5	28	0.632	0.92	0.69	0.45	0.25	0.12	0.04	0.01
5	30	0.572	0.94	0.73	0.49	0.28	0.14	0.05	0.01
5	32	0.513	0.95	0.77	0.54	0.32	0.16	0.06	0.02
5	34	0.455	0.96	0.81	0.58	0.36	0.18	0.07	0.02
5	36	0.397	0.97	0.84	0.63	0.40	0.21	0.09	0.03
5	38	0.339	0.98	0.87	0.67	0.44	0.25	0.11	0.03
5	40	0.282	0.99	0.90	0.71	0.49	0.28	0.13	0.04
5	42	0.225	0.99	0.92	0.75	0.54	0.32	0.15	0.05
5	44	0.169	0.99	0.93	0.79	0.58	0.36	0.18	0.06
6	24	0.740	0.89	0.62	0.35	0.17	0.06	0.02	0.00
6	26	0.678	0.92	0.67	0.40	0.20	0.08	0.02	0.00
6	28	0.616	0.94	0.71	0.45	0.23	0.10	0.03	0.01
6	30	0.558	0.95	0.76	0.49	0.27	0.11	0.04	0.01
6	32	0.500	0.97	0.80	0.55	0.31	0.14	0.05	0.01
6	34	0.442	0.98	0.84	0.60	0.35	0.16	0.06	0.02
6	36	0.386	0.98	0.87	0.64	0.39	0.19	0.07	0.02
6	38	0.329	0.99	0.90	0.69	0.44	0.23	0.09	0.02
6	40	0.274	0.99	0.92	0.74	0.49	0.27	0.11	0.03
6	42	0.219	1.00	0.94	0.78	0.54	0.31	0.13	0.04
7	22	0.796	0.88	0.57	0.29	0.12	0.04	0.01	0.00
7	24	0.732	0.91	0.63	0.34	0.15	0.05	0.01	0.00
7	26	0.670	0.93	0.68	0.39	0.18	0.06	0.02	0.00
7	28	0.610	0.95	0.73	0.44	0.21	0.08	0.02	0.00
7	30	0.550	0.97	0.78	0.50	0.25	0.10	0.03	0.00
7	32	0.492	0.98	0.82	0.55	0.29	0.12	0.04	0.01
7	34	0.435	0.98	0.86	0.61	0.34	0.15	0.05	0.01
7	36	0.379	0.99	0.89	0.66	0.39	0.18	0.06	0.01
7	38	0.324	0.99	0.91	0.71	0.44	0.21	0.07	0.02
7	40	0.269	1.00	0.93	0.75	0.49	0.25	0.09	0.02
8	22	0.792	0.90	0.58	0.28	0.11	0.03	0.01	0.00
8	24	0.727	0.92	0.64	0.33	0.13	0.04	0.01	0.00
8	26	0.665	0.95	0.70	0.38	0.16	0.05	0.01	0.00
8	28	0.604	0.96	0.75	0.44	0.20	0.07	0.01	0.00
8	30	0.545	0.98	0.80	0.50	0.24	0.08	0.02	0.00
8	32	0.488	0.98	0.84	0.56	0.28	0.11	0.03	0.00
8	34	0.431	0.99	0.87	0.62	0.33	0.13	0.04	0.01
8	36	0.375	0.99	0.90	0.67	0.38	0.16	0.05	0.01
8	38	0.320	1.00	0.93	0.72	0.44	0.20	0.06	0.01
9	20	0.855	0.87	0.52	0.22	0.07	0.02	0.00	0.00
9	22	0.788	0.91	0.58	0.27	0.09	0.02	0.00	0.00
9	24	0.724	0.94	0.65	0.32	0.12	0.03	0.01	0.00
9	26	0.661	0.96	0.71	0.38	0.15	0.04	0.01	0.00
9	28	0.601	0.97	0.76	0.44	0.18	0.05	0.01	0.00
9	30	0.542	0.98	0.81	0.50	0.22	0.07	0.01	0.00
9	32	0.484	0.99	0.85	0.56	0.27	0.09	0.02	0.00
9	34	0.428	0.99	0.89	0.62	0.32	0.12	0.03	0.00
9	36	0.373	1.00	0.92	0.68	0.38	0.15	0.04	0.01
10	20	0.853	0.89	0.51	0.21	0.06	0.01	0.00	0.00
10	22	0.786	0.92	0.59	0.26	0.08	0.02	0.00	0.00
10	24	0.721	0.95	0.65	0.31	0.11	0.02	0.00	0.00
10	26	0.659	0.97	0.72	0.37	0.13	0.03	0.01	0.00
10	28	0.598	0.98	0.77	0.43	0.17	0.05	0.01	0.00
10	30	0.539	0.99	0.82	0.50	0.21	0.06	0.01	0.00
10	32	0.482	0.99	0.87	0.57	0.26	0.08	0.02	0.00
10	34	0.426	1.00	0.90	0.63	0.31	0.11	0.02	0.00

THE ACCEPTANCE PROBABILITIES IN THIS TABLE ARE ACCURATE FOR SINGLE-LIMIT PLANS AND ARE APPROXIMATELY CORRECT FOR DOUBLE-LIMIT PLANS. FOR SINGLE-LIMIT PLANS, EITHER THE MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M) OR THE MINIMUM ALLOWABLE QUALITY INDEX (k) MAY BE SPECIFIED. FOR DOUBLE-LIMIT PLANS, ONLY THE MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE SHOULD BE USED.

Table 3. Operating characteristics of variables acceptance plans (standard deviation method).

Because variables plans deal with continuous data, there are an infinite number of plans that might be used and it will occasionally be necessary to interpolate between the acceptance parameters listed in Table 3. The operating characteristic tables for variables plans include a wide range of acceptance plans and, like the attributes tables, not all plans will be suitable for all situations.

Appendix C of the new standard provides a more complete table for the estimation of lot percent defective (standard deviation method). This table is the equivalent of Table B5 in Military Standard 414 on variables sampling (8) except that it includes several useful sample sizes that were omitted in both Military Standard 414 and AASHTO Standard R9. The new table consists of five sections, one of which is presented as Table 4.

The percent defective estimation tables in Appendix C of the new standard cover a wide range of sample sizes, considerably more than would ever be used in a single acceptance procedure. For acceptance procedures that make use of only one or two sample sizes, it is possible to construct much more compact tables such as Table 5. With this format, there is a separate short table for each sample size.

Appendix D of the new standard contains two tables that have been developed for use with variables procedures based on the range as the measure of variability. The first, presented as Table 6, gives the operating characteristics for a wide selection of range plans. The largest sample size included in this table is n = 15 because, above that sample size, range plans are considerably less efficient than standard deviation plans. The second, presented as Table 7, gives the estimate of lot percent defective associated with the quality index (Q) computed by the

VARIABILITY-UNKNOWN PROCEDURE — STANDARD DEVIATION METHOD

QUALITY INDEX (Q)	ESTIMATED LOT PERCENT DEFECTIVE FOR SELECTED SAMPLE SIZES												
	3	4	5	6	7	8	9	10	15	20	30	50	100
0.0	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00
0.01	49.72	49.67	49.64	49.63	49.63	49.62	49.62	49.62	49.61	49.61	49.60	49.60	49.60
0.02	49.45	49.33	49.28	49.27	49.25	49.24	49.24	49.23	49.22	49.21	49.21	49.21	49.20
0.03	49.17	49.00	48.93	48.90	48.88	48.86	48.85	48.85	48.83	48.82	48.81	48.81	48.81
0.04	48.90	48.67	48.58	48.53	48.50	48.49	48.47	48.46	48.44	48.43	48.42	48.41	48.41
0.05	48.62	48.33	48.22	48.16	48.13	48.11	48.09	48.08	48.05	48.04	48.02	48.02	48.01
0.06	48.35	48.00	47.86	47.80	47.75	47.73	47.71	47.70	47.66	47.64	47.63	47.62	47.61
0.07	48.07	47.67	47.51	47.43	47.38	47.35	47.33	47.31	47.27	47.25	47.24	47.22	47.22
0.08	47.79	47.33	47.15	47.06	47.01	46.97	46.95	46.93	46.88	46.86	46.84	46.83	46.82
0.09	47.52	47.00	46.80	46.70	46.63	46.59	46.57	46.54	46.49	46.47	46.45	46.43	46.42
0.10	47.24	46.67	46.44	46.33	46.26	46.22	46.18	46.16	46.10	46.08	46.05	46.04	46.03
0.11	46.96	46.33	46.09	45.96	45.89	45.84	45.80	45.78	45.71	45.69	45.66	45.64	45.63
0.12	46.69	46.00	45.73	45.60	45.51	45.46	45.42	45.40	45.32	45.29	45.27	45.25	45.24
0.13	46.41	45.67	45.38	45.23	45.14	45.08	45.04	45.01	44.94	44.90	44.88	44.86	44.84
0.14	46.13	45.33	45.02	44.86	44.77	44.71	44.66	44.63	44.55	44.51	44.48	44.46	44.45
0.15	45.85	45.00	44.67	44.50	44.40	44.33	44.29	44.25	44.16	44.13	44.09	44.07	44.05
0.16	45.58	44.67	44.31	44.13	44.03	43.96	43.91	43.87	43.76	43.74	43.70	43.68	43.66
0.17	45.30	44.33	43.96	43.77	43.65	43.58	43.53	43.49	43.39	43.35	43.31	43.29	43.27
0.18	45.02	44.00	43.60	43.40	43.28	43.21	43.15	43.11	43.01	42.96	42.92	42.89	42.88
0.19	44.74	43.67	43.25	43.04	42.91	42.83	42.77	42.73	42.62	42.57	42.53	42.50	42.48
0.20	44.46	43.33	42.90	42.68	42.54	42.46	42.40	42.35	42.24	42.19	42.15	42.11	42.09
0.21	44.18	43.00	42.54	42.31	42.17	42.08	42.02	41.97	41.85	41.80	41.76	41.73	41.70
0.22	43.90	42.67	42.19	41.95	41.80	41.71	41.64	41.60	41.47	41.42	41.37	41.34	41.31
0.23	43.62	42.33	41.84	41.59	41.44	41.34	41.27	41.22	41.09	41.03	40.98	40.95	40.93
0.24	43.34	42.00	41.48	41.22	41.07	40.97	40.89	40.84	40.71	40.65	40.60	40.56	40.54
0.25	43.05	41.67	41.13	40.86	40.70	40.59	40.52	40.47	40.33	40.27	40.22	40.18	40.15
0.26	42.77	41.33	40.78	40.50	40.33	40.22	40.15	40.09	39.95	39.89	39.83	39.79	39.77
0.27	42.49	41.00	40.43	40.14	39.97	39.85	39.77	39.72	39.57	39.50	39.45	39.41	39.38
0.28	42.20	40.67	40.08	39.78	39.60	39.48	39.40	39.34	39.19	39.12	39.07	39.03	39.00
0.29	41.92	40.33	39.72	39.42	39.23	39.11	39.03	38.97	38.81	38.75	38.69	38.65	38.62
0.30	41.63	40.00	39.37	39.06	38.87	38.75	38.66	38.60	38.44	38.37	38.31	38.26	38.24
0.31	41.35	39.67	39.02	38.70	38.50	38.38	38.29	38.23	38.06	37.99	37.93	37.89	37.86
0.32	41.06	39.33	38.67	38.34	38.14	38.01	37.92	37.86	37.69	37.61	37.55	37.51	37.48
0.33	40.77	39.00	38.32	37.98	37.78	37.65	37.55	37.49	37.31	37.24	37.18	37.13	37.10
0.34	40.49	38.67	37.97	37.62	37.42	37.28	37.19	37.12	36.94	36.87	36.80	36.75	36.72
0.35	40.20	38.33	37.62	37.27	37.05	36.92	36.82	36.75	36.57	36.49	36.43	36.38	36.35
0.36	39.91	38.00	37.26	36.91	36.69	36.55	36.46	36.38	36.20	36.12	36.05	36.01	35.97
0.37	39.62	37.67	36.93	36.55	36.33	36.19	36.09	36.02	35.83	35.75	35.68	35.63	35.60
0.38	39.33	37.33	36.58	36.20	35.98	35.83	35.73	35.65	35.46	35.38	35.31	35.26	35.23
0.39	39.03	37.00	36.23	35.84	35.62	35.47	35.37	35.29	35.10	35.01	34.94	34.89	34.86
0.40	38.74	36.67	35.86	35.49	35.26	35.11	35.00	34.93	34.73	34.65	34.58	34.52	34.49
0.41	38.45	36.33	35.54	35.14	34.90	34.75	34.64	34.57	34.37	34.28	34.21	34.16	34.12
0.42	38.15	36.00	35.19	34.79	34.55	34.39	34.29	34.21	34.00	33.92	33.85	33.79	33.76
0.43	37.85	35.67	34.85	34.43	34.19	34.04	33.93	33.85	33.64	33.56	33.49	33.43	33.39
0.44	37.56	35.33	34.50	34.08	33.84	33.68	33.57	33.49	33.28	33.20	33.12	33.07	33.03
0.45	37.26	35.00	34.16	33.73	33.49	33.33	33.21	33.13	32.92	32.84	32.76	32.71	32.67
0.46	36.96	34.67	33.81	33.38	33.13	32.97	32.86	32.78	32.57	32.48	32.40	32.35	32.31
0.47	36.66	34.33	33.47	33.04	32.78	32.62	32.51	32.42	32.21	32.12	32.04	31.99	31.95
0.48	36.35	34.00	33.12	32.69	32.43	32.27	32.15	32.07	31.85	31.77	31.69	31.63	31.60
0.49	36.05	33.67	32.78	32.34	32.08	31.92	31.80	31.72	31.50	31.41	31.33	31.28	31.24
0.50	35.75	33.33	32.44	32.00	31.74	31.57	31.45	31.37	31.15	31.06	30.98	30.93	30.89
0.51	35.44	33.00	32.10	31.65	31.39	31.22	31.10	31.02	30.80	30.71	30.63	30.57	30.54
0.52	35.13	32.67	31.76	31.31	31.04	30.87	30.76	30.67	30.45	30.36	30.28	30.23	30.19
0.53	34.82	32.33	31.42	30.96	30.70	30.53	30.41	30.32	30.10	30.01	29.93	29.88	29.84
0.54	34.51	32.00	31.08	30.62	30.36	30.18	30.07	29.98	29.76	29.67	29.59	29.53	29.49
0.55	34.20	31.67	30.74	30.28	30.01	29.84	29.72	29.64	29.41	29.32	29.24	29.19	29.15
0.56	33.88	31.33	30.40	29.94	29.67	29.50	29.38	29.29	29.07	28.98	28.90	28.85	28.81
0.57	33.57	31.00	30.06	29.60	29.33	29.16	29.04	28.95	28.73	28.64	28.56	28.51	28.47
0.58	33.25	30.67	29.73	29.26	28.99	28.82	28.70	28.61	28.39	28.30	28.22	28.17	28.13
0.59	32.93	30.33	29.39	28.93	28.66	28.48	28.36	28.28	28.05	27.96	27.89	27.83	27.79
0.60	32.61	30.00	29.05	28.59	28.32	28.15	28.03	27.94	27.72	27.63	27.55	27.50	27.46
0.61	32.28	29.67	28.72	28.25	27.98	27.81	27.69	27.60	27.38	27.29	27.22	27.16	27.13
0.62	31.96	29.33	28.39	27.92	27.65	27.48	27.36	27.27	27.05	26.96	26.89	26.83	26.80
0.63	31.63	29.00	28.05	27.59	27.32	27.15	27.03	26.94	26.72	26.63	26.56	26.50	26.47
0.64	31.30	28.67	27.72	27.26	26.99	26.82	26.70	26.61	26.39	26.31	26.23	26.18	26.14
0.65	30.97	28.33	27.39	26.93	26.66	26.49	26.37	26.28	26.07	25.98	25.90	25.85	25.82
0.66	30.63	28.00	27.06	26.60	26.33	26.16	26.04	25.96	25.74	25.66	25.58	25.53	25.49
0.67	30.30	27.67	26.73	26.27	26.00	25.83	25.72	25.63	25.42	25.33	25.26	25.21	25.17
0.68	29.96	27.33	26.40	25.94	25.68	25.51	25.39	25.31	25.10	25.01	24.94	24.89	24.86
0.69	29.61	27.00	26.07	25.61	25.35	25.19	25.07	24.99	24.78	24.69	24.62	24.57	24.54
0.70	29.27	26.67	25.74	25.29	25.03	24.86	24.75	24.67	24.46	24.38	24.31	24.26	24.23
0.71	28.92	26.33	25.41	24.96	24.71	24.54	24.43	24.35	24.15	24.08	23.99	23.95	23.91
0.72	28.57	26.00	25.08	24.64	24.39	24.23	24.11	24.03	23.83	23.75	23.68	23.64	23.60
0.73	28.22	25.67	24.76	24.32	24.07	23.91	23.80	23.72	23.52	23.44	23.37	23.33	23.30
0.74	27.86	25.33	24.44	24.00	23.75	23.59	23.49	23.41	23.21	23.13	23.07	23.02	22.99
0.75	27.50	25.00	24.11	23.68	23.44	23.28	23.17	23.10	22.90	22.83	22.76	22.72	22.69
0.76	27.13	24.67	23.78	23.37	23.13	22.97	22.86	22.79	22.60	22.53	22.46	22.42	22.39
0.77	26.76	24.33	23.47	23.05	22.81	22.66	22.56	22.48	22.30	22.23	22.16	22.12	22.09
0.78	26.39	24.00	23.15	22.74	22.51	22.35	22.25	22.18	21.99	21.93	21.86	21.82	21.79
0.79	26.02	23.67	22.83	22.42	22.19	22.04	21.94	21.87	21.70	21.63	21.57	21.53	21.50

NUMBERS IN BODY OF TABLE ARE ESTIMATED LOT PERCENT DEFECTIVE CORRESPONDING TO SPECIFIC VALUES OF QUALITY INDEX AND SAMPLE SIZE. FOR Q VALUES GREATER THAN OR EQUAL TO ZERO, THE PERCENT DEFECTIVE ESTIMATE MAY BE READ DIRECTLY FROM THE TABLE. FOR Q VALUES LESS THAN ZERO, THE TABLE VALUE MUST BE SUBTRACTED FROM 100.

Table 4. First of five tables for estimation of percent defective (standard deviation method).

VARIABILITY-UNKNOWN PROCEDURE — SAMPLE SIZE 5 — STANDARD DEVIATION METHOD

Q	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	50.00	49.64	49.29	48.93	48.58	48.22	47.86	47.51	47.15	46.80
0.1	46.44	46.09	45.73	45.38	45.02	44.67	44.31	43.96	43.60	43.25
0.2	42.90	42.54	42.19	41.84	41.48	41.13	40.78	40.43	40.08	39.72
0.3	39.37	39.02	38.67	38.32	37.97	37.62	37.28	36.93	36.58	36.23
0.4	35.88	35.54	35.19	34.85	34.50	34.16	33.81	33.47	33.12	32.78
0.5	32.44	32.10	31.76	31.42	31.08	30.74	30.40	30.06	29.73	29.39
0.6	29.05	28.72	28.39	28.05	27.72	27.39	27.06	26.73	26.40	26.07
0.7	25.74	25.41	25.09	24.76	24.44	24.11	23.79	23.47	23.15	22.83
0.8	22.51	22.19	21.87	21.56	21.24	20.93	20.62	20.31	20.00	19.69
0.9	19.38	19.07	18.77	18.46	18.16	17.86	17.55	17.25	16.96	16.66
1.0	16.36	16.07	15.78	15.48	15.19	14.91	14.62	14.33	14.05	13.76
1.1	13.48	13.20	12.93	12.65	12.37	12.10	11.83	11.56	11.29	11.02
1.2	10.76	10.50	10.23	9.97	9.72	9.46	9.21	8.96	8.71	8.46
1.3	8.21	7.97	7.73	7.49	7.25	7.02	6.79	6.56	6.33	6.10
1.4	5.88	5.66	5.44	5.23	5.02	4.81	4.60	4.39	4.19	3.99
1.5	3.80	3.61	3.42	3.23	3.05	2.87	2.69	2.52	2.35	2.19
1.6	2.03	1.87	1.72	1.57	1.42	1.28	1.15	1.02	0.89	0.77
1.7	0.66	0.55	0.45	0.36	0.27	0.19	0.12	0.06	0.02	0.00

NUMBERS IN THE BODY OF THE TABLE ARE ESTIMATES OF LOT PERCENT DEFECTIVE CORRESPONDING TO SPECIFIC VALUES OF Q, THE QUALITY INDEX. FOR VALUES OF Q GREATER THAN OR EQUAL TO ZERO, THE ESTIMATE OF PERCENT DEFECTIVE IS READ DIRECTLY FROM THE TABLE. FOR VALUES OF Q LESS THAN ZERO, THE TABLE VALUE MUST BE SUBTRACTED FROM 100.

Table 5. Alternate format for individual tables for estimation of percent defective.

VARIABLES ACCEPTANCE PLANS ### VARIABILITY-UNKNOWN PROCEDURE ### RANGE METHOD

SAMPLE SIZE (n)	MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M)	MINIMUM ALLOWABLE QUALITY INDEX (k)	PROBABILITY OF ACCEPTANCE FOR SELECTED LEVELS OF LOT PERCENT DEFECTIVE						
			10	20	30	40	50	60	70
3	34	0.293	0.89	0.71	0.55	0.35	0.22	0.12	0.05
3	36	0.259	0.91	0.74	0.56	0.39	0.24	0.13	0.06
3	38	0.224	0.93	0.78	0.60	0.42	0.27	0.15	0.07
3	40	0.188	0.94	0.81	0.63	0.46	0.30	0.17	0.08
3	42	0.151	0.96	0.84	0.67	0.50	0.33	0.20	0.09
3	44	0.114	0.97	0.87	0.71	0.54	0.37	0.22	0.11
3	46	0.076	0.98	0.89	0.75	0.58	0.41	0.26	0.13
3	48	0.038	0.98	0.91	0.79	0.63	0.46	0.29	0.16
4	32	0.269	0.90	0.70	0.48	0.30	0.16	0.07	0.03
4	34	0.242	0.92	0.73	0.52	0.33	0.18	0.08	0.03
4	36	0.216	0.94	0.77	0.56	0.36	0.21	0.10	0.04
4	38	0.189	0.95	0.80	0.60	0.40	0.23	0.11	0.04
4	40	0.162	0.96	0.83	0.64	0.44	0.26	0.13	0.05
4	42	0.135	0.97	0.86	0.68	0.48	0.30	0.15	0.06
4	44	0.108	0.98	0.89	0.72	0.52	0.33	0.18	0.07
4	46	0.081	0.99	0.91	0.76	0.57	0.37	0.21	0.09
5	30	0.280	0.89	0.65	0.40	0.22	0.10	0.04	0.01
5	32	0.256	0.91	0.69	0.45	0.26	0.12	0.05	0.01
5	34	0.232	0.93	0.73	0.49	0.28	0.14	0.06	0.02
5	36	0.208	0.95	0.77	0.53	0.32	0.16	0.07	0.02
5	38	0.184	0.96	0.80	0.58	0.36	0.19	0.08	0.02
5	40	0.161	0.97	0.84	0.62	0.40	0.22	0.09	0.03
5	42	0.138	0.98	0.87	0.67	0.44	0.25	0.11	0.04
5	44	0.115	0.98	0.89	0.71	0.49	0.28	0.13	0.05
5	46	0.092	0.99	0.91	0.75	0.53	0.32	0.16	0.06
6	28	0.278	0.88	0.61	0.34	0.17	0.07	0.02	0.01
6	30	0.255	0.91	0.66	0.40	0.20	0.09	0.03	0.01
6	32	0.233	0.93	0.71	0.45	0.24	0.10	0.03	0.01
6	34	0.210	0.95	0.75	0.49	0.27	0.12	0.04	0.01
6	36	0.188	0.96	0.79	0.54	0.31	0.14	0.05	0.01
6	38	0.167	0.97	0.83	0.59	0.35	0.17	0.06	0.02
6	40	0.145	0.98	0.86	0.64	0.40	0.20	0.08	0.02
6	42	0.124	0.99	0.89	0.69	0.44	0.23	0.10	0.03
7	26	0.260	0.90	0.62	0.35	0.16	0.06	0.02	0.00
7	28	0.238	0.93	0.67	0.39	0.19	0.07	0.03	0.00
7	30	0.217	0.95	0.72	0.44	0.22	0.09	0.03	0.01
7	32	0.196	0.96	0.77	0.50	0.26	0.11	0.03	0.01
7	34	0.175	0.97	0.81	0.55	0.30	0.13	0.04	0.01
7	36	0.155	0.98	0.85	0.60	0.34	0.16	0.05	0.01
7	38	0.135	0.99	0.88	0.65	0.39	0.19	0.07	0.01
8	32	0.269	0.88	0.57	0.29	0.12	0.04	0.01	0.00
8	34	0.247	0.91	0.63	0.34	0.14	0.05	0.01	0.00
8	36	0.226	0.94	0.69	0.39	0.17	0.06	0.02	0.00
8	38	0.205	0.96	0.74	0.44	0.21	0.08	0.02	0.00
8	40	0.185	0.97	0.78	0.50	0.25	0.09	0.03	0.00
8	42	0.164	0.98	0.82	0.55	0.29	0.12	0.03	0.01
8	44	0.143	0.99	0.86	0.61	0.34	0.14	0.04	0.01
9	32	0.257	0.90	0.58	0.28	0.11	0.03	0.01	0.00
9	34	0.236	0.92	0.64	0.33	0.13	0.04	0.01	0.00
9	36	0.216	0.95	0.69	0.38	0.16	0.05	0.01	0.00
9	38	0.196	0.96	0.75	0.44	0.20	0.07	0.02	0.00
9	40	0.177	0.98	0.80	0.50	0.24	0.08	0.02	0.00
9	42	0.158	0.98	0.84	0.56	0.28	0.11	0.03	0.00
9	44	0.140	0.99	0.87	0.61	0.33	0.13	0.04	0.01
10	20	0.249	0.87	0.52	0.23	0.07	0.02	0.00	0.00
10	22	0.248	0.91	0.58	0.27	0.10	0.03	0.00	0.00
10	24	0.228	0.93	0.64	0.32	0.12	0.03	0.01	0.00
10	26	0.208	0.95	0.70	0.38	0.15	0.04	0.01	0.00
10	28	0.189	0.97	0.76	0.44	0.19	0.06	0.01	0.00
10	30	0.171	0.98	0.81	0.50	0.23	0.08	0.02	0.00
10	32	0.153	0.99	0.85	0.56	0.27	0.10	0.02	0.00
11	20	0.241	0.88	0.52	0.21	0.07	0.01	0.00	0.00
11	22	0.241	0.91	0.58	0.26	0.09	0.02	0.00	0.00
11	24	0.221	0.94	0.65	0.32	0.11	0.03	0.00	0.00
11	26	0.202	0.96	0.71	0.38	0.14	0.04	0.01	0.00
11	28	0.183	0.97	0.77	0.44	0.18	0.05	0.01	0.00
11	30	0.165	0.98	0.82	0.50	0.22	0.07	0.01	0.00
11	32	0.148	0.99	0.86	0.56	0.27	0.09	0.02	0.00
12	20	0.235	0.89	0.51	0.21	0.06	0.01	0.00	0.00
12	22	0.235	0.92	0.59	0.26	0.09	0.02	0.00	0.00
12	24	0.215	0.95	0.65	0.31	0.11	0.03	0.00	0.00
12	26	0.197	0.97	0.72	0.37	0.13	0.03	0.01	0.00
12	28	0.179	0.98	0.77	0.43	0.17	0.05	0.01	0.00
12	30	0.161	0.99	0.82	0.50	0.21	0.06	0.01	0.00
13	20	0.249	0.89	0.51	0.20	0.05	0.01	0.00	0.00
13	22	0.229	0.93	0.59	0.25	0.07	0.01	0.00	0.00
13	24	0.210	0.95	0.66	0.31	0.10	0.02	0.00	0.00
13	26	0.192	0.97	0.72	0.37	0.13	0.03	0.00	0.00
13	28	0.175	0.98	0.78	0.43	0.16	0.04	0.01	0.00
13	30	0.157	0.99	0.83	0.50	0.21	0.06	0.01	0.00
14	20	0.244	0.90	0.51	0.19	0.05	0.01	0.00	0.00
14	22	0.225	0.93	0.59	0.24	0.07	0.01	0.00	0.00
14	24	0.206	0.96	0.66	0.30	0.09	0.02	0.00	0.00
14	26	0.188	0.97	0.73	0.36	0.12	0.03	0.00	0.00
14	28	0.171	0.98	0.79	0.43	0.16	0.04	0.01	0.00
14	30	0.154	0.99	0.84	0.50	0.21	0.05	0.01	0.00
15	20	0.240	0.90	0.51	0.19	0.04	0.01	0.00	0.00
15	22	0.221	0.94	0.59	0.24	0.06	0.01	0.00	0.00
15	24	0.202	0.96	0.67	0.30	0.08	0.02	0.00	0.00
15	26	0.185	0.98	0.73	0.36	0.11	0.03	0.00	0.00
15	28	0.168	0.99	0.79	0.43	0.15	0.03	0.00	0.00

THE ACCEPTANCE PROBABILITIES IN THIS TABLE HAVE BEEN COMPUTED BY INTERPOLATION IN THE NONCENTRAL T DISTRIBUTION USING MOMINTEGER DEGREES OF FREEDOM ASSOCIATED WITH RANGE ESTIMATES OF VARIABILITY. THESE PROBABILITY VALUES ARE QUITE ACCURATE FOR SINGLE-LIMIT PLANS AND APPROXIMATELY CORRECT FOR DOUBLE-LIMIT PLANS. FOR SINGLE-LIMIT PLANS, EITHER THE MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M) OR THE MINIMUM ALLOWABLE QUALITY INDEX (k) MAY BE SPECIFIED. FOR DOUBLE-LIMIT PLANS, ONLY THE MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE SHOULD BE USED.

Table 6. Operating characteristics of variables acceptance plans (range method).

VARIABILITY-UNKNOWN PROCEDURE — RANGE METHOD

QUALITY INDEX (Q)	ESTIMATED LOT PERCENT DEFECTIVE FOR SELECTED SAMPLE SIZES												
	3	4	5	6	7	8	9	10	11	12	13	14	15
0.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
0.01	49.5	49.3	49.1	49.0	48.9	48.9	48.8	48.8	48.7	48.7	48.7	48.7	48.6
0.02	49.0	48.5	48.2	48.1	47.9	47.8	47.7	47.6	47.5	47.4	47.4	47.3	47.3
0.03	48.4	47.8	47.4	47.1	46.9	46.7	46.5	46.4	46.2	46.1	46.1	46.0	45.9
0.04	47.9	47.0	46.5	46.1	45.8	45.6	45.3	45.2	45.0	44.9	44.7	44.6	44.5
0.05	47.4	46.3	45.6	45.1	44.8	44.5	44.2	44.0	43.8	43.6	43.4	43.3	43.2
0.06	46.9	45.6	44.8	44.2	43.7	43.4	43.0	42.8	42.5	42.3	42.1	42.0	41.8
0.07	46.3	44.8	43.9	43.2	42.7	42.3	41.9	41.6	41.3	41.0	40.8	40.6	40.5
0.08	45.8	44.1	43.0	42.2	41.6	41.2	40.7	40.4	40.1	39.8	39.6	39.3	39.1
0.09	45.3	43.3	42.1	41.3	40.6	40.1	39.6	39.2	38.8	38.5	38.3	38.0	37.8
0.10	44.7	42.6	41.3	40.3	39.6	39.0	38.5	38.0	37.6	37.3	37.0	36.7	36.5
0.11	44.2	41.8	40.4	39.4	38.5	37.9	37.3	36.9	36.4	36.1	35.8	35.4	35.2
0.12	43.7	41.1	39.5	38.4	37.5	36.8	36.2	35.7	35.2	34.9	34.5	34.2	33.9
0.13	43.1	40.4	38.7	37.5	36.5	35.8	35.1	34.6	34.1	33.6	33.3	32.9	32.6
0.14	42.6	39.6	37.8	36.5	35.5	34.7	34.0	33.4	32.9	32.5	32.1	31.7	31.4
0.15	42.1	38.9	36.9	35.6	34.5	33.7	32.9	32.3	31.7	31.3	30.9	30.5	30.1
0.16	41.5	38.1	36.1	34.6	33.5	32.6	31.8	31.2	30.6	30.1	29.7	29.3	28.9
0.17	41.0	37.4	35.2	33.7	32.5	31.6	30.8	30.1	29.5	29.0	28.5	28.1	27.7
0.18	40.4	36.6	34.4	32.8	31.5	30.6	29.7	29.0	28.4	27.8	27.4	26.9	26.5
0.19	39.9	35.9	33.5	31.9	30.6	29.5	28.6	27.9	27.3	26.7	26.2	25.8	25.4
0.20	39.3	35.2	32.7	30.9	29.6	28.5	27.6	26.9	26.2	25.6	25.1	24.7	24.2
0.21	38.8	34.4	31.8	30.0	28.6	27.5	26.6	25.8	25.1	24.6	24.1	23.6	23.1
0.22	38.2	33.7	31.0	29.1	27.7	26.6	25.6	24.8	24.1	23.5	23.0	22.5	22.1
0.23	37.7	32.9	30.2	28.2	26.8	25.6	24.6	23.8	23.1	22.5	21.9	21.4	21.0
0.24	37.1	32.2	29.3	27.3	25.8	24.6	23.6	22.8	22.1	21.5	20.9	20.4	20.0
0.25	36.5	31.4	28.5	26.5	24.9	23.7	22.7	21.8	21.1	20.5	19.9	19.4	19.0
0.26	35.9	30.7	27.7	25.6	24.0	22.8	21.7	20.9	20.1	19.5	18.9	18.4	18.0
0.27	35.4	29.9	26.9	24.7	23.1	21.9	20.8	19.9	19.2	18.5	18.0	17.5	17.0
0.28	34.8	29.2	26.0	23.9	22.2	21.0	19.9	19.0	18.3	17.6	17.1	16.5	16.1
0.29	34.2	28.4	25.2	23.0	21.4	20.1	19.0	18.1	17.4	16.7	16.2	15.6	15.2
0.30	33.6	27.7	24.4	22.2	20.5	19.2	18.1	17.2	16.5	15.8	15.3	14.8	14.3
0.31	33.0	27.0	23.6	21.4	19.6	18.4	17.3	16.4	15.6	15.0	14.5	13.9	13.5
0.32	32.3	26.2	22.8	20.5	18.8	17.5	16.4	15.6	14.8	14.2	13.6	13.1	12.7
0.33	31.7	25.5	22.0	19.7	18.0	16.7	15.6	14.7	14.0	13.4	12.8	12.3	11.9
0.34	31.1	24.7	21.2	18.9	17.2	15.9	14.8	14.0	13.2	12.6	12.1	11.6	11.1
0.35	30.4	24.0	20.5	18.1	16.4	15.1	14.0	13.2	12.5	11.8	11.3	10.8	10.4
0.36	29.8	23.2	19.7	17.4	15.6	14.4	13.3	12.4	11.7	11.1	10.6	10.1	9.7
0.37	29.1	22.5	18.9	16.6	14.9	13.6	12.5	11.7	11.0	10.4	9.9	9.4	9.1
0.38	28.5	21.7	18.1	15.8	14.1	12.9	11.8	11.0	10.3	9.7	9.3	8.8	8.4
0.39	27.8	21.0	17.4	15.1	13.4	12.2	11.1	10.3	9.7	9.1	8.6	8.2	7.8
0.40	27.1	20.2	16.6	14.4	12.7	11.5	10.5	9.7	9.0	8.5	8.0	7.6	7.2
0.41	26.4	19.4	15.9	13.6	12.0	10.8	9.8	9.1	8.4	7.9	7.4	7.0	6.7
0.42	25.6	18.7	15.2	12.9	11.3	10.2	9.2	8.5	7.8	7.3	6.9	6.5	6.2
0.43	24.9	17.9	14.4	12.2	10.7	9.5	8.6	7.9	7.3	6.8	6.4	6.0	5.7
0.44	24.1	17.2	13.7	11.6	10.0	8.9	8.0	7.3	6.7	6.3	5.9	5.5	5.2
0.45	23.3	16.4	13.0	10.9	9.4	8.3	7.4	6.8	6.2	5.8	5.4	5.0	4.8
0.46	22.5	15.7	12.3	10.2	8.8	7.8	6.9	6.3	5.7	5.3	4.9	4.6	4.3
0.47	21.7	14.9	11.6	9.6	8.2	7.2	6.4	5.8	5.3	4.9	4.5	4.2	4.0
0.48	20.8	14.1	11.0	9.0	7.6	6.7	5.9	5.3	4.8	4.4	4.1	3.8	3.6
0.49	19.9	13.4	10.3	8.4	7.1	6.2	5.4	4.9	4.4	4.0	3.7	3.5	3.2
0.50	19.0	12.6	9.6	7.8	6.5	5.7	5.0	4.4	4.0	3.7	3.4	3.1	2.9
0.51	18.0	11.9	9.0	7.2	6.0	5.2	4.5	4.0	3.7	3.3	3.1	2.8	2.6
0.52	17.0	11.1	8.3	6.7	5.5	4.8	4.1	3.7	3.3	3.0	2.8	2.5	2.4
0.53	15.9	10.3	7.7	6.2	5.1	4.3	3.8	3.3	3.0	2.7	2.5	2.3	2.1
0.54	14.7	9.6	7.1	5.6	4.6	3.9	3.4	3.0	2.7	2.4	2.2	2.0	1.9
0.55	13.5	8.8	6.5	5.1	4.2	3.6	3.0	2.7	2.4	2.2	2.0	1.8	1.6
0.56	12.1	8.0	5.9	4.7	3.8	3.2	2.7	2.4	2.1	1.9	1.7	1.6	1.5
0.57	10.5	7.2	5.4	4.2	3.4	2.9	2.4	2.1	1.9	1.7	1.5	1.4	1.3
0.58	8.6	6.5	4.8	3.8	3.0	2.6	2.2	1.9	1.7	1.5	1.3	1.2	1.1
0.59	6.2	5.7	4.3	3.3	2.7	2.3	1.9	1.6	1.5	1.3	1.2	1.0	1.0
0.60	1.4	4.9	3.7	2.9	2.4	2.1	1.7	1.4	1.3	1.1	1.0	0.9	0.8
0.61	0.0	4.1	3.2	2.6	2.1	1.7	1.4	1.2	1.1	1.0	0.9	0.8	0.7
0.62	0.0	3.3	2.8	2.2	1.8	1.5	1.2	1.1	0.9	0.8	0.7	0.7	0.6
0.63	0.0	2.5	2.3	1.9	1.5	1.3	1.1	0.9	0.8	0.7	0.6	0.6	0.5
0.64	0.0	1.7	1.9	1.6	1.3	1.1	0.9	0.8	0.7	0.6	0.5	0.5	0.4
0.65	0.0	0.9	1.5	1.3	1.1	0.9	0.7	0.6	0.6	0.5	0.4	0.4	0.4
0.66	0.0	0.0	1.1	1.0	0.9	0.7	0.6	0.5	0.5	0.4	0.4	0.3	0.3
0.67	0.0	0.0	0.8	0.8	0.7	0.6	0.5	0.4	0.4	0.3	0.3	0.3	0.2
0.68	0.0	0.0	0.4	0.6	0.5	0.5	0.4	0.3	0.3	0.3	0.2	0.2	0.2
0.69	0.0	0.0	0.2	0.4	0.4	0.4	0.3	0.3	0.2	0.2	0.2	0.2	0.2
0.70	0.0	0.0	0.0	0.3	0.3	0.3	0.2	0.2	0.2	0.2	0.2	0.1	0.1
0.71	0.0	0.0	0.0	0.1	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.1
0.72	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.73	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.74	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.0	0.0	0.0
0.75	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.76	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.77	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.78	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.79	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0

NUMBERS IN BODY OF TABLE ARE ESTIMATES OF LOT PERCENT DEFECTIVE CORRESPONDING TO SPECIFIC VALUES OF QUALITY INDEX AND SAMPLE SIZE. FOR Q VALUES GREATER THAN OR EQUAL TO ZERO, THE PERCENT DEFECTIVE ESTIMATE MAY BE READ DIRECTLY FROM THE TABLE. FOR Q VALUES LESS THAN ZERO, THE TABLE VALUE MUST BE SUBTRACTED FROM 100.

Table 7. Table for estimation of percent defective (range method).

range method in accordance with Equations 3 and 4. Because the range tends to be larger than the standard deviation, the Q values tend to be smaller, and the table is more compact than its counterpart for the standard deviation method. Also, because it is believed that some precision is lost in adapting the standard deviation algorithms to construct the range table, the percent defective estimates in the body of the table have been printed to only a single decimal place.

$$Q_L = \frac{\bar{X} - L}{R} \tag{3}$$

$$Q_U = \frac{U - \bar{X}}{R} \tag{4}$$

in which

Q = quality index

$\bar{X}$ = sample mean

R = sample range, difference between largest and smallest values in the sample

L, U = lower and upper specification limits outside of which the material or work is defined to be defective

Still another useful format for operating characteristic tables is presented in Table 8, although this particular version has not been included in the new standard. Whereas the more customary format lists lot percent defective in the heading of the table and probability of acceptance in the body of the table, this version does just the opposite. The advantage of this format is that it always provides an ample number of

VARIABLES ACCEPTANCE PLANS *** VARIABILITY-UNKNOWN PROCEDURE *** STANDARD DEVIATION METHOD

SAMPLE SIZE (n)	MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M)	MINIMUM ALLOWABLE QUALITY INDEX (k)	LOT PERCENT DEFECTIVE VALUES PRODUCING THE LISTED ACCEPTANCE PROBABILITIES								
			0.99	0.95	0.90	0.80	0.50	0.20	0.10	0.05	0.01
3	30	0.679	1	4	7	12	28	48	59	68	81
3	35	0.524	3	7	10	16	32	53	63	71	84
3	40	0.357	4	10	14	21	38	58	67	75	86
3	45	0.181	6	13	18	26	44	63	72	79	88
4	25	0.750	2	5	8	12	25	42	52	60	74
4	30	0.600	3	7	10	15	29	47	56	64	76
4	35	0.450	5	10	14	19	34	51	60	68	79
4	40	0.300	7	13	17	24	39	56	65	71	82
4	45	0.150	9	17	22	29	44	61	69	76	85
5	25	0.723	3	6	9	13	25	40	49	56	69
5	30	0.572	5	9	12	17	30	45	53	60	72
5	35	0.426	7	12	16	21	34	50	58	65	76
5	40	0.282	9	15	20	26	40	55	63	69	79
5	45	0.141	12	19	24	30	45	60	67	73	82
6	20	0.869	2	5	7	11	21	34	42	49	61
6	25	0.709	4	7	10	14	25	39	47	53	65
6	30	0.558	6	10	14	18	30	44	51	58	69
6	35	0.414	8	14	17	23	35	49	56	62	73
6	40	0.274	11	17	21	27	40	54	61	66	76
7	20	0.862	3	6	8	11	21	33	40	46	58
7	25	0.701	5	8	11	15	25	38	45	51	62
7	30	0.550	7	11	15	19	30	43	50	56	66
7	35	0.407	9	15	18	23	35	48	55	60	70
7	40	0.269	12	19	23	28	40	53	59	65	74
8	20	0.858	3	6	9	12	20	32	38	44	55
8	25	0.696	5	9	12	16	25	37	44	49	60
8	30	0.545	8	12	15	20	30	42	48	54	64
8	35	0.403	11	16	19	24	35	47	53	59	68
9	20	0.855	4	7	9	12	20	31	37	43	53
9	25	0.692	6	10	12	16	25	36	42	48	58
9	30	0.542	9	13	16	20	30	41	47	53	62
9	35	0.400	12	17	20	25	35	46	52	57	66
10	20	0.853	4	7	10	13	20	30	36	41	51
10	25	0.690	7	10	13	17	25	35	41	46	56
10	30	0.539	9	14	17	21	30	41	46	51	60
10	35	0.398	12	18	21	25	35	46	51	56	65
15	15	1.037	4	6	8	10	15	23	27	31	39
15	20	0.848	6	9	11	14	20	28	33	37	45
15	25	0.683	9	13	15	18	25	33	38	42	50
15	30	0.533	12	16	19	22	30	39	43	47	55
20	15	1.036	5	7	8	10	15	22	25	29	35
20	20	0.846	7	10	12	15	20	27	31	34	41
20	25	0.680	10	14	16	19	25	32	36	40	46
30	15	1.036	6	8	9	11	15	20	23	26	31
30	20	0.844	9	12	13	15	20	26	29	31	37
30	25	0.678	13	16	18	20	25	31	34	37	42
50	10	1.277	4	6	6	8	10	13	15	17	20
50	15	1.036	7	9	10	12	15	19	21	23	27
50	20	0.843	11	13	15	16	20	24	27	29	32
100	10	1.279	5	7	7	8	10	12	14	15	17
100	15	1.036	9	11	12	13	15	18	19	20	23
100	20	0.842	13	15	16	17	20	23	25	26	29

THE ACCEPTANCE PROBABILITIES IN THE HEADING OF THIS TABLE ARE ACCURATE FOR SINGLE-LIMIT PLANS AND ARE APPROXIMATELY CORRECT FOR DOUBLE-LIMIT PLANS. FOR SINGLE LIMIT APPLICATIONS, EITHER THE MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M) OR THE MINIMUM ALLOWABLE QUALITY INDEX (k) MAY BE SPECIFIED. FOR DOUBLE-LIMIT APPLICATIONS, ONLY THE MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE SHOULD BE USED.

Table 8. Alternate format for operating characteristic table for variables plans.

plotting points spaced conveniently throughout the length of each OC curve, a refinement that is especially useful when a wide range of sample sizes is included in a single table. This approach is appropriate primarily for variables plans, but is also suitable for attributes plans when the lot size is divisible by 100.

OPERATING CHARACTERISTICS OF DOUBLE-LIMIT PLANS

Acceptance plans having both lower and upper limits are referred to as double-limit plans. The operating characteristics for attributes plans provided by Tables 1 and 2 are correct for both single-limit and double-limit plans. For double-limit variables plans, there is no unique operating characteristic curve because probability of acceptance is influenced in part by the manner in which the percent defective is distributed between the two tails of the population. There exists, instead, a band of OC curves for each double-limit variables plan. It has been found (11, p. 246), however, that this band is quite narrow and that the single-limit OC curves are sufficiently accurate for most double-limit applications. Table 9, generated by computer simulation, provides a convincing demonstration of this fortunate property.

UNDERLYING THEORETICAL PRINCIPLES

The operating characteristics for attributes acceptance plans are computed by means of the hypergeometric formula:

$$P = \frac{\sum_{x=0}^{x=C} C_{d,x} C_{N-d,n-x}}{C_{N,n}} \qquad (5)$$

VARIABILITY-UNKNOWN PROCEDURE — STANDARD DEVIATION METHOD

SAMPLE SIZE	MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE	PERCENT DEFECTIVE LOWER TAIL	PERCENT DEFECTIVE UPPER TAIL	PERCENT DEFECTIVE TOTAL	PROBABILITY OF ACCEPTANCE SINGLE LIMIT (COMPUTED)	PROBABILITY OF ACCEPTANCE DOUBLE LIMIT (SIMULATED)
3	42	0	10	10	0.96	0.96
3	42	5	5	10	0.96	0.96
3	38	20	0	20	0.78	0.78
3	38	10	10	20	0.78	0.77
3	34	10	50	60	0.12	0.11
3	34	25	35	60	0.12	0.12
5	36	10	0	10	0.97	0.97
5	36	5	5	10	0.97	0.98
5	32	0	30	30	0.54	0.54
5	32	15	15	30	0.54	0.52
5	26	60	0	60	0.04	0.04
5	26	30	30	60	0.04	0.03
10	22	0	10	10	0.92	0.93
10	22	5	5	10	0.92	0.92
10	24	20	0	20	0.65	0.65
10	24	10	10	20	0.65	0.65
10	28	10	30	40	0.17	0.15
10	28	20	20	40	0.17	0.15

EACH SIMULATION RESULT WAS OBTAINED BY INDEPENDENTLY GENERATING 5000 RANDOM SAMPLES OF THE APPROPRIATE SIZE FROM A CONTINUOUS NORMAL POPULATION.

Table 9. Demonstration that single-limit operating characteristic curves are sufficiently accurate for most double-limit variables acceptance plans.

in which

P = probability of acceptance

N = population (lot) size

n = sample size

d = number of defects in the population

c = acceptance number, maximum allowable number of defective items in the sample

$C_{m,n}$ = number of possible combinations of m items taken n at a time = $m!/(n!(m - n)!)$

x = summation variable

In terms of the hypergeometric distribution, the lot percent defective would be expressed as 100d/N. This distribution was used to develop Table 1.

As the population size increases, the hypergeometric distribution approaches the binomial distribution as a limit. For very large or infinite lot sizes, the operating characteristics for attributes acceptance plans are computed as follows:

$$P = \sum_{x=0}^{x=c} C_{n,x} p^{x} (1 - p)^{n-x} \qquad (6)$$

in which

P = probability of acceptance

n = sample size

p = fraction defective of the population

c = acceptance number, maximum allowable number of defective items in the sample

$C_{m,n}$ = number of possible combinations of m items taken n at a time = m!/(n!(m - n)!)

x = summation variable

In terms of the binomial distribution, the lot percent defective would be expressed as 100p. This distribution was used to develop Table 2.

The estimates of lot percent defective for the standard deviation method contained in Table 4 are obtained by numerically integrating the beta distribution function (13):

$$p = \int_{x=0}^{x=\text{Max}(0,\ 1/2 - Q\sqrt{n}/(2(n-1)))} \beta(a,b,x)\,dx \qquad (7)$$

in which

p = fraction defective of the population for single-limit applications (for double-limit applications, two separate integration steps must be performed and the results added to obtain the total fraction defective)

$\beta(a,b,x)$ = beta distribution function

a,b = parameters of the beta distribution = n/2 - 1

n = sample size

Q = quality index, $(\bar{X} - L)/S$ or $(U - \bar{X})/S$ for single-limit applications, both required for double-limit applications

$\bar{X}$ = sample mean

S = sample standard deviation

L,U = lower and upper specification limits

x = integration variable

The area under the beta distribution obtained in this manner is the fraction defective which must be multiplied by 100 to yield the estimate of percent defective. Although this integration can be done manually using tables of the beta function (14), it is far more practical to use computer assistance with subroutines developed specifically for this purpose.

The operating characteristics for variables plans based on the standard deviation are obtained by numerically integrating the noncentral t distribution function (12):

$$P = 1 - \int_{x=-\infty}^{x=k\sqrt{n}} t(v,\delta,x)\,dx \qquad (8)$$

in which

P	= probability of acceptance
$t(v,\delta,x)$	= noncentral t distribution function
v	= degrees of freedom = $n - 1$
n	= sample size
δ	= noncentrality parameter = $K_p \sqrt{n}$
K_p	= normal z-score associated with each level of population percent defective for which the computation is made
k	= acceptance constant
x	= integration variable

If the acceptance procedure is stated in terms of the maximum allowable estimated percent defective (M) rather than the minimum allowable value (k) of the quality index (Q), this must first be converted to a k

value using tables such as Tables 4, 5, or 7. The integration step indicated in Equation 8 may be performed manually using tables of the noncentral t distribution (12) although, like the integration of the beta distribution in Equation 7, it is much more practical to use computer assistance. Table 3 was generated in this manner.

When these same operations are to be performed for acceptance plans based on the range (R), minor modifications must be made to account for the reduced degrees of freedom associated with range estimates of variability. The following values are obtained from Duncan (11):

SAMPLE SIZE	CONVERSION FACTOR (d_2^*)	DEGREES OF FREEDOM (RANGE METHOD)
3	1.91	2.0
4	2.24	2.9
5	2.48	3.8
6	2.67	4.7
7	2.83	5.5
8	2.96	6.3
9	3.08	7.0
10	3.18	7.7
11	3.27	8.4
12	3.35	9.0
13	3.42	9.6
14	3.49	10.2
15	3.55	10.8

To obtain estimates of lot percent defective using the range method, the upper integration limit in Equation 7 must be changed (13) to:

$$x = \text{Max}(0, 1/2 - d_2^* Q\sqrt{(v + 1)}/2v) \quad (9)$$

in which

- x = integration variable
- Q = quality index computed by the range method, $(\bar{X} - L)/R$ or $(U - \bar{X})/R$ for single-limit applications, both required for double-limit applications
- d_2^* = factor that, when divided into the range computed from the sample, converts it into an estimate of the standard deviation
- v = degrees of freedom (the appropriate noninteger values associated with the range method must be used)

To develop operating characteristic curves for variables acceptance plans based on the range, Equation 8 may be used except that it is necessary to account for the appropriate noninteger degrees of freedom associated with range estimates of variability (15). In this case, it is necessary to compute two probability values for integral degrees of freedom in order to obtain the desired value by interpolation.

A POTENTIAL PROBLEM WITH VARIABLES PLANS

Although such occurrences are rare, it is possible when using variables acceptance plans that a lot may be judged rejectable even though none of the individual test results falls outside the specification limits. Provided no fundamental assumptions (normal population, random sampling,

etc.) have been violated, this is a theoretically correct result. The proper inference is that, based on the mean and standard deviation (or range) estimated from the sample, the population percent defective is unacceptably large.

This same result may also be caused by one or more outliers, test results that deviate unusually far from the norm due to some assignable cause such as equipment malfunction or operator error. Because such a result may be challenged by a contractor who is unfamiliar with its theoretical basis, and may in fact be an indication of a breakdown in the sampling and testing process, it is advisable to investigate and reevaluate any lot rejected in this manner.

PAVEMENT THICKNESS EXAMPLE

A highway agency wishes to develop an acceptance procedure for pavement thickness that is as uncomplicated as possible, involving no statistical calculations or special tables. The pavement will be considered satisfactory if at least 90 percent of it has a thickness greater than the design value. Therefore, the acceptable quality level (AQL) may be considered to be 10 percent defective and it is desired that this level of quality have a relatively high probability of acceptance. At the other extreme, if 40 percent or more of the pavement is less than the design thickness, it has been decided that this will be defined as the rejectable quality level (RQL) and a correspondingly low probability of acceptance is desired.

For purposes of this example, suppose that a seller's risk of $\alpha = 0.05$ and a buyer's risk of $\beta = 0.10$ are desired. The corresponding

probabilities of acceptance are P = 0.95 at the AQL and P = 0.10 at the RQL.

The requirement for simplicity dictates an attributes plan. When attributes acceptance procedures are applied to continuous data (thickness in this case), the lot size is considered to be infinite. By scanning the rows and columns of Table 2, it is observed that a plan with a sample size of n = 15 and an acceptance number of c = 3 produces very nearly the desired risk levels. (Because the sample size and acceptance number are discrete values, it is not possible to match the risks exactly.) The following values are obtained:

LOT PERCENT DEFECTIVE	PROBABILITY OF ACCEPTANCE
10 (AQL)	0.94
20	0.65
30	0.30
40 (RQL)	0.09
50	0.02

It can be seen from these values that the basic objectives have been well satisfied. A good quality pavement having 10 percent defective or less will have a probability of acceptance of at least P = 0.94. If the pavement is 40 percent defective or more, the probability of acceptance will be P = 0.09 or less.

The completed acceptance procedure will require that n = 15 cores be taken at random locations within a specified lot size. Because attributes acceptance theory makes no assumptions about the distributional form of the

population, there is considerable latitude to define the lot size in any manner that the highway agency feels is appropriate. Provided that no more than c = 3 cores are less than the design thickness, the lot will be judged to be acceptable.

GRADATION EXAMPLE

An acceptance procedure is to be prepared for a crushed stone base course. The percentage by weight of material passing the #200 sieve is known to be a significant performance characteristic. Experience has shown that bases having 7.0 percent or less of minus #200 material have performed well but bases exceeding 10.0 percent of minus #200 material have poor stability and drainage and tend to be frost susceptible. For this example, it is assumed that an analysis of historical data has shown the test results on minus #200 material to be approximately normally distributed with a typical standard deviation of about σ = 1.0 percent.

The information provided in this example is sufficient to develop a workable acceptance plan but it is not in the most useful form. For the types of acceptance plans covered in Standard R9, definitions of acceptable and unacceptable quality must be stated in terms of the percentage of material falling outside some specification limit (or pair of limits). Instead, the information is presented in terms of two average levels of minus #200 material that experience has shown have produced satisfactory and unsatisfactory results, respectively. As a reasonable approximation, these average values can be associated with the typical standard deviation of σ = 1.0 percent by means of normal distribution theory to provide guidance in establishing both the AQL and the RQL in terms of percent

defective. The acceptance plan will then perform as desired as long as the standard deviation is reasonably close to the typical value and, if conservatively designed, it should provide ample protection even when the standard deviation is larger than usual.

Since there is no reason to impose a lower limit on minus #200 material, this will be a single-limit specification. A logical choice for this limit is 7.0 percent, the level of minus #200 material that is known to be clearly satisfactory. It is believed that the base will perform well as long as 90 percent or more of the material has a minus #200 value of 7.0 percent or less. Therefore, the AQL is defined as 10 percent defective above the limit of 7.0 percent. This is a relatively conservative definition because, even if the standard deviation were considerably larger than the typical value, there is little chance that any of the material in the normal distribution representing AQL quality would reach the known critical value of 10.0 percent minus #200 material. The AQL is illustrated in the upper diagram in Figure 3.

To determine the level of percent defective to be defined as the RQL, it is noted that if this same distribution had 50 percent of its material above the limit of 7.0 percent, its upper tail would extend just to the critical value of 10.0 percent minus #200 material. On those few occasions that the standard deviation was substantially larger than the typical value of σ = 1.0 percent, a relatively small portion of the distribution would extend above the critical value of 10.0 percent. As the amount of material exceeding 7.0 percent minus #200 material increases above 50 percent, however, progressively more will exceed the critical value of 10.0 percent

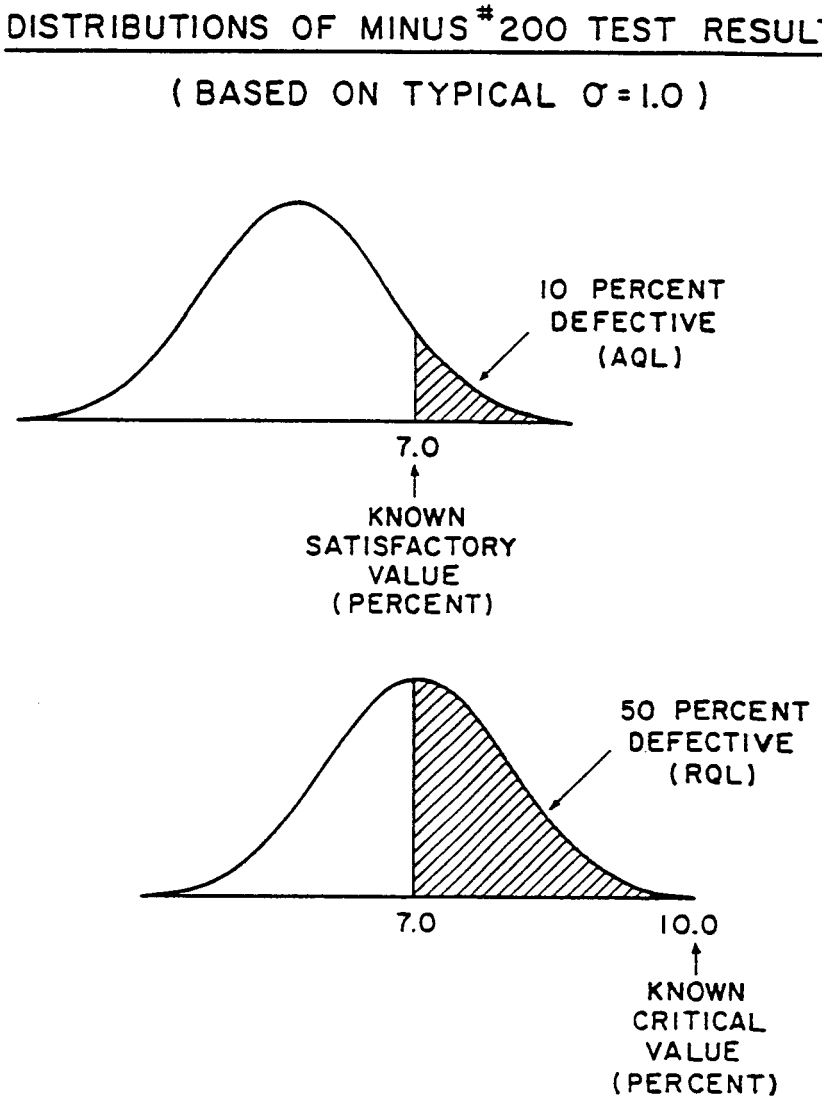

Figure 3. Illustration of definitions of AQL and RQL for gradation example.

and performance problems might be expected to develop. This provides a rational basis for defining the RQL as 50 percent defective above the limit of 7.0 percent minus #200 material, as illustrated in the lower diagram in Figure 3.

For this example, it will be assumed that the highway agency wishes to control both the seller's risk and the buyer's risk at $\alpha = \beta = 0.05$. The required acceptance probabilities at the AQL and RQL are P = 0.95 and P = 0.05, respectively. It is seen from Table 3 that a variables plan with a sample size of n = 8 and a maximum allowable estimated percent defective of M = 26 meets these requirements.

LOT PERCENT DEFECTIVE	PROBABILITY OF ACCEPTANCE
10 (AQL)	0.95
20	0.70
30	0.38
40	0.16
50 (RQL)	0.05
60	0.01

A suitable lot size must be chosen and the method of test specified. Because variables acceptance theory assumes sampling from a normal population, care must be taken not to combine distinctly different populations into a single lot. The acceptance procedure will require that the mean ($\bar{X}$) and standard deviation (S) be calculated from n = 8 random samples and used to compute the Q statistic in Equation 10. The corresponding percent defective estimate is obtained from tables such as Table 4

or the type presented as Table 5. For the lot to be judged acceptable, the estimated percent defective must be no larger than M = 26. (Alternatively, it could be required that the Q statistic be equal to or greater than k = 0.665.)

$$Q = \frac{7.0 - \bar{X}}{S} \qquad (10)$$

PAY ADJUSTMENT CLAUSES

Because it is seldom possible to define a single level of quality that differentiates between satisfactory and unsatisfactory work, it has become customary to define two distinctly different quality levels -- the AQL and the RQL -- when developing statistical acceptance procedures. The AQL represents a clearly acceptable level of quality that the highway agency expects the contractor to deliver. The RQL represents a much lower level of quality that, when detected, requires some sort of remedial action.

In actual practice, highway agencies are often faced with the dilemma of having to deal with marginal quality, items of work that fall between the AQL and the RQL. Many agencies have found the use of adjusted pay schedules, which award payment in proportion to the quality received, to be a practical and effective solution. The percent defective parameter, upon which the revised version of Standard R9 is based, is particularly well suited for this purpose. For the reader interested in pursuing this

refinement, the development of pay adjustment clauses is extensively covered in the recent literature (1 - 3, 16 - 22).

SUMMARY AND CONCLUSIONS

A major revision of AASHTO Standard R9, Acceptance Sampling Plans for Highway Construction, was described. The primary goals were to correct several technical flaws and to reduce the level of complexity of the standard. The new version is oriented around the concept of percent defective as the quality measure and advocates the standard deviation method in favor of the less efficient range method for variables acceptance plans. Several new tables were developed, including operating characteristic tables for a wide range of both attributes and variables acceptance plans, and it was demonstrated by computer simulation that single-limit variables operating characteristic curves are sufficiently accurate for most double-limit applications. Finally, two examples were presented to illustrate the use of the revised standard.

ACKNOWLEDGMENT

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ACCEPTANCE SAMPLING PLANS

FOR

HIGHWAY CONSTRUCTION

Interim Recommended Practice for

ACCEPTANCE SAMPLING PLANS FOR HIGHWAY CONSTRUCTION

AASHTO DESIGNATION: R9 - 85

1 SCOPE

This recommended practice provides guidance in the preparation and application of acceptance plans for highway materials and construction items.

2 GENERAL PROVISIONS

2.1 Requirements of Acceptance Plans

2.1.1 The primary objective is to communicate in a clear and unambiguous manner exactly what is desired. A particular specification requirement must be explicit and subject to one, and only one, interpretation by the engineer, the contractor, the materials supplier, the accountant, and the lawyer.

2.1.2 To avoid potential legal disputes, the contractor should be given most of the responsibility for controlling the construction process while the highway agency should be primarily responsible for judging the acceptability of the finished work through the use of properly designed acceptance plans.

2.1.3 The acceptance plan should be realistic in defining acceptable (AQL) and rejectable (RQL) quality levels. The AQL should be set high enough to satisfy design requirements but not so high that extraordinary methods or materials will be required. The RQL should be set low enough so that the decision to reject is truly justified.

2.1.4 All acceptance plans based on fractional sampling and testing are subject to the risk of making incorrect decisions. It is the responsibility of the designer of an acceptance plan to control the risks at suitably low levels.

2.1.5 The choice of sample size must strike a balance between practical limitations on the number of samples that can be processed and the need for larger samples to obtain more precise information (lower risks).

2.1.6 Mathematical and statistical principles must be applied correctly but the actual mechanics of the acceptance procedure should be simple and straightforward, not requiring an understanding of the underlying theory.

2.1.7 The plan must be suitable for use by the highway industry. It must be applicable to the various types of materials and construction procedures and must be adaptable to varying rates of production.

2.2 Measure of Quality Used in this Guideline

2.2.1 The measure of quality used in this guideline is the LOT PERCENT DEFECTIVE. This represents the percentage of the population (lot) that falls outside a single lower or upper specification limit, or outside both lower and upper limits, as illustrated in Figure 2.1.

2.2.2 This approach is appropriate when the percent defective is believed to be more strongly associated with the ultimate performance of the product than the mean or some other statistical parameter.

3 TYPES OF ACCEPTANCE PLANS

3.1 Acceptance Plans for Inspection by Attributes

3.1.1 Acceptance plans for inspection by attributes are used when it is impractical or undesirable to measure the specific value of a quality characteristic and, instead, each inspected item is classified as either satisfactory or unsatisfactory. The typical application involves the COUNTING of some type of defect, such as the number of chipped bells in a lot of concrete pipe.

3.1.2 Attributes procedures may also be used with continuous data when the failure to satisfy the normality requirement precludes the use of variables plans.

3.1.3 Like the variables acceptance procedure described in Section 3.2, attributes plans are designed to control the percent defective of the population (lot). However, there are the following distinct differences:

3.1.3.1 The decision reached is ACCEPT or REJECT, no estimate of lot percent defective is made.

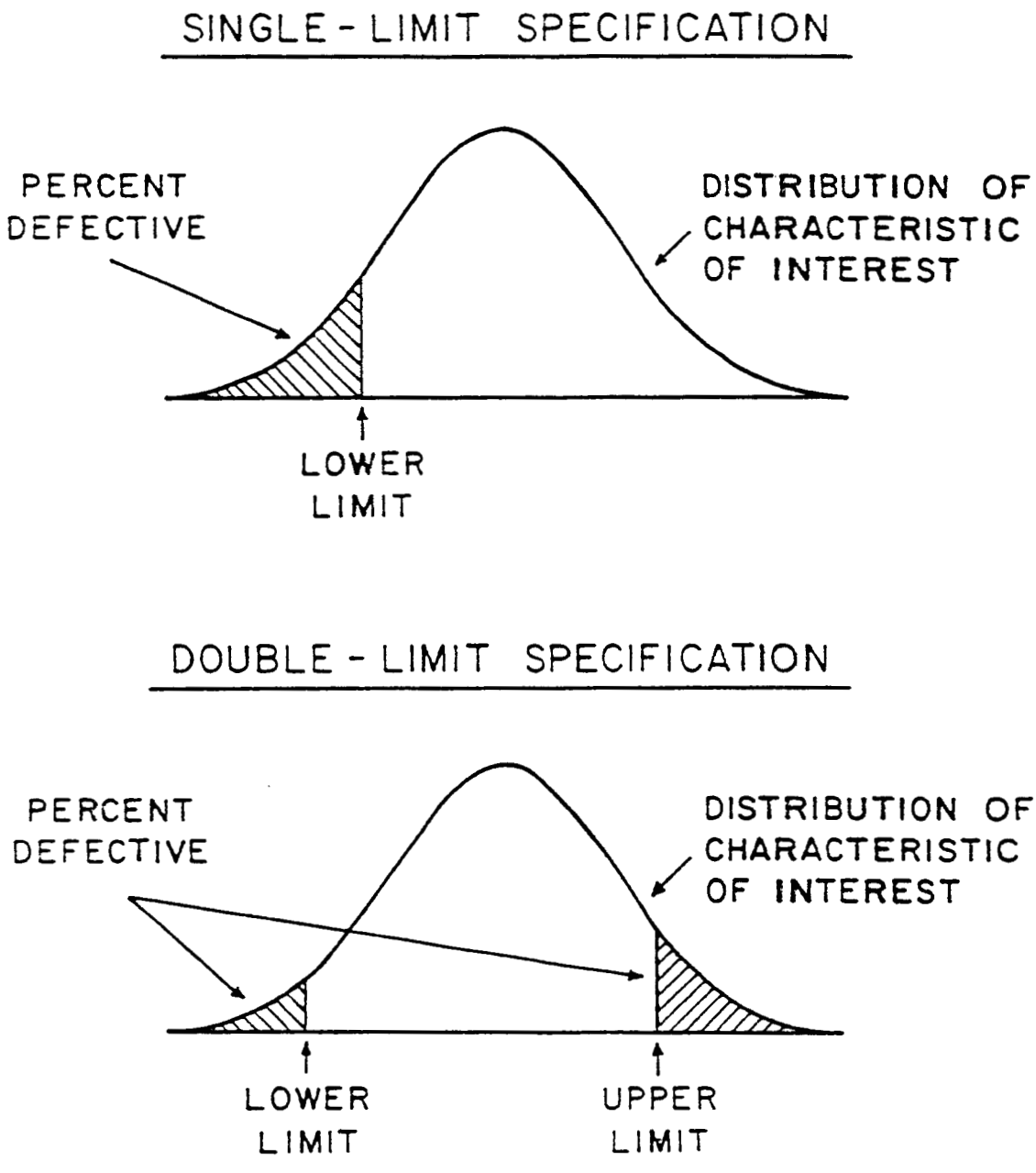


FIGURE 2.1 Illustration of the Concept of Percent Defective

3.1.3.2 The population being sampled need not be normally distributed as it must be for variables procedures.

3.1.3.3 The attributes procedure is less statistically efficient, requiring a larger sample size for a given discriminating power.

3.1.3.4 An attributes plan may be based on a single sample, if desired. The variables plans described in this standard require a sample size of at least n = 3.

3.2 Acceptance Plans for Inspection by Variables

3.2.1 Acceptance plans for inspection by variables usually apply to quality characteristics that can be MEASURED rather than COUNTED and involve the computation of statistical parameters.

3.2.2 Like the attributes acceptance procedures described in Section 3.1, variables plans are designed to control the percent defective of the population (lot). Unlike the attributes procedure, an estimate of percent defective can be made which provides the basis for an ACCEPT or REJECT decision.

3.2.3 As a general rule, variables plans are more statistically efficient than attributes plans, providing greater discriminating power for a given sample size.

3.2.4 Fundamental statistical parameters required for the application of the variables procedure presented in this section are the MEAN and the STANDARD DEVIATION, computed as shown in Equations 3.1 and 3.2.

$$\bar{X} = \frac{\sum_{i=1}^{i=n} X_i}{n} \qquad (3.1)$$

$$S = \sqrt{\frac{\sum_{i=1}^{i=n} (X_i - \bar{X})^2}{n - 1}} \qquad (3.2)$$

in which

$\bar{X}$ = sample mean

S = sample standard deviation

X_i = individual sample values

n = sample size

3.2.5 Because the variability of the population is being estimated by the standard deviation computed from the sample, acceptance plans of this type are often referred to as "variability-unknown" plans.

3.2.6 In order to estimate the LOT PERCENT DEFECTIVE, it is first necessary to compute one or two QUALITY INDEX (Q) values, using either or both of Equations 3.3 and 3.4, as appropriate.

$$Q_L = \frac{\bar{X} - L}{S} \qquad (3.3)$$

$$Q_U = \frac{U - \bar{X}}{S} \qquad (3.4)$$

in which

Q = quality index

$\bar{X}$ = sample mean

S = sample standard deviation

L, U = lower and upper specification limits outside of which the material or work is defined to be defective

3.2.7 The estimate of LOT PERCENT DEFECTIVE associated with any specific Q value and sample size is obtained from the table in Appendix C. It will be observed when using this table that a sample size of at least n = 3 is required for this type of variables acceptance procedure.

3.2.8 For cases in which there are BOTH LOWER AND UPPER SPECIFICATION LIMITS, both Equations 3.3 and 3.4 are used and the INDIVIDUAL ESTIMATES OF PERCENT DEFECTIVE ARE ADDED to obtain the LOT PERCENT DEFECTIVE. (Note that it is NOT correct to add the individual Q values and then determine the percent defective.)

3.2.9 Normality Tests for Variables Data

3.2.9.1 A basic assumption of variables acceptance theory is that the population (lot) that is being sampled is normally distributed. Many construction characteristics have been found to closely approximate this distribution. However, before deciding to use a variables acceptance procedure, an attempt should be made to verify that the assumption of normality is reasonably satisfied. Once this determination has been made, it is assumed to apply to all future production and further normality tests would not routinely be performed.

3.2.9.2 It may be sufficient to visually check histograms of construction data to verify that the population being sampled is approximately normally distributed, or to rely on various published studies.

3.2.9.3 If a quantitative procedure capable of producing a conclusion at a prescribed level of confidence is desired, the chi-square goodness-of-fit test is recommended. The details of this test may be found in many standard statistical texts.

3.2.9.4 If the assumption of normality is not met, two remedies are possible. A greater number of acceptance tests can be performed

so that averages of two or more tests can be used in place of individual tests. This strongly increases the tendency toward normality. Alternatively, an attributes procedure can be used which is uninfluenced by the distributional form of the population.

3.3 Other Types of Variables Plans

3.3.1 Variables plans may also be of the "variability-known" type, in which the standard deviation of the product is assumed to be constant and known in advance. However, since it is usually one of the objectives of a statistical specification to encourage and reward a greater degree of uniformity, this approach may be counterproductive. Since conditions warranting the use of the "variability-known" procedure rarely exist in the highway field, this method has not been developed in this standard.

3.3.2 Variables plans may also be based on the range (R) as the measure of variability and many satisfactory plans of this type are in existence. The steps necessary to develop acceptance plans based on the range are outlined in Appendix D. However, it is expected that the standard deviation method will be the method of choice for most future applications for the following reasons:

3.3.2.1 The standard deviation method is more statistically efficient, requiring smaller sample sizes to provide an equivalent degree of quality assurance. This translates directly into cost savings.

3.3.2.2 The standard deviation has better mathematical properties than the range for various subsequent statistical analyses that might be performed.

3.3.2.3 Because the standard deviation is computed from all the data values, rather than the two extreme values, it is less sensitive than the range to possible outliers.

3.3.2.4 The standard deviation method is administratively simpler, avoiding the need for the subgrouping rules advocated with the range method.

3.3.2.5 Most scientific calculators now enable the standard deviation to be computed with a single keystroke, negating the previous computational advantage of the range.

4 GUIDELINES FOR DEFINING AND MEASURING QUALITY

4.1 General Considerations

4.1.1 In the highway construction field, it is seldom possible to define a single level of quality that distinctly differentiates between satisfactory and unsatisfactory work. However, it usually is possible to define a desired level of quality that is clearly acceptable, and another lower level of quality that is considered to be so poor that it would be totally unacceptable. This provides two convenient reference points from which an acceptance procedure can be developed.

4.1.2 For purposes of this standard, quality is defined in terms of PERCENT DEFECTIVE, i.e., the percentage of material or work falling outside stated specification limits.

4.1.3 When statistical limits are derived from historical data, care must be exercised to assure that the data reflect the normal variations of the measured property. In some cases, current specification limits have evolved from nonrandom sampling techniques (perhaps resampling) or they may represent different test methods. For example, when applying Marshall voids criteria, limits based on the bulk specific gravity of aggregates would not be appropriate for apparent specific gravity.

4.2 Characteristics Used for Acceptance

4.2.1 A SIGNIFICANT CHARACTERISTIC is one that directly affects the performance or appearance of a material or item of construction. As such, it is an indicator of quality.

4.2.2 A practical acceptance plan should define quality in terms of realistic levels of percent defective of significant characteristics.

4.2.3 If a well established performance-versus-quality relationship does not exist, or if the characteristic of interest is not readily measurable by a standard acceptance test, it may be necessary to base the acceptance plan on the percent defective of some other measurable property that is correlated with the characteristic of interest.

4.3 Selection of Test Method

4.3.1 If possible, a standard test method should be used. Both the procedure and any necessary equipment must be described or cited in the acceptance specification.

4.3.2 The test method must be practical and economical. This involves consideration of equipment costs, level of operator skill required, the length of time required to perform a test, and the possibility of obtaining timely results when remedial action is possible.

4.3.3 The test method should be both ACCURATE and PRECISE to qualify as the basis for acceptance decisions.

4.3.3.1 Individual test results will always stray from the true population parameter to some extent. If a large number of tests could be performed on identically the same product, the test results would not be identical but would be spread out in some form of statistical distribution.

4.3.3.2 The term ACCURACY refers to the degree to

which the mean of this distribution tends to coincide with the true population parameter.

4.3.3.3 The term PRECISION refers to the degree of dispersion of this distribution. The concept of good precision implies a narrow distribution of replicate test results.

4.3.3.4 A test method may be accurate without being precise, and vice versa.

4.3.3.5 A method with less inherent precision may be preferable to a more precise method if a large number of test results can be obtained quickly and inexpensively. The overall precision improves as the square root of the number of individual test results that are averaged together.

4.3.3.6 For some types of tests, it is impossible or impractical to ever know the true value of the population parameter. In these cases, the accuracy of an individual inspector or laboratory can be approximately judged by comparing the individual results with the grand mean obtained from many participants performing the test under carefully controlled conditions. This is the basis for some laboratory accreditation tests.

4.3.3.7 A test method that is not accurate is said to be BIASED. Steps should be taken to avoid the potential for bias that might result from nonstandard equipment or procedures or from untrained operators.

4.3.3.8 The percent defective estimated from a sample by the method described in Section 3.2 is known to be an accurate (mathematically unbiased) estimate of the percent defective of the population being sampled.

4.4 Definition of Quality Levels

4.4.1 The ACCEPTABLE QUALITY LEVEL (AQL) and REJECTABLE QUALITY LEVEL (RQL) are specific population (lot) characteristics.

4.4.2 The AQL is that level of percent defective at or below which the work is considered to be completely acceptable.

4.4.3 The RQL is that level of percent defective at or above which the work is considered to be sufficiently unacceptable that repair or replacement is warranted.

4.4.4 The basis for these definitions may be the subjective assessment of a panel of knowledgeable engineers, historical records of quality received and subsequent performance, or theoretical considerations.

5 ACCEPTANCE CRITERIA, RISKS, AND CRITICALITY

5.1 General Considerations

5.1.1 The primary reason for including numerical limits in an acceptance plan is to assure conformance with design requirements. The expected benefits of more stringent specification requirements must be carefully weighed against the probable increase in costs.

5.1.2 The exact quality for which the buyer is willing to pay should be specified in terms of realistic and enforceable limits of significant characteristics.

5.1.3 The acceptance criteria must be stated in terms of lower, upper, or both lower and upper limits of significant characteristics outside of which no more than a specified percentage of the work (or a maximum number of test results in the case of attributes acceptance plans) shall be allowed to fall.

5.2 Acceptance Criteria

5.2.1 For ATTRIBUTES plans, the lot size, sample size, and maximum allowable number of unsatisfactory test results is specified.

5.2.2 For VARIABLES plans, the lot size, sample size, and

maximum allowable estimated percent defective (M) is specified. (Because it is necessary to allow for random variability of the sampling process, neither the AQL nor the RQL is likely to be suitable as the specified maximum allowable estimated percent defective of the acceptance requirement. A satisfactory value will usually be found to lie somewhere between the AQL and the RQL. This is illustrated in the examples in Section 12.)

5.2.3 Alternatively, in place of maximum allowable estimated percent defective, a VARIABLES plan with a single lower or upper limit may specify the minimum acceptable value (k) of the quality index (Q) computed from the sample in accordance with Equation 3.3 or 3.4. For double-limit plans, it is mathematically preferable and administratively simpler to use only the maximum allowable estimated percent defective (M).

5.3 Types of Risks

5.3.1 BUYER'S RISK (β) is the risk of erroneously accepting unsatisfactory (RQL) work.

5.3.2 SELLER'S RISK (α) is the risk of erroneously rejecting satisfactory (AQL) work.

5.4 Factors Affecting Criticality

5.4.1 From the engineering viewpoint, each acceptance plan should be based on the CRITICALITY of the measured property as it affects safety, performance, or durability. In addition, legal and contractual requirements should be considered. The word "criticality" has been selected for use in this guideline to express the overall concept of relative importance of various factors. The factors to be considered in determining criticality are:

Safety — danger to human life.

Serviceability -- inconvenience and other consequences, including military, of disruption of service or use of the road or bridge.

Cost -- for construction, control, and maintenance or replacement.

Legal and Contractual Requirements -- clarity of directions and interpretations of requirements, responsibilities and obligations; recognition of customary practice; fair and equitable bidding and acceptance procedures.

5.5 Criticality Ratings

5.5.1 For classification purposes, the following ratings of criticality are suggested:

Critical — when the requirement is essential to preservation of life.

Major -- when the requirement is necessary for the prevention of substantial economic loss.

Minor — when the requirement does not materially affect performance.

Contractual -- when the requirement is established only to provide uniform standards for bidding.

5.6 Guidelines for Choosing Risks According to Criticality

5.6.1 A large buyer's risk indicates a high probability of acceptance for poor (RQL) or marginal quality work. Although this may encourage lower bid prices, it greatly increases the likelihood for premature failures and higher maintenance costs.

5.6.2 A large seller's risk will result in the unnecessary rejection of satisfactory (AQL) work and may eventually drive both the quality and the price to unnecessarily high levels.

5.6.3 It is the responsibility of the highway agency to design acceptance plans that control the risks at suitable levels appropriate for the

degree of criticality of each application. The following table gives probability values that have been suggested for specific degrees of criticality, although it is recognized that IT MAY NOT ALWAYS BE NECESSARY OR PRACTICAL TO MATCH THESE IDEALIZED VALUES.

Classification	Probability of Acceptance at RQL (Buyer's Risk)	Probability of Acceptance at AQL	Seller's Risk at AQL
Critical	0.005	0.950	0.050
Major	0.050	0.990	0.010
Minor	0.100	0.995	0.005
Contractual	0.200	0.999	0.001

5.6.4 Analyzing the risks associated with any particular acceptance scheme is usually accomplished by constructing operating characteristic (OC) curves. Methods to develop OC curves are discussed in Section 7.

5.6.5 Either or both the buyer's and seller's risks may be specified in selecting a suitable acceptance plan although, because the sample size is a discrete variable, it may not always be possible to obtain precisely the desired risk levels.

6 SELECTION OF LOT SIZE AND SAMPLE SIZE

6.1 The selection of lot size is dictated primarily by practicality and convenience although, for variables acceptance procedures, it is important to avoid combining two distinctly different populations. The variables procedures described in this standard assume that the lot being sampled is a homogeneous normal distribution. Attributes acceptance procedures make no distributional assumptions and require less care in defining lot sizes.

6.2 Some practical limitations that have been used to define maximum lot sizes are a day's production of concrete, a particular concrete item or structure, a mile of roadway, a specified area or a day's worth of paving, a production run of a prefabricated or manufactured item, etc.

6.3 In the development of a statistical acceptance procedure, the sample size is generally a more important consideration than the lot size because this strongly influences the risks involved and, consequently, the overall effectiveness of the quality assurance program.

6.4 The minimum sample size that may be used depends upon the type of acceptance procedure. The variables acceptance procedures described in this standard require a sample size of at least $n = 3$. Attributes procedures can be based on a single sample, if desired.

6.5 A more important consideration in the selection of sample size is the analysis of the risks associated with the acceptance plan. Only by constructing the operating characteristic curves, as described in Section 7, can it be determined if the sample size is sufficient to enable the procedure to properly discriminate between acceptable and unacceptable work.

7 CONSTRUCTION OF OPERATING CHARACTERISTIC CURVES

7.1 Definition and Uses

7.1.1 An operating characteristic (OC) curve is a graphical representation of an acceptance plan's ability to discriminate between acceptable and unacceptable work. A typical example is shown in Figure 7.1.

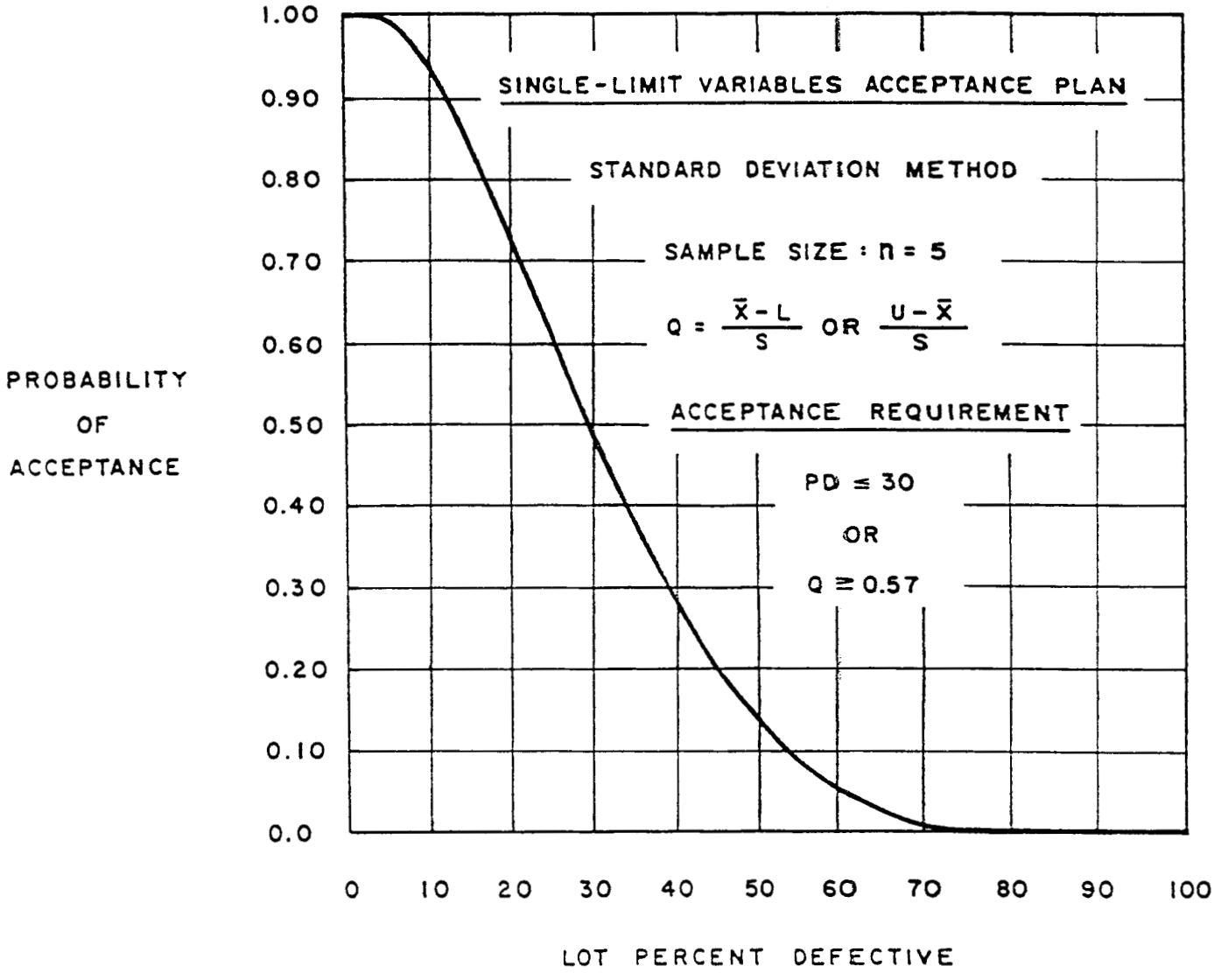


FIGURE 7.1 Typical Operating Characteristic Curve for a Variables Acceptance Plan

7.1.2 The importance of constructing OC curves cannot be over-emphasized. In this manner, the risks to both the highway agency and the contractor can be determined in advance and, if necessary, modifications of the acceptance plan can be made before troublesome situations arise in the field. Several examples are presented in Section 12.

7.1.3 The construction of OC curves for attributes acceptance plans is relatively straightforward and may be accomplished with the tables in Appendix A or the computational procedure in Appendix E.

7.1.4 The theory underlying the construction of OC curves for variables acceptance plans is considerably more complex and is beyond the technical scope of this standard. Consequently, only a tabular method is provided in Appendix B. An outline of the procedures used to construct the tables is given in Appendix E.

7.2 Operating Characteristic Tables

7.2.1 The tables in Appendix A and Appendix B provide the data necessary to construct OC curves for attributes and variables plans, respectively.

7.2.2 The tables span a wide range of potentially useful acceptance plans. Plans having relatively undesirable OC curves have been omitted and not all plans in these tables will be suitable for all situations.

7.2.3 Because attributes plans deal with discrete data, there are a finite number of practical plans, many of which are listed in Appendix A. The tables are constructed so that it will never be necessary to interpolate between acceptance numbers (c) or between sample sizes up to a sample size of n = 10. Some interpolation may be necessary for larger sample sizes or for specific lot sizes, although the OC curves are relatively insensitive to lot size.

7.2.4 Because variables plans deal with continuous data, there are an infinite number of possible plans that might be used. For either evaluating an existing plan or choosing a new plan, it may be appropriate to interpolate between the variables plans listed in Appendix B.

7.3 Procedure for Using Tables in Appendix A and Appendix B

7.3.1 Select either Appendix A for attributes plans or Appendix B for variables plans.

7.3.2 Select the risk considered more critical (buyer's or seller's) and scan the appropriate table to determine those plans that approximately match this risk.

7.3.3 Determine from the table which of these plans comes the closest to matching the less critical risk.

7.3.4 If necessary, interpolate between plans to match the desired risks as closely as possible.

8 ATTRIBUTES PROCEDURES FOR SMALL LOTS

Appendix A provides operating characteristic information for lot sizes down to N = 20. If necessary, acceptance procedures for smaller lot sizes may be developed in one of the following ways:

8.1 Specify an acceptance plan and use the hypergeometric formula in Appendix E to develop the OC curve. Repeat this procedure until a satisfactory OC curve is obtained.

8.2 Specify 100 percent inspection and state the number of defective items that will be tolerated in the lot.

8.3 Specify 100 percent inspection and require that any defective items be repaired or replaced.

9 RANDOM SAMPLING

9.1 Importance of Random Sampling

9.1.1 Of the various theoretical conditions upon which statistical acceptance procedures are based, the assumption of random sampling is one of the most important. Only when all vestiges of personal bias are removed can the laws of statistical probability be relied upon to function properly.

9.2 Types of Random Sampling

9.2.1 Random sampling is often defined as a manner of sampling which allows every member of the population (lot) to have an equal opportunity of appearing in the sample.

9.2.2 The most basic form of random sampling, sometimes called simple random sampling, allows every possible subset of the required sample size to have an equal chance of being selected. This is the least restrictive definition but it has the drawback that the sample locations occasionally tend to be clustered. For this reason, most highway agencies use some method of stratification to assure that the sample locations will be spread more uniformly throughout the work.

9.2.3 Stratified random sampling satisfies the fundamental requirement of Section 9.2.1 while avoiding the clustering problem by dividing the lot into as many equal-sized sublots (strata) as there are samples to be drawn. A single random sample is then obtained from each sublot.

9.3 Random Sampling Procedures

9.3.1 Stratified sampling plans for highway construction items are designed to spread the samples throughout the work and tend to be quite similar. First, most plans divide the lot into equal-sized sublots on the basis of

area, weight, or other appropriate measure. Then, within each sublot, provisions are made to select a single random sample. A typical example of this approach is illustrated in Figure 9.1. The uniform random numbers between zero and one are obtained from standard tables or may be generated by computer.

9.3.2 In practice, some agencies carry this method one step further. In sampling bituminous concrete, for example, it may be more convenient to sample directly from the appropriate trucks rather than to wait until after the material has been placed. In this case, the random locations in Figure 9.1 are used to determine which trucks are to be sampled. This is normally done in advance based on known total quantities and truck capacities.

9.3.3 For the sampling of discrete items, such as sections of pipe, it may be desirable to devise variations of stratified random sampling.

10 RETESTING AND CORRECTIVE ACTION

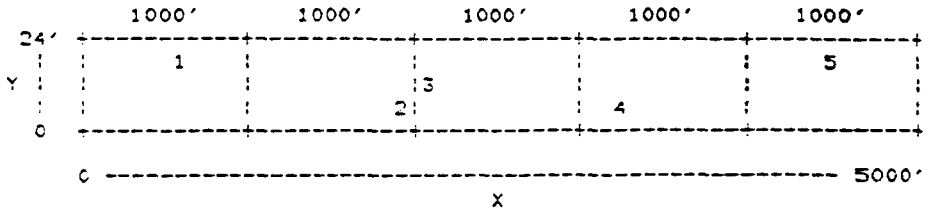
10.1 Retesting Provisions

10.1.1 When the original tests indicate that a large or costly item of work may warrant rejection, it is common practice to perform a series of retests before making the final decision. This constitutes a sequential acceptance scheme and will be dealt with in more detail in a forthcoming standard.

10.1.2 The use of a retesting provision, and the exact manner in which the retest results are processed, both affect the risks associated with the acceptance procedure.

10.1.3 If a retesting provision is to be used, it should be described explicitly in the contract documents.

(NUMERALS 1 - 5 INDICATE SAMPLING LOCATIONS)



DETERMINATION OF RANDOM X COORDINATES

SAMPLE NUMBER	RANDOM NUMBER	MULTIPLICATION TERM (SUBLOT LENGTH)	ADDITION TERM (CUMULATIVE LENGTH TO THIS SUBLOT)	X
1	0.603	× 1000	+ 0	= 603
2	0.992	× 1000	+ 1000	= 1992
3	0.086	× 1000	+ 2000	= 2086
4	0.214	× 1000	+ 3000	= 3214
5	0.551	× 1000	+ 4000	= 4551

DETERMINATION OF RANDOM Y COORDINATES

SAMPLE NUMBER	RANDOM NUMBER	MULTIPLICATION TERM (PAVEMENT WIDTH)	Y
1	0.750	× 24	= 18
2	0.286	× 24	= 7
3	0.542	× 24	= 13
4	0.081	× 24	= 2
5	0.877	× 24	= 21

FIGURE 9.1 Basic Stratified Random Sampling Procedure Applied to Highway Pavement

10.2 Corrective Action

10.2.1 When the acceptance procedure leads to the conclusion that a particular lot of material or item of work is to be rejected, the specification must spell out whatever corrective actions are to be taken. These actions will usually take the form of options at the discretion of the highway agency.

10.2.2 The most obvious corrective action is to require the contractor to remove and replace the defective material or item at no additional expense to the highway agency. This will not always be a practical course of action, however, and other options must be provided.

10.2.3 Many highway agencies have elected to include pay adjustment clauses in the specification so that, at their option, defective work may be accepted at a predetermined level of reduced payment.

10.2.4 Another method of dealing with rejectable work is to require the contractor to submit a written plan for corrective action to be accomplished at no additional expense to the highway agency.

10.2.5 As a last resort, it may be necessary to negotiate a settlement when an item of work is determined to be rejectable.

11 A POTENTIAL PROBLEM WITH VARIABLES PLANS

11.1 Although such occurrences are rare, it is possible when using variables acceptance procedures that a lot may be judged rejectable even though NONE OF THE INDIVIDUAL TEST RESULTS FALLS OUTSIDE THE SPECIFICATION LIMITS. Provided no fundamental assumptions (normal population, random sampling, etc.) have been violated, this is a theoretically correct result. The

proper inference is that, based on the mean and standard deviation estimated from the sample, the population PERCENT DEFECTIVE is unacceptably large.

11.2 This same result may also be caused by one or more outliers, test results that deviate unusually far from the norm due to some assignable cause such as equipment malfunction or operator error.

11.3 Because the result described in Section 11.1 may be challenged by a contractor who is unfamiliar with its theoretical basis, and may in fact be an indication of a breakdown in the sampling and testing process, it is advisable to investigate and reevaluate any lot rejected in this manner.

12 EXAMPLES

12.1 Analysis of Attributes Acceptance Plan for Precast Concrete Center Barrier

12.1.1 An existing acceptance plan for precast concrete center barrier requires that n = 10 sections be selected at random and that various dimensional checks be performed. The plan further stipulates that, if more than c = 2 of the n = 10 test sections fail to meet the prescribed dimensional tolerances, the entire lot must be tested at the supplier's expense and that all defective sections shall be replaced at no extra cost to the highway agency. The maximum lot size is specified to be a single production run or N = 100 sections, whichever is smaller. There is no minimum lot size but it is known from past experience that it is rarely smaller than N = 20.

12.1.2 To analyze this acceptance procedure, it is desired to construct the operating characteristic (OC) curves for the bounding lot sizes of

N = 20 and N = 100. Because it is possible that there might occasionally be lot sizes smaller than N = 20, the OC curve for a lot size of N = 10 will also be plotted.

12.1.3 The following information is obtained from Appendix A for a sample size of n = 10 and an acceptance number of c = 2:

LOT PERCENT DEFECTIVE	PROBABILITY OF ACCEPTANCE	
	LOT SIZE = 20	LOT SIZE = 100
5	1.00	0.99
10	1.00	0.94
15	0.89	0.83
20	0.71	0.68
25	0.50	0.52
30	0.31	0.37
35	0.17	0.25
40	0.08	0.15
45	0.03	0.09
50	0.01	0.05
55	0.0	0.02
60	0.0	0.01

12.1.4 Although lot percent defective is not truly a continuous variable when the lot size is finite, it is still useful to plot the OC curves as if they were continuous as has been done in Figure 12.1. Given the fact that most lot sizes will fall between N = 20 and N = 100, these two curves provide practical bounds for the probability of acceptance for any level of submitted quality. (For lot sizes of N = 10 or fewer, the lot is 100 percent inspected and the OC curve plots as a vertical line.)

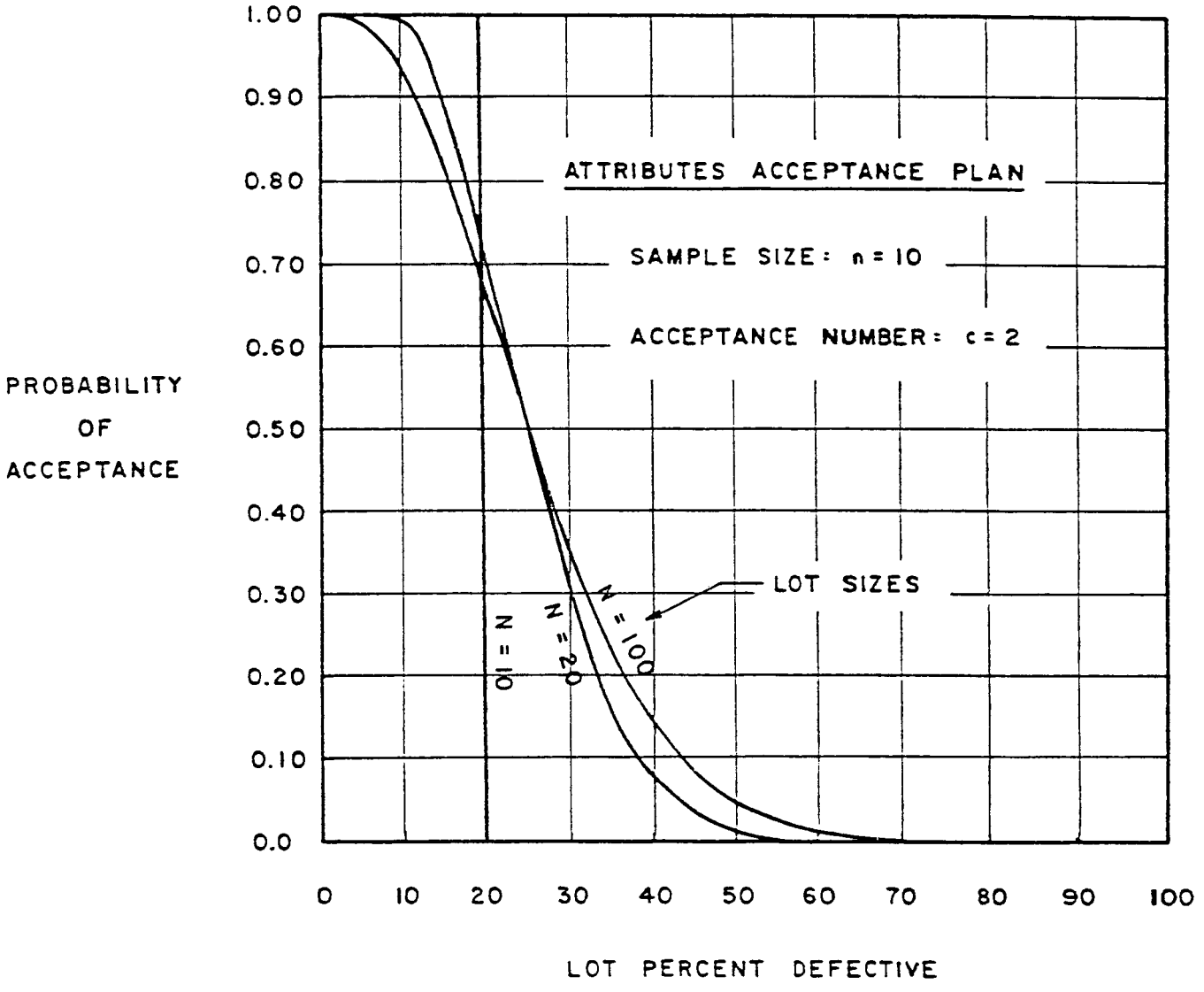


FIGURE 12.1 Operating Characteristic Curves for Acceptance Plan for Precast Concrete Center Barrier Described in Example 12.1

12.1.5 Although an acceptable quality level (AQL) has not been defined in this example, a value that has often been used in highway applications is 10 percent defective. In this example, it is seen that the probability of acceptance for a lot that is truly 10 percent defective lies between P = 1.00 and P = 0.94, depending upon whether the lot size is N = 20 or N = 100, respectively. This would probably be regarded as satisfactory, although this decision would be up to the highway agency. The purpose of the OC curves is to provide the information upon which this decision can logically be made.

12.1.6 Another factor that must be considered is the amount of unnecessary retesting that can be expected to occur when the process is truly under control. It can be seen from Figure 12.1 that, if most of the lot sizes tend to run close to N = 20, then lots that are truly 10 percent defective will almost always be accepted and the requirement for 100 percent inspection will almost never be triggered. If the lot sizes tend to run closer to N = 100, then 100 percent inspection will be required about 6.0 percent of the time when the lot is 10 percent defective.

12.1.7 Like the AQL, no RQL (rejectable quality level) has been explicitly defined for this acceptance plan. If, for example, the highway agency considered the dimensions of the center barrier to be of moderate but not critical importance, it is possible that the lot would not be regarded as rejectable unless 50 percent or more of it was defective. If this were the case, the present plan would probably be considered quite satisfactory since, at 50 percent defective, there is at most a P = 0.05 probability of acceptance. On the other hand, if 30 percent defective were regarded as seriously deficient, then the present plan might not be satisfactory since the probability of acceptance at that level of quality lies between P = 0.31 and P = 0.37.

12.1.8 In order to analyze how the plan will perform with the occasional lot sizes that fall between N = 10 and N = 20, the OC curve for a lot size of N = 10 has been plotted in Figure 12.1. This appears as a step function because it represents 100 percent sampling. The smaller lots are not likely to pose a problem because those having 10 percent defective or less will always be accepted and those having more than 20 percent defective will always be rejected and subjected to 100 percent inspection.

12.2 Analysis of Variables Acceptance Plan for Pavement Thickness

12.2.1 An existing acceptance plan for pavement thickness requires that n = 5 cores be taken at random locations from each 5000 linear feet of paving, similar to the diagram in Figure 9.1. The following acceptance requirement is specified:

$$\bar{X} \geq \text{Design Thickness} + kS \qquad (12.1)$$

in which

$\bar{X}$ = sample mean

S = sample standard deviation

k = 0.6 (the acceptance constant)

12.2.2 To evaluate the suitability of this acceptance plan, it is first necessary to construct the OC curve. To do this, it is observed that the acceptance requirement in Equation 12.1 can be rewritten in the following form:

$$\frac{\bar{X} - \text{Design Thickness}}{S} \geq k \qquad (12.2)$$

In terms of the quality index (Q), it is seen to be of the form described in Section 5.2.3:

$$Q \geq k \qquad (12.3)$$

12.2.3 The following information is obtained from Appendix B by interpolating for k = 0.6 between table values of k = 0.572 and k = 0.632 for a sample size of n = 5. For variables acceptance procedures, probability of acceptance is a continuous variable and has been plotted as such in Figure 12.2.

LOT PERCENT DEFECTIVE	PROBABILITY OF ACCEPTANCE
10	0.930
20	0.710
30	0.470
40	0.265
50	0.130
60	0.045
70	0.010

12.2.4 Like the previous example, neither the AQL nor the RQL has been specified. If it is assumed that the highway agency would be willing to tolerate up to 10 percent of the pavement having a thickness less than the design value, this implies that the AQL is 10 percent defective, and the corresponding probability of acceptance is P = 0.93. Consequently, approximately 7.0 percent of the lots of this (presumably satisfactory) quality will be rejected by this acceptance procedure. However, if rejected lots are subjected to further testing before the final decision is made, and a 7.0 percent retesting rate is not

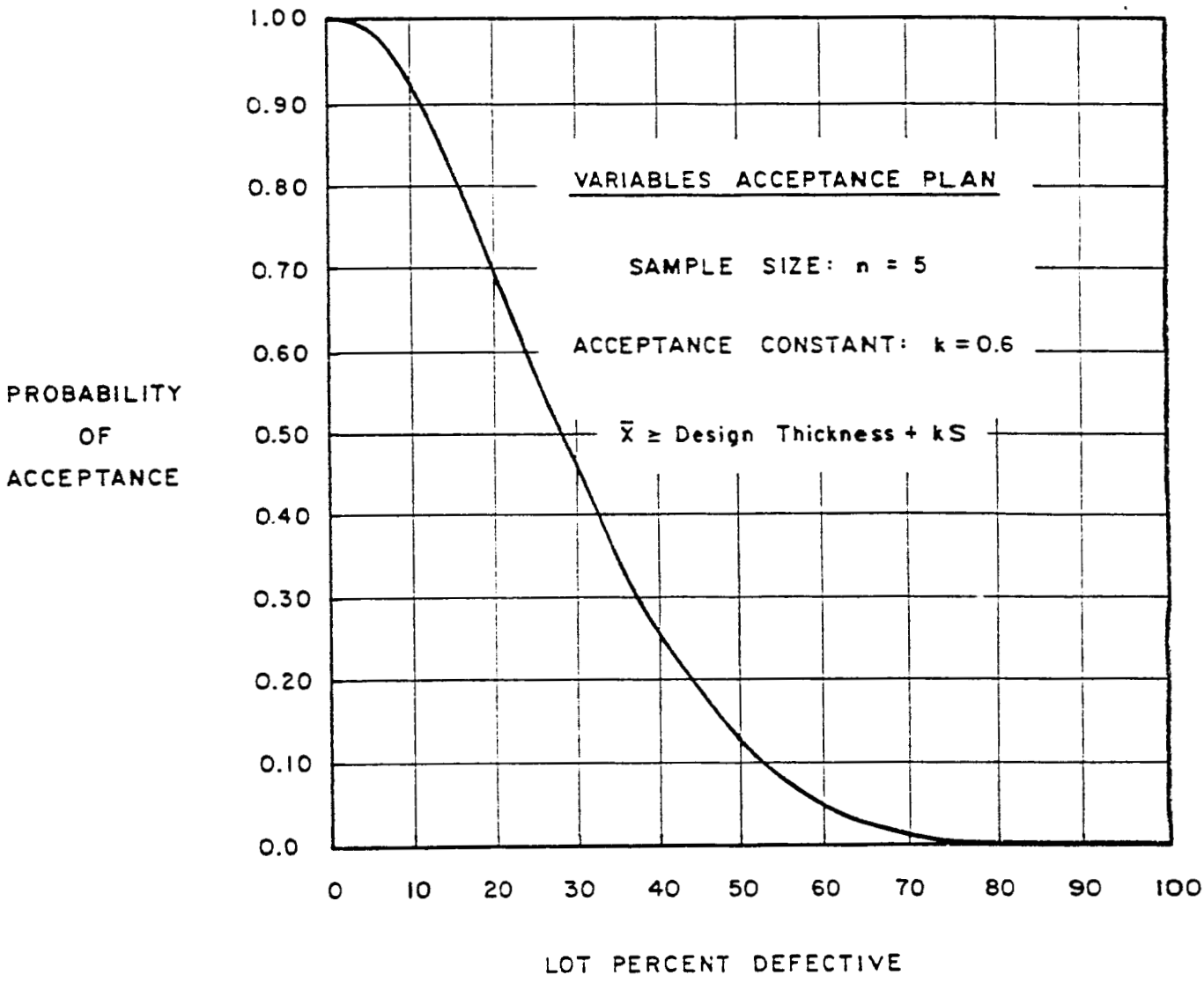


FIGURE 12.2 Operating Characteristic Curve for Acceptance Plan for Pavement Thickness Described in Example 12.2

considered objectionable, this may be regarded as a satisfactory acceptance plan.

12.2.5 At the other extreme, if 50 percent or more of the pavement had a thickness less than the design value, this might be considered unacceptable and would imply that the RQL is 50 percent defective. If so, it can be seen from Figure 12.2 that, if such a defective pavement were constructed, the highway agency's risk of accepting it would be only $P = 0.13$.

12.2.6 As in the previous example, the AQL and RQL values of 10 and 50 pecent defective, respectively, have been assumed only for discussion purposes. It is the responsibility of the highway agency to decide what levels are appropriate for each particular application.

12.2.7 The acceptance requirement, given in Equations 12.1 through 12.3, is presented in the form of a minimum acceptable value (k) of the quality index (Q). It could also be given as a maximum allowable estimated percent defective (M). To determine what the appropriate value would be, it is necessary to interpolate in Appendix B to determine that a minimum acceptable value of $k = 0.6$ corresponds to a maximum allowable estimated percent defective of $M = 29.1$. For practical purposes, this would probably be rounded off to $M = 29$. The acceptance procedure utilizing this approach would read as follows:

12.2.7.1 Take $n = 5$ randomly located cores and determine the corresponding thickness values. Compute the mean ($\bar{X}$) and standard deviation (S) and, from these, the Q value in accordance with Equation 12.4.

$$Q = \frac{\bar{X} - \text{Design Thickness}}{S} \qquad (12.4)$$

Consult Appendix C to determine the estimated lot percent defective (PD). For the lot to be judged acceptable, it is required that PD ≤ M, i.e., PD ≤ 29.

12.2.7.2 This plan is identical to the plan requiring that Q ≥ k, i.e., Q ≥ 0.6. It is important to understand that, although the AQL might be considered to be 10 percent defective, the acceptance procedure must allow the sample estimate of percent defective to be as large as PD = M = 29. This allowance takes into account sampling variability. When the lot is truly 10 percent defective, there is a small probability of P = 0.07 that the estimated percent defective will be equal to or greater than PD = 29. Consequently, when the acceptance requirement is set at M = 29, there is a probability of P = 0.93 that the lot will be accepted, as illustrated by the OC curve in Figure 12.2.

12.3 Analysis of Variables Acceptance Plan for Air Voids of Bituminous Concrete

12.3.1 An existing acceptance plan for bituminous concrete requires that, for each 10,000 square yards (or less) of paving, n = 10 cores are to be taken at random locations. The percentage of air voids for each core is to be determined by a standard test. The mean ($\overline{X}$) and standard deviation (S) of the test results are to be calculated and, from these, the quality indexes (Q) associated with lower and upper limits of L = 2.0 and U = 7.0 are to be computed with equations 12.5 and 12.6.

$$Q_L = \frac{\overline{X} - 2.0}{S} \tag{12.5}$$

$$Q_U = \frac{7.0 - \overline{X}}{S} \tag{12.6}$$

The percent defective estimates associated with these two Q values are obtained from the appropriate tables in Appendix C and added together to obtain the total

lot percent defective. For the lot to be judged acceptable, it is required that the total lot percent defective (PD) estimated in this manner be no larger than M = 25.

12.3.2 It is stated in the footnote of the tables in Appendix B that the acceptance probabilities are approximately correct for double-limit acceptance procedures such as this. For a sample size of n = 10 and a maximum allowable estimated percent defective of M = 25, the following values are interpolated from the table:

LOT PERCENT DEFECTIVE	PROBABILITY OF ACCEPTANCE
10	0.960
20	0.685
30	0.340
40	0.115
50	0.025

12.3.3 It can be seen by scanning these values that this acceptance plan will accept most lots having a percent defective of 10 or less and will usually reject lots having much more than 40 percent defective. This plan would probably be regarded as satisfactory.

12.4 Development of Attributes Acceptance Plan for Depth of Cover Over Reinforcing Steel

12.4.1 It is known that the spalling of portland cement concrete bridge decks is attributable primarily to corrosion of the top mat of reinforcing steel and that a clear depth of cover of 2.0 inches or more is one method of preventing or delaying this form of distress. Instruments utilizing magnetic flux are available that can accurately measure the depth of cover through the

hardened concrete. It is desired to develop an acceptance plan to assure that at least 90 percent of the reinforcing steel has a depth of cover of 2.0 inches, or more.

12.4.2 If only a single-sided specification is written to assure adequate cover over the reinforcing steel, this will encourage contractors to set the steel deeper than desired, reducing the moment arm from the neutral axis and threatening the structural capacity of the bridge deck. Consequently, an upper limit on the depth of steel is also required. Since the location of the steel can be controlled quite accurately with the proper number and type of chairs and ties, it is considered reasonable to set the lower and upper limits fairly close to each other. It is decided to develop a plan that would assure that at least 90 percent of the steel lies between depths of 2.0 and 3.0 inches or, conversely, that no more than 10 percent falls outside these limits. Therefore, by definition, the acceptable quality level (AQL) is 10 percent defective.

12.4.3 It is important to note that this would be an unreasonable and unfair requirement if data existed showing that contractors were incapable of achieving sufficient control to keep 90 percent of the lot within these two limits. For example, if depth of cover were normally distributed with a typical standard deviation of σ = 0.25 inches, and the contractor were capable of controlling the mean depth exactly at the midrange value of 2.5 inches, then the limits would fall at plus and minus two standard deviations and the lot percent defective would be about 5.0 percent. This degree of control on the part of the contractor would be barely adequate, not allowing much margin for error. If depth of cover were not normally distributed, the degree of control that can reasonably be expected would be determined by a direct count of the available data.

12.4.4 Whether or not the highway agency chooses to define a specific level of percent defective as rejectable (RQL), it must recognize that larger values of lot percent defective are very undesirable for two reasons. First, as the percentage of reinforcing steel having less than 2.0 inches of cover increases, so does the likelihood of premature deck failure and the attendant costly repairs. Second, if the reinforcing steel is placed too deep in the deck slab, the deck may not be able to develop the load-carrying capacity for which it was designed. Consequently, when developing this acceptance procedure, an OC curve that drops off rapidly as percent defective increases will be desired.

12.4.5 Normally, because the depth measurements are continuous and variables procedures make more efficient use of the data, it would be desirable to use a variables acceptance procedure. However, for purposes of this illustration, it is assumed that the highway agency has examined many sets of data and has found the distributions of depth of cover over reinforcing steel to be quite skewed. Since variables acceptance theory assumes that the population being sampled is normally distributed, while attributes acceptance procedures are uninfluenced by distributional form, an attributes plan is considered appropriate for this application.

12.4.6 When an attributes type of acceptance procedure is applied to continuous data, the lot size is considered to be infinite. Accordingly, the last table in Appendix A has been constructed for an infinite lot size.

12.4.7 Since measurements of depth of cover are nondestructive and fairly easy to obtain, it is decided to try a sample size of $n = 20$. For an acceptance number of $c = 4$, the following values are obtained for an infinite lot size in Appendix A:

LOT PERCENT DEFECTIVE	PROBABILITY OF ACCEPTANCE
5	1.00
10 (AQL)	0.96
15	0.83
20	0.63
25	0.41
30	0.24
35	0.12
40	0.05
45	0.02
50	0.01

The AQL of 10 percent defective will be accepted approximately 96 percent of the time. For lots having greater amounts of percent defective, the OC curve drops quite rapidly, as desired. Decks having 40 percent defective will be accepted only about 5.0 percent of the time while those with 50 percent defective or more will almost always be rejected.

12.5 Development of Attributes Acceptance Plan for Pavement Thickness

12.5.1 A highway agency wishes to develop an acceptance procedure for pavement thickness that is as uncomplicated as possible, involving no statistical calculations or special tables. The pavement will be considered satisfactory if at least 90 percent of it has a thickness greater than the design value. Therefore, the AQL may be considered to be 10 percent defective and it is desired that this level of quality have a relatively high probability of acceptance. At the other extreme, if 40 percent or more of the pavement is less than the design thickness, it has been decided that this will be regarded as rejectable (RQL) and a correspondingly low probability of acceptance is desired.

12.5.2 For purposes of this example, suppose that a seller's risk of $\alpha = 0.05$ and a buyer's risk of $\beta = 0.10$ are desired. The corresponding acceptance probabilities are P = 0.95 at the AQL and P = 0.10 at the RQL.

12.5.3 The requirement for simplicity dictates an attributes plan. When attributes acceptance procedures are applied to continuous data (thickness in this case), the lot size is considered to be infinite. By scanning the rows and columns of the last table in Appendix A, it is observed that a plan with a sample size of n = 15 and an acceptance number of c = 3 produces very nearly the desired risk levels. (Because the sample size and acceptance number are discrete values, it is not possible to match the risks exactly.) The following values are obtained:

LOT PERCENT DEFECTIVE	PROBABILITY OF ACCEPTANCE
10 (AQL)	0.94
20	0.65
30	0.30
40 (RQL)	0.09
50	0.02

12.5.4 It can be seen from these values that the basic objectives have been well satisfied. A good quality pavement having 10 percent defective or less will have a probability of acceptance of at least P = 0.94. If the pavement is 40 percent defective or more, the probability of acceptance will be P = 0.09 or less.

12.5.5 The completed acceptance procedure will require that n = 15 cores be taken at random locations within a specified lot size. Provided that no more than c = 3 cores are less than the design thickness, the lot will be judged acceptable.

12.6 Development of Variables Acceptance Procedure for Percent Passing #200 Sieve

12.6.1 An acceptance procedure is to be prepared for a crushed-stone base course. The percentage by weight of material passing the #200 sieve is known to be a significant performance characteristic. Experience has shown that bases having 7.0 percent or less of minus #200 material have performed well but bases exceeding 10.0 percent of minus #200 material have poor stability and drainage and tend to be frost susceptible. For this example, it is assumed that an analysis of historical data has shown the test results on minus #200 material to be approximately normally distributed with a typical standard deviation of about σ = 1.0 percent.

12.6.2 The information provided in this example is sufficient to develop a workable acceptance plan but it is not in the most useful form. For the types of acceptance plans covered in this standard, definitions of acceptable and unacceptable quality must be stated in terms of the percentage of material falling outside some specification limit (or pair of limits). Instead, the information provided in Section 12.6.1 is presented in terms of two average levels of minus #200 material that experience has shown have produced satisfactory and unsatisfactory results, respectively. As a reasonable approximation, these average values can be associated with the typical standard deviation of σ = 1.0 percent by means of normal distribution theory to provide guidance in establishing both the acceptable quality level (AQL) and the rejectable quality level (RQL) in terms of percent defective. The acceptance plan will then perform as desired as long as the standard deviation is reasonably close to the typical value and, if conservatively designed, it should provide ample protection even when the standard deviation is larger than usual.

12.6.3 Since there is no reason to impose a lower limit on minus #200 material, this will be a single-limit specification. A logical choice for this

limit is 7.0 percent, the level of minus #200 material that is known to be clearly satisfactory. It is believed that the base will perform well as long as 90 percent or more of the material has a minus #200 value of 7.0 percent or less. Therefore, the AQL is defined as 10 percent defective above the limit of 7.0 percent. This is a relatively conservative definition because, even if the standard deviation were considerably larger than the typical value, there is little chance that any of the material in the normal distribution representing AQL quality would reach the known critical value of 10.0 percent minus #200 material. The AQL is illustrated in the upper diagram in Figure 12.3.

12.6.4 To determine the level of percent defective to be defined as the RQL, it is noted that if this same distribution had 50 percent of its material above the limit of 7.0 percent, its upper tail would extend just to the critical value of 10.0 percent minus #200 material. On those few occasions that the standard deviation was substantially larger than the typical value of $\sigma = 1.0$ percent, a relatively small portion of the distribution would extend above the critical value of 10.0 percent. As the amount of material exceeding 7.0 percent minus #200 material increases above 50 percent, however, progressively more will exceed the critical value of 10.0 percent and performance problems might be expected to develop. This provides a rational basis for defining the RQL as 50 percent defective above the limit of 7.0 percent minus #200 material, as illustrated in the lower diagram in Figure 12.3.

12.6.5 For this example, it will be assumed that the highway agency wishes to control both the seller's risk and the buyer's risk at $\alpha = \beta = 0.05$. The required acceptance probabilities at the AQL and RQL must then be $P = 0.95$ and $P = 0.05$, respectively. It is seen from Appendix B that a variables plan with a sample size of $n = 8$ and a maximum allowable estimated percent defective of $M = 26$ meets these requirements exactly:

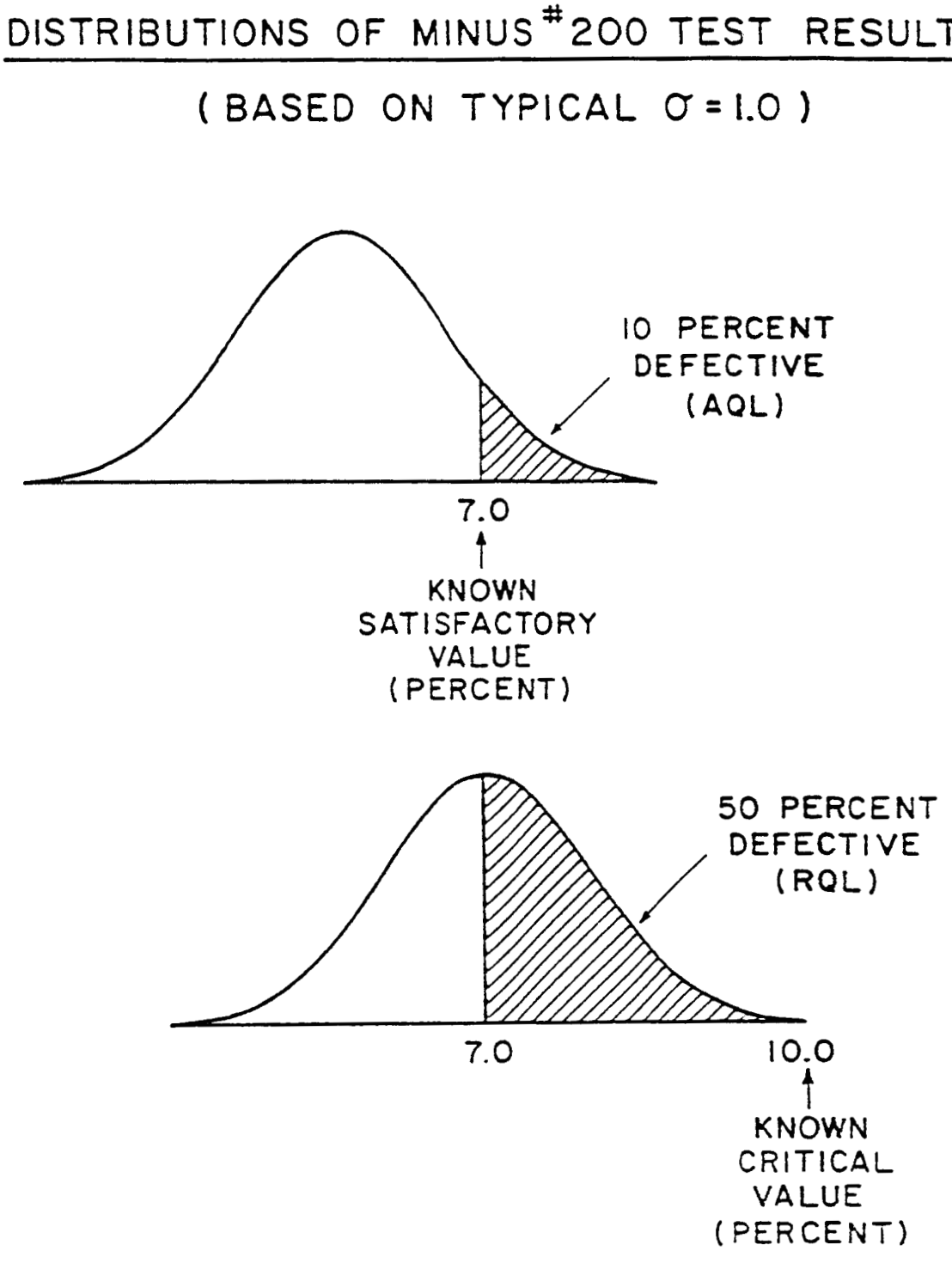


FIGURE 12.3 Illustration of Definitions of AQL and RQL for Acceptance Plan for Minus #200 Material Described in Example 12.6

LOT PERCENT DEFECTIVE	PROBABILITY OF ACCEPTANCE
10 (AQL)	0.95
20	0.70
30	0.38
40	0.16
50 (RQL)	0.05
60	0.01

12.6.6 A suitable lot size must be chosen and the method of test specified. The acceptance procedure will require that the mean ($\bar{X}$) and standard deviation (S) be calculated from n = 8 random samples and used to compute the Q statistic in Equation 12.7. The corresponding percent defective estimate is obtained from the appropriate table in Appendix C. For the lot to be judged acceptable, the percent defective estimate must be no larger than M = 26. (Alternatively, it could be required that the Q statistic be equal to or greater than k = 0.665.)

$$Q = \frac{7.0 - \bar{X}}{S} \tag{12.7}$$

12.7 Development of Variables Acceptance Procedure for Asphalt Cement Content of Bituminous Concrete

12.7.1 Asphalt cement (AC) content is known to be an extremely important quality characteristic of bituminous concrete. If the AC content is too low, the pavement will not be properly bound together and may dry out and crack prematurely. If the AC content is too high, the pavement may be unstable and excess AC may be flushed out onto the surface, producing a dangerously slippery condition. A two-sided specification is desired to control the AC content within acceptable limits.

12.7.2 Experience has shown that an optimal AC content with a particular pavement mix is about 6.0 percent, that values as low as 5.0 percent and as high as 7.0 percent are about as great a departure from the optimum value that should be tolerated, and that AC content tends to be normally distributed with a typical within-lot standard deviation of approximately $\sigma = 0.25$ percent.

12.7.3 The highway agency is considering setting the AQL at 10 percent defective outside the limits of L = 5.0 percent and U = 7.0 percent. Figure 12.4 illustrates that, for a typical standard deviation of $\sigma = 0.25$ percent, the bituminous concrete supplier would have to control his AC content between mean values of 5.32 and 6.68 percent. Since this appears to be well within the capability of most suppliers, an AQL of 10 percent defective is regarded as reasonable and practical.

12.7.4 It is believed that, if the test results indicate that 50 percent or more of a lot has AC values falling outside the limits of L = 5.0 percent and U = 7.0 percent, the performance of the pavement is likely to be seriously impaired. Accordingly, the RQL is defined as 50 percent defective.

12.7.5 Since AC content is a highly important characteristic that cannot readily be corrected once the material has been delivered and placed, it is desired to develop an acceptance procedure that can be relied upon to make a high percentage of correct decisions. The risk of falsely rejecting a bituminous concrete lot that is truly at the AQL of 10 percent defective must be very low. Conversely, the probability of accepting a lot that is 50 percent or more defective should also be quite small.

12.7.6 By consulting the operating characteristic tables for variables acceptance plans in Appendix B, it is seen that a plan using a sample size of n = 15 and a maximum allowable estimated percent defective of M = 30 is satisfactory. The following values are obtained:

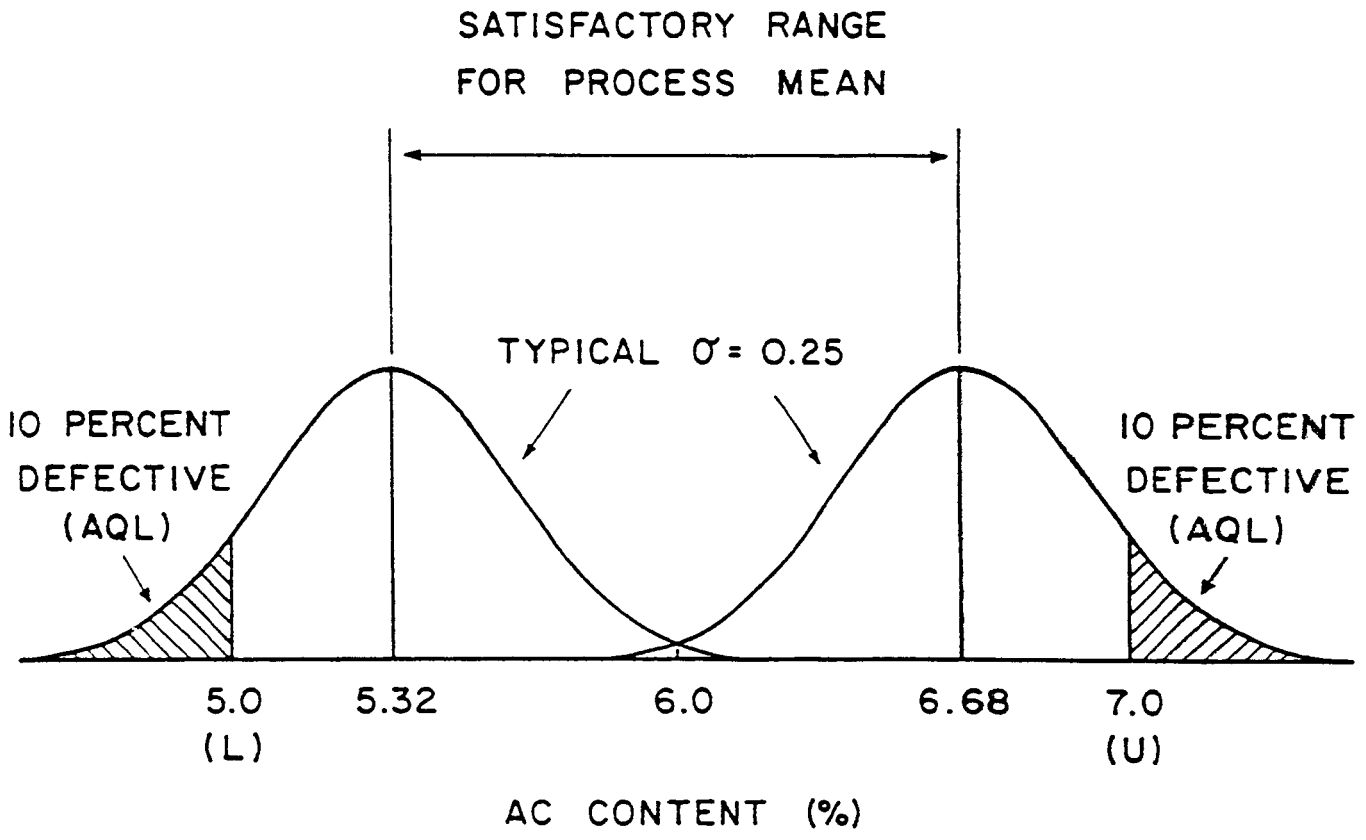


FIGURE 12.4 Illustration of Latitude Permitted in Setting of Process Mean Described in Example 12.7

LOT PERCENT DEFECTIVE	PROBABILITY OF ACCEPTANCE
10 (AQL)	1.00
20	0.87
30	0.50
40	0.16
50 (RQL)	0.03

12.7.7 The acceptance procedure will require that 15 random samples be obtained from each lot and the AC contents determined by a standard test. The mean ($\overline{X}$) and standard deviation (S) are to be calculated and the Q values computed with Equations 12.8 and 12.9 using limits of L = 5.0 and U = 7.0.

$$Q_L = \frac{\overline{X} - 5.0}{S} \qquad (12.8)$$

$$Q_U = \frac{7.0 - \overline{X}}{S} \qquad (12.9)$$

The percent defective estimates associated with these two Q values are to be obtained from the appropriate table in Appendix C and added together to obtain the total lot percent defective. For the lot to be judged acceptable, the total lot percent defective estimated in this manner must be no larger than M = 30.

12.8 Use of Percent Defective Estimation Tables (Appendix C)

12.8.1 If the acceptance procedure is a single-limit plan that specifies a minimum value (k) that the quality index (Q) must equal or exceed, then it is not actually necessary to use the percent defective estimation tables in Appendix C. However, both highway agency and contractor's personnel may find

it easier to relate to the concept of percent defective than to the more abstract Q statistic. For double-limit specifications, it is recommended that the acceptance requirements be stated only in terms of the maximum allowable percent defective (M).

12.8.2 The percent defective estimation tables in Appendix C cover a wide range of sample sizes, considerably more than would ever be used in a single acceptance procedure. For acceptance procedures that make use of only one or two sample sizes, it is possible to construct much more compact tables such as that shown in Figure 12.5. With this format, there is a separate short table for each sample size.

12.8.3 Example Based on Pavement Thickness Specification

12.8.3.1 Consider the variables acceptance procedure for pavement thickness presented in Example 12.2. The sample size is n = 5 and the maximum allowable estimated percent defective was determined in Section 12.2.7 to be M = 29 percent. For a design thickness of 9.0 inches, the Q statistic to be computed is given by Equation 12.10.

$$Q = \frac{\overline{X} - 9.0}{S} \qquad (12.10)$$

12.8.3.2 Suppose the n = 5 core thicknesses obtained for a paving lot are 8.9, 9.1, 9.1, 9.2, and 9.3 inches. The mean and standard deviation are $\overline{X} = 9.12$ and $S = 0.148$ and the Q statistic is computed to be $Q = (9.12 - 9.0)/0.148 = 0.81$. The estimated percent defective may be obtained from either Figure 12.5 or Appendix C and is found to be PD = 22.19 percent. Since this is less than the maximum allowable estimated percent defective of M = 29, the lot is acceptable.

VARIABILITY-UNKNOWN PROCEDURE — SAMPLE SIZE 5 — STANDARD DEVIATION METHOD

Q	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	50.00	49.64	49.29	48.93	48.58	48.22	47.86	47.51	47.15	46.80
0.1	46.44	46.09	45.73	45.38	45.02	44.67	44.31	43.96	43.60	43.25
0.2	42.90	42.54	42.19	41.84	41.48	41.13	40.78	40.43	40.08	39.72
0.3	39.37	39.02	38.67	38.32	37.97	37.62	37.28	36.93	36.58	36.23
0.4	35.88	35.54	35.19	34.85	34.50	34.16	33.81	33.47	33.12	32.78
0.5	32.44	32.10	31.76	31.42	31.08	30.74	30.40	30.06	29.73	29.39
0.6	29.05	28.72	28.39	28.05	27.72	27.39	27.06	26.73	26.40	26.07
0.7	25.74	25.41	25.09	24.76	24.44	24.11	23.79	23.47	23.15	22.83
0.8	22.51	22.19	21.87	21.56	21.24	20.93	20.62	20.31	20.00	19.69
0.9	19.38	19.07	18.77	18.46	18.16	17.86	17.55	17.25	16.96	16.66
1.0	16.36	16.07	15.78	15.48	15.19	14.91	14.62	14.33	14.05	13.76
1.1	13.48	13.20	12.93	12.65	12.37	12.10	11.83	11.56	11.29	11.02
1.2	10.76	10.50	10.23	9.97	9.72	9.46	9.21	8.96	8.71	8.46
1.3	8.21	7.97	7.73	7.49	7.25	7.02	6.79	6.56	6.33	6.10
1.4	5.88	5.66	5.44	5.23	5.02	4.81	4.60	4.39	4.19	3.99
1.5	3.80	3.61	3.42	3.23	3.05	2.87	2.69	2.52	2.35	2.19
1.6	2.03	1.87	1.72	1.57	1.42	1.28	1.15	1.02	0.89	0.77
1.7	0.66	0.55	0.45	0.36	0.27	0.19	0.12	0.06	0.02	0.00

NUMBERS IN THE BODY OF THE TABLE ARE ESTIMATES OF LOT PERCENT DEFECTIVE CORRESPONDING TO SPECIFIC VALUES OF Q, THE QUALITY INDEX. FOR VALUES OF Q GREATER THAN OR EQUAL TO ZERO, THE ESTIMATE OF PERCENT DEFECTIVE IS READ DIRECTLY FROM THE TABLE. FOR VALUES OF Q LESS THAN ZERO, THE TABLE VALUE MUST BE SUBTRACTED FROM 100.

FIGURE 12.5 Format for Individual Tables for Estimation of Percent Defective

A P P E N D I X A

OPERATING CHARACTERISTIC TABLES FOR ATTRIBUTES PLANS

TABLE A Operating Characteristics of Attributes Acceptance Plans

ATTRIBUTES ACCEPTANCE PLANS — LOT SIZE = 20

SAMPLE SIZE (n)	ACCEPTANCE NUMBER (c)	PROBABILITY OF ACCEPTANCE FOR SELECTED LEVELS OF LOT PERCENT DEFECTIVE													
		5	10	15	20	25	30	35	40	45	50	55	60	65	70
1	0	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30
2	0	0.90	0.81	0.72	0.63	0.55	0.48	0.41	0.35	0.29	0.24	0.19	0.15	0.11	0.08
3	0	0.85	0.72	0.60	0.49	0.40	0.32	0.25	0.19	0.14	0.11	0.07	0.05	0.03	0.02
3	1	1.00	0.98	0.95	0.91	0.86	0.80	0.73	0.66	0.58	0.50	0.42	0.34	0.27	0.20
4	0	0.80	0.63	0.49	0.38	0.28	0.21	0.15	0.10	0.07	0.04	0.03	0.01	0.01	0.00
4	1	1.00	0.97	0.91	0.84	0.75	0.66	0.56	0.47	0.37	0.29	0.22	0.15	0.10	0.06
5	1	1.00	0.95	0.86	0.75	0.63	0.52	0.41	0.31	0.22	0.15	0.10	0.06	0.03	0.01
5	2	1.00	1.00	0.99	0.97	0.93	0.87	0.79	0.70	0.60	0.50	0.40	0.30	0.21	0.13
6	1	1.00	0.92	0.80	0.66	0.52	0.39	0.28	0.19	0.12	0.07	0.04	0.02	0.01	0.00
6	2	1.00	1.00	0.98	0.94	0.87	0.77	0.66	0.54	0.43	0.31	0.22	0.14	0.08	0.04
7	1	1.00	0.89	0.73	0.56	0.41	0.28	0.18	0.11	0.06	0.03	0.01	0.00	0.00	0.00
7	2	1.00	1.00	0.97	0.90	0.79	0.66	0.53	0.39	0.27	0.17	0.10	0.05	0.02	0.01
8	1	1.00	0.85	0.66	0.47	0.31	0.19	0.11	0.05	0.02	0.01	0.00	0.00	0.00	0.00
8	2	1.00	1.00	0.95	0.85	0.70	0.54	0.38	0.26	0.16	0.08	0.04	0.02	0.00	0.00
8	3	1.00	1.00	1.00	0.99	0.94	0.86	0.75	0.61	0.46	0.32	0.20	0.11	0.05	0.02
9	1	1.00	0.81	0.58	0.37	0.22	0.12	0.06	0.02	0.01	0.00	0.00	0.00	0.00	0.00
9	2	1.00	1.00	0.93	0.78	0.60	0.43	0.27	0.16	0.08	0.03	0.01	0.00	0.00	0.00
9	3	1.00	1.00	1.00	0.97	0.90	0.78	0.63	0.46	0.31	0.18	0.09	0.04	0.01	0.00
10	1	1.00	0.76	0.50	0.29	0.15	0.07	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00
10	2	1.00	1.00	0.89	0.71	0.50	0.31	0.17	0.08	0.03	0.01	0.00	0.00	0.00	0.00
10	3	1.00	1.00	1.00	0.96	0.85	0.69	0.50	0.32	0.18	0.09	0.03	0.01	0.00	0.00
15	2	1.00	1.00	0.60	0.25	0.07	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15	3	1.00	1.00	1.00	0.72	0.37	0.13	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
15	4	1.00	1.00	1.00	1.00	0.81	0.48	0.21	0.06	0.01	0.00	0.00	0.00	0.00	0.00
15	5	1.00	1.00	1.00	1.00	1.00	0.87	0.59	0.30	0.10	0.02	0.00	0.00	0.00	0.00
20	2	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	3	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	4	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	5	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	6	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

PROBABILITY OF ACCEPTANCE IS A FUNCTION OF LOT SIZE FOR ATTRIBUTES PLANS USING DISCRETE DATA. FOR VARIABLE LOT SIZES, IT WILL BE NECESSARY TO PLOT BOUNDING OPERATING CHARACTERISTIC CURVES. THE VALUES IN THIS TABLE ARE APPROPRIATE FOR BOTH SINGLE-LIMIT AND DOUBLE-LIMIT APPLICATIONS AND ARE UNINFLUENCED BY THE DISTRIBUTIONAL FORM OF THE POPULATION.

TABLE A Operating Characteristics of Attributes Acceptance Plans (Continued)

ATTRIBUTES ACCEPTANCE PLANS — LOT SIZE = 100

SAMPLE SIZE (n)	ACCEPTANCE NUMBER (c)	PROBABILITY OF ACCEPTANCE FOR SELECTED LEVELS OF LOT PERCENT DEFECTIVE													
		5	10	15	20	25	30	35	40	45	50	55	60	65	70
1	0	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30
2	0	0.90	0.81	0.72	0.64	0.56	0.49	0.42	0.36	0.30	0.25	0.20	0.16	0.12	0.09
3	0	0.86	0.73	0.61	0.51	0.42	0.34	0.27	0.21	0.16	0.12	0.09	0.06	0.04	0.03
3	1	0.99	0.97	0.94	0.90	0.85	0.79	0.72	0.65	0.58	0.50	0.42	0.35	0.28	0.21
4	0	0.81	0.65	0.52	0.40	0.31	0.23	0.17	0.12	0.09	0.06	0.04	0.02	0.01	0.01
4	1	0.99	0.95	0.89	0.82	0.74	0.65	0.56	0.47	0.39	0.31	0.24	0.17	0.12	0.08
5	1	0.98	0.92	0.84	0.74	0.63	0.53	0.42	0.33	0.25	0.18	0.13	0.08	0.05	0.03
5	2	1.00	0.99	0.98	0.95	0.90	0.84	0.77	0.69	0.60	0.50	0.40	0.31	0.23	0.16
6	1	0.97	0.89	0.78	0.66	0.53	0.41	0.31	0.23	0.16	0.10	0.06	0.04	0.02	0.01
6	2	1.00	0.99	0.96	0.91	0.84	0.75	0.65	0.54	0.44	0.34	0.25	0.17	0.11	0.06
7	1	0.96	0.86	0.72	0.57	0.44	0.32	0.22	0.15	0.09	0.06	0.03	0.02	0.01	0.00
7	2	1.00	0.98	0.93	0.86	0.76	0.65	0.53	0.42	0.31	0.22	0.14	0.09	0.05	0.02
8	1	0.95	0.82	0.66	0.50	0.36	0.24	0.16	0.10	0.06	0.03	0.01	0.01	0.00	0.00
8	2	1.00	0.97	0.90	0.80	0.68	0.55	0.42	0.31	0.21	0.13	0.08	0.04	0.02	0.01
8	3	1.00	1.00	0.98	0.95	0.90	0.81	0.71	0.60	0.48	0.36	0.25	0.16	0.10	0.05
9	1	0.94	0.78	0.60	0.43	0.29	0.18	0.11	0.06	0.03	0.02	0.01	0.00	0.00	0.00
9	2	1.00	0.96	0.87	0.74	0.60	0.46	0.33	0.22	0.14	0.08	0.04	0.02	0.01	0.00
9	3	1.00	0.99	0.97	0.92	0.84	0.74	0.61	0.48	0.35	0.24	0.15	0.09	0.05	0.02
10	1	0.92	0.74	0.54	0.36	0.23	0.14	0.07	0.04	0.02	0.01	0.00	0.00	0.00	0.00
10	2	0.99	0.94	0.83	0.68	0.52	0.37	0.25	0.15	0.09	0.05	0.02	0.01	0.00	0.00
10	3	1.00	0.99	0.96	0.89	0.79	0.65	0.51	0.37	0.25	0.16	0.09	0.05	0.02	0.01
15	2	0.98	0.83	0.60	0.38	0.21	0.11	0.05	0.02	0.01	0.00	0.00	0.00	0.00	0.00
15	3	1.00	0.96	0.84	0.65	0.45	0.28	0.15	0.07	0.03	0.01	0.00	0.00	0.00	0.00
15	4	1.00	0.99	0.95	0.85	0.70	0.51	0.34	0.20	0.10	0.05	0.02	0.01	0.00	0.00
15	5	1.00	1.00	0.99	0.95	0.87	0.73	0.57	0.39	0.24	0.13	0.06	0.02	0.01	0.00
20	2	0.95	0.68	0.38	0.18	0.07	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	3	0.99	0.89	0.65	0.39	0.20	0.08	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00
20	4	1.00	0.97	0.85	0.64	0.40	0.21	0.09	0.03	0.01	0.00	0.00	0.00	0.00	0.00
20	5	1.00	1.00	0.95	0.83	0.62	0.40	0.22	0.10	0.04	0.01	0.00	0.00	0.00	0.00
20	6	1.00	1.00	0.99	0.94	0.81	0.62	0.40	0.22	0.10	0.04	0.01	0.00	0.00	0.00
30	3	0.97	0.65	0.28	0.08	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	4	1.00	0.86	0.51	0.21	0.06	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	5	1.00	0.96	0.73	0.40	0.16	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	6	1.00	0.99	0.89	0.62	0.31	0.12	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00
30	7	1.00	1.00	0.96	0.80	0.51	0.24	0.08	0.02	0.00	0.00	0.00	0.00	0.00	0.00
30	8	1.00	1.00	0.99	0.91	0.70	0.41	0.18	0.06	0.01	0.00	0.00	0.00	0.00	0.00
50	5	1.00	0.63	0.13	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	6	1.00	0.84	0.29	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	7	1.00	0.95	0.50	0.11	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	8	1.00	0.99	0.71	0.23	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	9	1.00	1.00	0.87	0.40	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	10	1.00	1.00	0.95	0.60	0.18	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	11	1.00	1.00	0.99	0.77	0.32	0.06	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	8	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	9	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	10	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	11	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	12	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	13	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	14	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	15	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	16	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	17	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	18	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

PROBABILITY OF ACCEPTANCE IS A FUNCTION OF LOT SIZE FOR ATTRIBUTES PLANS USING DISCRETE DATA. FOR VARIABLE LOT SIZES, IT WILL BE NECESSARY TO PLOT BOUNDING OPERATING CHARACTERISTIC CURVES. THE VALUES IN THIS TABLE ARE APPROPRIATE FOR BOTH SINGLE-LIMIT AND DOUBLE-LIMIT APPLICATIONS AND ARE UNINFLUENCED BY THE DISTRIBUTIONAL FORM OF THE POPULATION.

TABLE A Operating Characteristics of Attributes Acceptance Plans (Continued)

ATTRIBUTES ACCEPTANCE PLANS — LOT SIZE = 500

SAMPLE SIZE (n)	ACCEPTANCE NUMBER (c)	PROBABILITY OF ACCEPTANCE FOR SELECTED LEVELS OF LOT PERCENT DEFECTIVE													
		5	10	15	20	25	30	35	40	45	50	55	60	65	70
1	0	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30
2	0	0.90	0.81	0.72	0.64	0.56	0.49	0.42	0.36	0.30	0.25	0.20	0.16	0.12	0.09
3	0	0.86	0.73	0.61	0.51	0.42	0.34	0.27	0.22	0.17	0.12	0.09	0.06	0.04	0.03
3	1	0.99	0.97	0.94	0.90	0.84	0.78	0.72	0.65	0.57	0.50	0.43	0.35	0.28	0.22
4	0	0.81	0.66	0.52	0.41	0.32	0.24	0.18	0.13	0.09	0.06	0.04	0.03	0.01	0.01
4	1	0.99	0.95	0.89	0.82	0.74	0.65	0.56	0.47	0.39	0.31	0.24	0.18	0.13	0.08
5	1	0.98	0.92	0.84	0.74	0.63	0.53	0.43	0.34	0.26	0.19	0.13	0.09	0.05	0.03
5	2	1.00	0.99	0.97	0.94	0.90	0.84	0.77	0.68	0.59	0.50	0.41	0.32	0.23	0.16
6	1	0.97	0.89	0.78	0.66	0.53	0.42	0.32	0.23	0.16	0.11	0.07	0.04	0.02	0.01
6	2	1.00	0.98	0.95	0.90	0.83	0.75	0.65	0.54	0.44	0.34	0.25	0.18	0.12	0.07
7	1	0.96	0.85	0.72	0.58	0.44	0.33	0.23	0.16	0.10	0.06	0.03	0.02	0.01	0.0
7	2	1.00	0.98	0.93	0.85	0.76	0.65	0.53	0.42	0.32	0.22	0.15	0.09	0.05	0.0
8	1	0.94	0.81	0.66	0.50	0.37	0.25	0.17	0.10	0.06	0.03	0.02	0.01	0.00	0.
8	2	0.99	0.96	0.90	0.80	0.68	0.55	0.43	0.31	0.22	0.14	0.09	0.05	0.02	0.
8	3	1.00	1.00	0.98	0.95	0.89	0.81	0.71	0.59	0.48	0.36	0.26	0.17	0.10	0.
9	1	0.93	0.78	0.60	0.43	0.30	0.19	0.12	0.07	0.04	0.02	0.01	0.00	0.00	0.0
9	2	0.99	0.95	0.86	0.74	0.60	0.46	0.34	0.23	0.15	0.09	0.05	0.02	0.01	0.00
9	3	1.00	0.99	0.97	0.92	0.84	0.73	0.61	0.48	0.36	0.25	0.16	0.10	0.05	0.02
10	1	0.92	0.74	0.54	0.37	0.24	0.15	0.08	0.04	0.02	0.01	0.00	0.00	0.00	0.00
10	2	0.99	0.93	0.82	0.68	0.52	0.38	0.26	0.16	0.10	0.05	0.03	0.01	0.00	0.00
10	3	1.00	0.99	0.95	0.88	0.78	0.65	0.51	0.38	0.26	0.17	0.10	0.05	0.02	0.01
15	2	0.97	0.82	0.60	0.39	0.23	0.12	0.06	0.03	0.01	0.00	0.00	0.00	0.00	0.00
15	3	1.00	0.95	0.83	0.65	0.46	0.29	0.17	0.09	0.04	0.02	0.01	0.00	0.00	0.00
15	4	1.00	0.99	0.94	0.84	0.69	0.51	0.35	0.21	0.12	0.06	0.02	0.01	0.00	0.00
15	5	1.00	1.00	0.98	0.94	0.85	0.72	0.56	0.40	0.26	0.15	0.07	0.03	0.01	0.00
20	2	0.93	0.68	0.40	0.20	0.09	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	3	0.99	0.87	0.65	0.41	0.22	0.10	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00
20	4	1.00	0.96	0.83	0.63	0.41	0.23	0.11	0.05	0.02	0.01	0.00	0.00	0.00	0.00
20	5	1.00	0.99	0.94	0.81	0.62	0.41	0.24	0.12	0.05	0.02	0.01	0.00	0.00	0.00
20	6	1.00	1.00	0.98	0.92	0.79	0.61	0.41	0.25	0.13	0.05	0.02	0.01	0.00	0.00
30	3	0.95	0.65	0.31	0.12	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	4	0.99	0.83	0.52	0.25	0.09	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	5	1.00	0.93	0.71	0.42	0.19	0.07	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	6	1.00	0.98	0.85	0.61	0.34	0.15	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00
30	7	1.00	0.99	0.94	0.77	0.51	0.27	0.12	0.04	0.01	0.00	0.00	0.00	0.00	0.00
30	8	1.00	1.00	0.98	0.88	0.68	0.43	0.22	0.09	0.03	0.01	0.00	0.00	0.00	0.00
50	5	0.97	0.62	0.21	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	6	0.99	0.78	0.35	0.09	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	7	1.00	0.89	0.52	0.18	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	8	1.00	0.95	0.67	0.30	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	9	1.00	0.98	0.80	0.44	0.15	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	10	1.00	0.99	0.89	0.59	0.25	0.07	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	11	1.00	1.00	0.95	0.72	0.37	0.13	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	8	0.96	0.30	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	9	0.99	0.44	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	10	1.00	0.59	0.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	11	1.00	0.72	0.14	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	12	1.00	0.83	0.22	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	13	1.00	0.90	0.33	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	14	1.00	0.95	0.45	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	15	1.00	0.98	0.57	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	16	1.00	0.99	0.69	0.16	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	17	1.00	1.00	0.79	0.25	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	18	1.00	1.00	0.86	0.34	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

PROBABILITY OF ACCEPTANCE IS A FUNCTION OF LOT SIZE FOR ATTRIBUTES PLANS USING DISCRETE DATA. FOR VARIABLE LOT SIZES, IT WILL BE NECESSARY TO PLOT BOUNDING OPERATING CHARACTERISTIC CURVES. THE VALUES IN THIS TABLE ARE APPROPRIATE FOR BOTH SINGLE-LIMIT AND DOUBLE-LIMIT APPLICATIONS AND ARE UNINFLUENCED BY THE DISTRIBUTIONAL FORM OF THE POPULATION.

TABLE A Operating Characteristics of Attributes Acceptance Plans (Continued)

ATTRIBUTES ACCEPTANCE PLANS — LOT SIZE = INFINITE

SAMPLE SIZE (n)	ACCEPTANCE NUMBER (c)	PROBABILITY OF ACCEPTANCE FOR SELECTED LEVELS OF LOT PERCENT DEFECTIVE													
		5	10	15	20	25	30	35	40	45	50	55	60	65	70
1	0	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30
2	0	0.90	0.81	0.72	0.64	0.56	0.49	0.42	0.36	0.30	0.25	0.20	0.16	0.12	0.09
3	0	0.86	0.73	0.61	0.51	0.42	0.34	0.27	0.22	0.17	0.13	0.09	0.06	0.04	0.03
3	1	0.99	0.97	0.94	0.90	0.84	0.78	0.72	0.65	0.57	0.50	0.43	0.35	0.28	0.22
4	0	0.81	0.66	0.52	0.41	0.32	0.24	0.18	0.13	0.09	0.06	0.04	0.03	0.02	0.01
4	1	0.99	0.95	0.89	0.82	0.74	0.65	0.56	0.48	0.39	0.31	0.24	0.18	0.13	0.08
5	1	0.98	0.92	0.84	0.74	0.63	0.53	0.43	0.34	0.26	0.19	0.13	0.09	0.05	0.03
5	2	1.00	0.99	0.97	0.94	0.90	0.84	0.76	0.68	0.59	0.50	0.41	0.32	0.24	0.16
6	1	0.97	0.89	0.78	0.66	0.53	0.42	0.32	0.23	0.16	0.11	0.07	0.04	0.02	0.01
6	2	1.00	0.98	0.95	0.90	0.83	0.74	0.65	0.54	0.44	0.34	0.26	0.18	0.12	0.07
7	1	0.96	0.85	0.72	0.58	0.44	0.33	0.23	0.16	0.10	0.06	0.04	0.02	0.01	0.00
7	2	1.00	0.97	0.93	0.85	0.76	0.65	0.53	0.42	0.32	0.23	0.15	0.10	0.06	0.03
8	1	0.94	0.81	0.66	0.50	0.37	0.26	0.17	0.11	0.06	0.04	0.02	0.01	0.00	0.00
8	2	0.99	0.96	0.89	0.80	0.68	0.55	0.43	0.32	0.22	0.14	0.09	0.05	0.03	0.01
8	3	1.00	0.99	0.98	0.94	0.89	0.81	0.71	0.59	0.48	0.36	0.26	0.17	0.11	0.06
9	1	0.93	0.77	0.60	0.44	0.30	0.20	0.12	0.07	0.04	0.02	0.01	0.00	0.00	0.00
9	2	0.99	0.95	0.86	0.74	0.60	0.46	0.34	0.23	0.15	0.09	0.05	0.03	0.01	0.00
9	3	1.00	0.99	0.97	0.91	0.83	0.73	0.61	0.48	0.36	0.25	0.17	0.10	0.05	0.03
10	1	0.91	0.74	0.54	0.38	0.24	0.15	0.09	0.05	0.02	0.01	0.00	0.00	0.00	0.00
10	2	0.99	0.93	0.82	0.68	0.53	0.38	0.26	0.17	0.10	0.05	0.03	0.01	0.00	0.00
10	3	1.00	0.99	0.95	0.88	0.78	0.65	0.51	0.38	0.27	0.17	0.10	0.05	0.03	0.01
15	2	0.96	0.82	0.60	0.40	0.24	0.13	0.06	0.03	0.01	0.00	0.00	0.00	0.00	0.00
15	3	0.99	0.94	0.82	0.65	0.46	0.30	0.17	0.09	0.04	0.02	0.01	0.00	0.00	0.00
15	4	1.00	0.99	0.94	0.84	0.69	0.52	0.35	0.22	0.12	0.06	0.03	0.01	0.00	0.00
15	5	1.00	1.00	0.98	0.94	0.85	0.72	0.56	0.40	0.26	0.15	0.08	0.03	0.01	0.00
20	2	0.92	0.68	0.40	0.21	0.09	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	3	0.98	0.87	0.65	0.41	0.23	0.11	0.04	0.02	0.00	0.00	0.00	0.00	0.00	0.00
20	4	1.00	0.96	0.83	0.63	0.41	0.24	0.12	0.05	0.02	0.01	0.00	0.00	0.00	0.00
20	5	1.00	0.99	0.93	0.80	0.62	0.42	0.25	0.13	0.06	0.02	0.01	0.00	0.00	0.00
20	6	1.00	1.00	0.98	0.91	0.79	0.61	0.42	0.25	0.13	0.06	0.02	0.01	0.00	0.00
30	3	0.94	0.65	0.32	0.12	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	4	0.98	0.82	0.52	0.26	0.10	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	5	1.00	0.93	0.71	0.43	0.20	0.08	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00
30	6	1.00	0.97	0.85	0.61	0.35	0.16	0.06	0.02	0.00	0.00	0.00	0.00	0.00	0.00
30	7	1.00	0.99	0.93	0.76	0.51	0.28	0.12	0.04	0.01	0.00	0.00	0.00	0.00	0.00
30	8	1.00	1.00	0.97	0.87	0.67	0.43	0.22	0.09	0.03	0.01	0.00	0.00	0.00	0.00
50	5	0.96	0.62	0.22	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	6	0.99	0.77	0.36	0.10	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	7	1.00	0.88	0.52	0.19	0.05	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	8	1.00	0.94	0.67	0.31	0.09	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	9	1.00	0.98	0.79	0.44	0.16	0.04	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	10	1.00	0.99	0.88	0.58	0.26	0.08	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	11	1.00	1.00	0.94	0.71	0.38	0.14	0.03	0.01	0.00	0.00	0.00	0.00	0.00	0.00
100	8	0.94	0.32	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	9	0.97	0.45	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	10	0.99	0.58	0.10	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	11	1.00	0.70	0.16	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	12	1.00	0.80	0.25	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	13	1.00	0.88	0.35	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	14	1.00	0.93	0.46	0.08	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	15	1.00	0.96	0.57	0.13	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	16	1.00	0.98	0.67	0.19	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	17	1.00	0.99	0.76	0.27	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
100	18	1.00	1.00	0.84	0.36	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

PROBABILITY OF ACCEPTANCE IS A FUNCTION OF LOT SIZE FOR ATTRIBUTES PLANS USING DISCRETE DATA. FOR VARIABLE LOT SIZES, IT WILL BE NECESSARY TO PLOT BOUNDING OPERATING CHARACTERISTIC CURVES. THE VALUES IN THIS TABLE ARE APPROPRIATE FOR BOTH SINGLE-LIMIT AND DOUBLE-LIMIT APPLICATIONS AND ARE UNINFLUENCED BY THE DISTRIBUTIONAL FORM OF THE POPULATION.

A P P E N D I X B

OPERATING CHARACTERISTIC TABLES FOR VARIABLES PLANS

TABLE B Operating Characteristics of Variables Acceptance Plans

VARIABLES ACCEPTANCE PLANS *** VARIABILITY-UNKNOWN PROCEDURE *** STANDARD DEVIATION METHOD

SAMPLE SIZE (n)	MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M)	MINIMUM ALLOWABLE QUALITY INDEX (k)	PROBABILITY OF ACCEPTANCE FOR SELECTED LEVELS OF LOT PERCENT DEFECTIVE						
			10	20	30	40	50	60	70
3	34	0.556	0.89	0.71	0.52	0.35	0.22	0.12	0.05
3	36	0.492	0.91	0.74	0.56	0.39	0.24	0.13	0.06
3	38	0.425	0.93	0.78	0.60	0.42	0.27	0.15	0.07
3	40	0.357	0.95	0.81	0.64	0.46	0.30	0.17	0.08
3	42	0.287	0.96	0.84	0.68	0.50	0.33	0.20	0.09
3	44	0.216	0.97	0.87	0.71	0.54	0.37	0.22	0.11
3	46	0.145	0.98	0.89	0.75	0.58	0.41	0.26	0.13
3	48	0.073	0.98	0.91	0.79	0.63	0.46	0.29	0.15
4	28	0.660	0.88	0.66	0.44	0.27	0.14	0.06	0.02
4	30	0.600	0.91	0.70	0.48	0.29	0.16	0.07	0.02
4	32	0.540	0.93	0.74	0.52	0.33	0.18	0.08	0.03
4	34	0.480	0.94	0.77	0.56	0.36	0.20	0.10	0.03
4	36	0.420	0.96	0.81	0.60	0.40	0.23	0.11	0.04
4	38	0.360	0.97	0.84	0.64	0.44	0.26	0.13	0.05
4	40	0.300	0.97	0.86	0.69	0.48	0.30	0.15	0.06
4	42	0.240	0.98	0.89	0.72	0.53	0.33	0.18	0.07
4	44	0.180	0.99	0.91	0.76	0.57	0.37	0.20	0.09
4	46	0.120	0.99	0.93	0.79	0.61	0.41	0.23	0.10
5	26	0.692	0.89	0.65	0.40	0.22	0.10	0.04	0.01
5	28	0.632	0.92	0.69	0.45	0.25	0.12	0.04	0.01
5	30	0.572	0.94	0.73	0.49	0.28	0.14	0.05	0.01
5	32	0.513	0.95	0.77	0.54	0.32	0.16	0.06	0.02
5	34	0.455	0.96	0.81	0.58	0.36	0.18	0.07	0.02
5	36	0.397	0.97	0.84	0.63	0.40	0.21	0.09	0.03
5	38	0.339	0.98	0.87	0.67	0.44	0.25	0.11	0.03
5	40	0.282	0.99	0.90	0.71	0.49	0.28	0.13	0.04
5	42	0.225	0.99	0.92	0.75	0.54	0.32	0.15	0.05
5	44	0.169	0.99	0.93	0.79	0.58	0.36	0.18	0.06
6	24	0.740	0.89	0.62	0.35	0.17	0.06	0.02	0.00
6	26	0.678	0.92	0.67	0.40	0.20	0.08	0.02	0.00
6	28	0.618	0.94	0.71	0.45	0.23	0.10	0.03	0.01
6	30	0.558	0.95	0.76	0.49	0.27	0.11	0.04	0.01
6	32	0.500	0.97	0.80	0.55	0.31	0.14	0.05	0.01
6	34	0.442	0.98	0.84	0.60	0.35	0.16	0.06	0.01
6	36	0.386	0.98	0.87	0.64	0.39	0.19	0.07	0.02
6	38	0.329	0.99	0.90	0.69	0.44	0.23	0.09	0.02
6	40	0.274	0.99	0.92	0.74	0.49	0.27	0.11	0.03
6	42	0.219	1.00	0.94	0.78	0.54	0.31	0.13	0.04
7	22	0.796	0.88	0.57	0.29	0.12	0.04	0.01	0.00
7	24	0.732	0.91	0.63	0.34	0.15	0.05	0.01	0.00
7	26	0.670	0.93	0.68	0.39	0.18	0.06	0.02	0.00
7	28	0.610	0.95	0.73	0.44	0.21	0.08	0.02	0.00
7	30	0.550	0.97	0.78	0.50	0.25	0.10	0.03	0.00
7	32	0.492	0.98	0.82	0.55	0.29	0.12	0.04	0.01
7	34	0.435	0.98	0.86	0.61	0.34	0.15	0.05	0.01
7	36	0.379	0.99	0.89	0.66	0.39	0.18	0.06	0.01
7	38	0.324	0.99	0.91	0.71	0.44	0.21	0.07	0.02
7	40	0.269	1.00	0.93	0.75	0.49	0.25	0.09	0.02
8	22	0.792	0.90	0.58	0.28	0.11	0.03	0.01	0.00
8	24	0.727	0.92	0.64	0.33	0.13	0.04	0.01	0.00
8	26	0.665	0.95	0.70	0.38	0.16	0.05	0.01	0.00
8	28	0.604	0.96	0.75	0.44	0.20	0.07	0.01	0.00
8	30	0.545	0.98	0.80	0.50	0.24	0.09	0.02	0.00
8	32	0.488	0.98	0.84	0.56	0.28	0.11	0.03	0.00
8	34	0.431	0.99	0.87	0.62	0.33	0.13	0.04	0.01
8	36	0.375	0.99	0.90	0.67	0.38	0.16	0.05	0.01
8	38	0.320	1.00	0.93	0.72	0.44	0.20	0.06	0.01
9	20	0.855	0.87	0.52	0.22	0.07	0.02	0.00	0.00
9	22	0.788	0.91	0.58	0.27	0.09	0.02	0.00	0.00
9	24	0.724	0.94	0.65	0.32	0.12	0.03	0.01	0.00
9	26	0.661	0.96	0.71	0.38	0.15	0.04	0.01	0.00
9	28	0.601	0.97	0.76	0.44	0.18	0.05	0.01	0.00
9	30	0.542	0.98	0.81	0.50	0.22	0.07	0.01	0.00
9	32	0.484	0.99	0.85	0.56	0.27	0.09	0.02	0.00
9	34	0.428	0.99	0.89	0.62	0.32	0.12	0.03	0.00
9	36	0.373	1.00	0.92	0.68	0.38	0.15	0.04	0.01
10	20	0.853	0.89	0.51	0.21	0.06	0.01	0.00	0.00
10	22	0.786	0.92	0.59	0.26	0.08	0.02	0.00	0.00
10	24	0.721	0.95	0.65	0.31	0.10	0.02	0.00	0.00
10	26	0.659	0.97	0.72	0.37	0.13	0.03	0.01	0.00
10	28	0.598	0.98	0.77	0.42	0.17	0.05	0.01	0.00
10	30	0.539	0.99	0.82	0.50	0.21	0.06	0.01	0.00
10	32	0.482	0.99	0.87	0.57	0.26	0.08	0.02	0.00
10	34	0.426	1.00	0.90	0.63	0.31	0.11	0.02	0.00

THE ACCEPTANCE PROBABILITIES IN THIS TABLE ARE ACCURATE FOR SINGLE-LIMIT PLANS AND ARE APPROXIMATELY CORRECT FOR DOUBLE-LIMIT PLANS. FOR SINGLE-LIMIT PLANS EITHER THE MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M) OR THE MINIMUM ALLOWABLE QUALITY INDEX (k) MAY BE SPECIFIED. FOR DOUBLE-LIMIT PLANS, ONLY THE MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE SHOULD BE USED.

TABLE B Operating Characteristics of Variables Acceptance Plans (Continued)

VARIABLES ACCEPTANCE PLANS *** VARIABILITY-UNKNOWN PROCEDURE *** STANDARD DEVIATION METHOD

SAMPLE SIZE (n)	MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M)	MINIMUM ALLOWABLE QUALITY INDEX (k)	PROBABILITY OF ACCEPTANCE FOR SELECTED LEVELS OF LOT PERCENT DEFECTIVE						
			5	10	15	20	25	30	35
15	11	1.216	0.90	0.60	0.33	0.15	0.06	0.02	0.01
15	12	1.168	0.93	0.66	0.37	0.18	0.08	0.03	0.01
15	13	1.122	0.95	0.71	0.42	0.22	0.10	0.04	0.01
15	14	1.079	0.96	0.75	0.47	0.25	0.12	0.05	0.02
15	15	1.037	0.97	0.79	0.53	0.29	0.14	0.06	0.02
15	16	0.996	0.98	0.83	0.57	0.34	0.17	0.08	0.03
15	17	0.958	0.99	0.86	0.62	0.38	0.20	0.09	0.04
15	18	0.920	0.99	0.89	0.67	0.42	0.23	0.11	0.05
15	19	0.883	0.99	0.91	0.71	0.47	0.26	0.13	0.06
15	20	0.848	1.00	0.93	0.75	0.51	0.30	0.15	0.07
15	21	0.813	1.00	0.95	0.79	0.56	0.34	0.18	0.08
15	22	0.780	1.00	0.96	0.82	0.60	0.38	0.21	0.10
15	23	0.747	1.00	0.97	0.85	0.64	0.42	0.24	0.12
15	24	0.715	1.00	0.98	0.87	0.69	0.46	0.27	0.14
15	25	0.683	1.00	0.98	0.90	0.72	0.51	0.31	0.16
15	26	0.652	1.00	0.99	0.92	0.76	0.55	0.34	0.19
15	27	0.622	1.00	0.99	0.93	0.79	0.59	0.38	0.21
15	28	0.592	1.00	0.99	0.95	0.82	0.63	0.42	0.24
15	29	0.562	1.00	1.00	0.96	0.85	0.67	0.46	0.27
20	10	1.271	0.90	0.54	0.24	0.09	0.03	0.01	0.00
20	11	1.219	0.93	0.61	0.29	0.11	0.04	0.01	0.00
20	12	1.170	0.95	0.67	0.35	0.14	0.05	0.01	0.00
20	13	1.123	0.97	0.73	0.40	0.18	0.06	0.02	0.01
20	14	1.079	0.98	0.78	0.46	0.22	0.08	0.03	0.01
20	15	1.036	0.99	0.82	0.52	0.26	0.11	0.04	0.01
20	16	0.996	0.99	0.86	0.58	0.31	0.13	0.05	0.01
20	17	0.956	0.99	0.89	0.64	0.35	0.16	0.06	0.02
20	18	0.918	1.00	0.92	0.69	0.41	0.19	0.08	0.03
20	19	0.882	1.00	0.94	0.74	0.46	0.23	0.10	0.03
20	20	0.846	1.00	0.96	0.78	0.51	0.27	0.12	0.04
20	21	0.811	1.00	0.97	0.82	0.56	0.31	0.14	0.05
20	22	0.777	1.00	0.98	0.85	0.62	0.36	0.17	0.07
20	23	0.744	1.00	0.98	0.88	0.66	0.41	0.20	0.08
20	24	0.712	1.00	0.99	0.91	0.71	0.46	0.24	0.10
20	25	0.680	1.00	0.99	0.93	0.75	0.51	0.28	0.12
20	26	0.649	1.00	1.00	0.95	0.79	0.55	0.32	0.15
30	9	1.332	0.90	0.44	0.13	0.03	0.00	0.00	0.00
30	10	1.275	0.94	0.53	0.18	0.04	0.01	0.00	0.00
30	11	1.221	0.96	0.62	0.24	0.06	0.01	0.00	0.00
30	12	1.171	0.98	0.69	0.30	0.09	0.02	0.00	0.00
30	13	1.124	0.99	0.76	0.37	0.12	0.03	0.01	0.00
30	14	1.079	0.99	0.82	0.44	0.16	0.04	0.01	0.00
30	15	1.036	1.00	0.87	0.52	0.21	0.06	0.01	0.00
30	16	0.995	1.00	0.91	0.59	0.26	0.08	0.02	0.00
30	17	0.955	1.00	0.93	0.66	0.32	0.11	0.03	0.01
30	18	0.917	1.00	0.96	0.72	0.38	0.14	0.04	0.01
30	19	0.880	1.00	0.97	0.78	0.44	0.18	0.05	0.01
30	20	0.844	1.00	0.98	0.83	0.51	0.22	0.07	0.02
30	21	0.809	1.00	0.99	0.87	0.57	0.27	0.09	0.02
30	22	0.775	1.00	0.99	0.90	0.64	0.33	0.12	0.03
30	23	0.742	1.00	1.00	0.93	0.70	0.38	0.15	0.04
50	8	1.398	0.90	0.29	0.04	0.00	0.00	0.00	0.00
50	9	1.335	0.95	0.41	0.07	0.01	0.00	0.00	0.00
50	10	1.277	0.97	0.52	0.11	0.01	0.00	0.00	0.00
50	11	1.223	0.99	0.64	0.17	0.02	0.00	0.00	0.00
50	12	1.173	0.99	0.73	0.24	0.04	0.00	0.00	0.00
50	13	1.125	1.00	0.82	0.33	0.06	0.01	0.00	0.00
50	14	1.080	1.00	0.88	0.42	0.10	0.01	0.00	0.00
50	15	1.036	1.00	0.92	0.51	0.14	0.02	0.00	0.00
50	16	0.995	1.00	0.95	0.61	0.20	0.03	0.00	0.00
50	17	0.955	1.00	0.97	0.69	0.26	0.05	0.01	0.00
50	18	0.916	1.00	0.99	0.77	0.34	0.08	0.01	0.00
50	19	0.879	1.00	0.99	0.83	0.42	0.11	0.02	0.00
50	20	0.843	1.00	1.00	0.88	0.51	0.16	0.03	0.00
100	7	1.471	0.89	0.10	0.00	0.00	0.00	0.00	0.00
100	8	1.402	0.96	0.21	0.00	0.00	0.00	0.00	0.00
100	9	1.338	0.99	0.35	0.01	0.00	0.00	0.00	0.00
100	10	1.279	1.00	0.52	0.04	0.00	0.00	0.00	0.00
100	11	1.225	1.00	0.68	0.09	0.00	0.00	0.00	0.00
100	12	1.174	1.00	0.80	0.15	0.01	0.00	0.00	0.00
100	13	1.126	1.00	0.89	0.25	0.01	0.00	0.00	0.00
100	14	1.080	1.00	0.95	0.37	0.03	0.00	0.00	0.00
100	15	1.036	1.00	0.98	0.51	0.06	0.00	0.00	0.00
100	16	0.995	1.00	0.99	0.64	0.11	0.00	0.00	0.00
100	17	0.954	1.00	1.00	0.76	0.18	0.01	0.00	0.00

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A P P E N D I X C

TABLES FOR ESTIMATION OF LOT PERCENT DEFECTIVE

TABLE C Estimation of Lot Percent Defective by Standard Deviation Method

VARIABILITY-UNKNOWN PROCEDURE — STANDARD DEVIATION METHOD

QUALITY INDEX (Q)	ESTIMATED LOT PERCENT DEFECTIVE FOR SELECTED SAMPLE SIZES												
	3	4	5	6	7	8	9	10	15	20	30	50	100
0.0	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00	50.00
0.01	49.72	49.67	49.64	49.63	49.63	49.62	49.62	49.62	49.61	49.61	49.60	49.60	49.60
0.02	49.45	49.33	49.29	49.27	49.25	49.24	49.24	49.23	49.22	49.21	49.21	49.21	49.20
0.03	49.17	49.00	48.93	48.90	48.88	48.86	48.85	48.85	48.83	48.82	48.81	48.81	48.81
0.04	48.90	48.67	48.58	48.53	48.50	48.49	48.47	48.46	48.44	48.42	48.42	48.41	48.41
0.05	48.62	48.33	48.22	48.16	48.13	48.11	48.09	48.08	48.05	48.04	48.02	48.02	48.01
0.06	48.35	48.00	47.86	47.80	47.75	47.73	47.71	47.70	47.66	47.64	47.63	47.62	47.61
0.07	48.07	47.67	47.51	47.43	47.38	47.35	47.33	47.31	47.27	47.25	47.24	47.22	47.22
0.08	47.79	47.33	47.15	47.06	47.01	46.97	46.95	46.93	46.88	46.86	46.84	46.83	46.82
0.09	47.52	47.00	46.80	46.70	46.63	46.59	46.57	46.54	46.49	46.47	46.45	46.43	46.42
0.10	47.24	46.67	46.44	46.33	46.26	46.22	46.18	46.16	46.10	46.08	46.05	46.04	46.03
0.11	46.96	46.33	46.09	45.96	45.89	45.84	45.80	45.78	45.71	45.69	45.66	45.64	45.63
0.12	46.69	46.00	45.73	45.60	45.51	45.46	45.42	45.40	45.33	45.29	45.27	45.25	45.24
0.13	46.41	45.67	45.38	45.23	45.14	45.08	45.04	45.01	44.94	44.90	44.88	44.86	44.84
0.14	46.13	45.33	45.02	44.86	44.77	44.71	44.66	44.63	44.55	44.51	44.48	44.46	44.45
0.15	45.85	45.00	44.67	44.50	44.40	44.33	44.29	44.25	44.16	44.13	44.09	44.07	44.05
0.16	45.58	44.67	44.31	44.13	44.03	43.96	43.91	43.87	43.78	43.74	43.70	43.68	43.66
0.17	45.30	44.33	43.96	43.77	43.65	43.58	43.53	43.49	43.39	43.35	43.31	43.29	43.27
0.18	45.02	44.00	43.60	43.40	43.28	43.21	43.15	43.11	43.01	42.96	42.92	42.89	42.88
0.19	44.74	43.67	43.25	43.04	42.91	42.83	42.77	42.73	42.62	42.57	42.53	42.50	42.48
0.20	44.46	43.33	42.90	42.68	42.54	42.46	42.40	42.35	42.24	42.19	42.15	42.11	42.09
0.21	44.18	43.00	42.54	42.31	42.17	42.08	42.02	41.97	41.85	41.80	41.76	41.73	41.70
0.22	43.90	42.67	42.19	41.95	41.80	41.71	41.64	41.60	41.47	41.42	41.37	41.34	41.31
0.23	43.62	42.33	41.84	41.59	41.44	41.34	41.27	41.22	41.09	41.03	40.98	40.95	40.93
0.24	43.34	42.00	41.48	41.22	41.07	40.97	40.89	40.84	40.71	40.65	40.60	40.56	40.54
0.25	43.05	41.67	41.13	40.86	40.70	40.59	40.52	40.47	40.33	40.27	40.22	40.18	40.15
0.26	42.77	41.33	40.78	40.50	40.33	40.22	40.15	40.09	39.95	39.89	39.83	39.79	39.77
0.27	42.49	41.00	40.43	40.14	39.97	39.85	39.77	39.72	39.57	39.50	39.45	39.41	39.38
0.28	42.20	40.67	40.08	39.78	39.60	39.48	39.40	39.34	39.19	39.12	39.07	39.03	39.00
0.29	41.92	40.33	39.72	39.42	39.23	39.11	39.03	38.97	38.81	38.75	38.69	38.65	38.62
0.30	41.63	40.00	39.37	39.06	38.87	38.75	38.66	38.60	38.44	38.37	38.31	38.26	38.24
0.31	41.35	39.67	39.02	38.70	38.50	38.38	38.29	38.23	38.06	37.99	37.93	37.89	37.86
0.32	41.06	39.33	38.67	38.34	38.14	38.01	37.92	37.86	37.69	37.61	37.55	37.51	37.48
0.33	40.77	39.00	38.32	37.98	37.78	37.65	37.55	37.49	37.31	37.24	37.18	37.13	37.10
0.34	40.48	38.67	37.97	37.62	37.42	37.28	37.19	37.12	36.94	36.87	36.80	36.75	36.72
0.35	40.20	38.33	37.62	37.27	37.05	36.92	36.82	36.75	36.57	36.49	36.43	36.38	36.35
0.36	39.91	38.00	37.28	36.91	36.69	36.55	36.46	36.38	36.20	36.12	36.05	36.01	35.97
0.37	39.62	37.67	36.93	36.55	36.33	36.19	36.09	36.02	35.83	35.75	35.68	35.63	35.60
0.38	39.33	37.33	36.58	36.20	35.98	35.83	35.73	35.65	35.46	35.38	35.31	35.26	35.23
0.39	39.03	37.00	36.23	35.84	35.62	35.47	35.37	35.29	35.10	35.01	34.94	34.89	34.86
0.40	38.74	36.67	35.88	35.49	35.26	35.11	35.00	34.93	34.73	34.65	34.58	34.53	34.49
0.41	38.45	36.33	35.54	35.14	34.90	34.75	34.64	34.57	34.37	34.28	34.21	34.16	34.12
0.42	38.15	36.00	35.19	34.79	34.55	34.39	34.29	34.21	34.00	33.92	33.85	33.79	33.76
0.43	37.85	35.67	34.85	34.43	34.19	34.04	33.93	33.85	33.64	33.56	33.48	33.43	33.39
0.44	37.56	35.33	34.50	34.08	33.84	33.68	33.57	33.49	33.28	33.20	33.12	33.07	33.03
0.45	37.26	35.00	34.16	33.73	33.49	33.33	33.21	33.13	32.92	32.84	32.76	32.71	32.67
0.46	36.96	34.67	33.81	33.38	33.13	32.97	32.86	32.78	32.57	32.48	32.40	32.35	32.31
0.47	36.66	34.33	33.47	33.04	32.78	32.62	32.51	32.42	32.21	32.12	32.04	31.99	31.95
0.48	36.35	34.00	33.12	32.69	32.43	32.27	32.15	32.07	31.85	31.77	31.69	31.63	31.60
0.49	36.05	33.67	32.78	32.34	32.08	31.92	31.80	31.72	31.50	31.41	31.33	31.28	31.24
0.50	35.75	33.33	32.44	32.00	31.74	31.57	31.45	31.37	31.15	31.06	30.98	30.93	30.89
0.51	35.44	33.00	32.10	31.65	31.39	31.22	31.10	31.02	30.80	30.71	30.63	30.57	30.54
0.52	35.13	32.67	31.76	31.31	31.04	30.87	30.76	30.67	30.45	30.36	30.28	30.23	30.19
0.53	34.83	32.33	31.42	30.96	30.70	30.53	30.41	30.32	30.10	30.01	29.93	29.88	29.84
0.54	34.51	32.00	31.08	30.62	30.36	30.18	30.07	29.98	29.76	29.67	29.59	29.53	29.49
0.55	34.20	31.67	30.74	30.28	30.01	29.84	29.72	29.64	29.41	29.32	29.24	29.19	29.15
0.56	33.88	31.33	30.40	29.94	29.67	29.50	29.38	29.29	29.07	28.98	28.90	28.85	28.81
0.57	33.57	31.00	30.06	29.60	29.33	29.16	29.04	28.95	28.73	28.64	28.56	28.51	28.47
0.58	33.25	30.67	29.73	29.26	28.99	28.82	28.70	28.61	28.39	28.30	28.22	28.17	28.13
0.59	32.93	30.33	29.39	28.93	28.66	28.48	28.36	28.28	28.05	27.96	27.89	27.83	27.79
0.60	32.61	30.00	29.05	28.59	28.32	28.15	28.03	27.94	27.72	27.63	27.55	27.50	27.46
0.61	32.28	29.67	28.72	28.25	27.98	27.81	27.69	27.60	27.38	27.30	27.22	27.16	27.13
0.62	31.96	29.33	28.39	27.92	27.65	27.48	27.36	27.27	27.05	26.96	26.89	26.83	26.80
0.63	31.63	29.00	28.05	27.59	27.32	27.15	27.03	26.94	26.72	26.63	26.56	26.50	26.47
0.64	31.31	28.67	27.72	27.26	26.99	26.82	26.70	26.61	26.39	26.31	26.23	26.18	26.14
0.65	30.98	28.33	27.39	26.92	26.66	26.49	26.37	26.28	26.07	25.98	25.90	25.85	25.82
0.66	30.65	28.00	27.06	26.60	26.33	26.16	26.04	25.96	25.74	25.66	25.58	25.53	25.49
0.67	30.32	27.67	26.73	26.27	26.00	25.83	25.72	25.63	25.42	25.33	25.26	25.21	25.17
0.68	29.98	27.33	26.40	25.94	25.68	25.51	25.39	25.31	25.10	25.01	24.94	24.89	24.86
0.69	29.64	27.00	26.07	25.61	25.35	25.19	25.07	24.99	24.78	24.69	24.62	24.57	24.54
0.70	29.29	26.67	25.74	25.29	25.03	24.86	24.75	24.67	24.46	24.38	24.31	24.26	24.23
0.71	28.93	26.33	25.41	24.96	24.71	24.54	24.43	24.35	24.15	24.06	23.99	23.95	23.91
0.72	28.57	26.00	25.09	24.64	24.39	24.23	24.11	24.03	23.83	23.75	23.68	23.64	23.60
0.73	28.20	25.67	24.76	24.32	24.07	23.91	23.80	23.72	23.52	23.44	23.37	23.33	23.30
0.74	27.83	25.33	24.44	24.00	23.75	23.59	23.48	23.41	23.21	23.13	23.07	23.02	22.99
0.75	27.45	25.00	24.11	23.68	23.44	23.28	23.17	23.10	22.90	22.83	22.76	22.72	22.69
0.76	27.07	24.67	23.79	23.37	23.12	22.97	22.86	22.79	22.60	22.52	22.46	22.42	22.39
0.77	26.68	24.33	23.47	23.05	22.81	22.66	22.56	22.48	22.30	22.22	22.16	22.12	22.09
0.78	26.29	24.00	23.15	22.74	22.50	22.35	22.25	22.18	21.99	21.92	21.86	21.82	21.79
0.79	25.89	23.67	22.83	22.42	22.19	22.04	21.94	21.87	21.70	21.62	21.57	21.53	21.50

NUMBERS IN BODY OF TABLE ARE ESTIMATES OF LOT PERCENT DEFECTIVE CORRESPONDING TO SPECIFIC VALUES OF QUALITY INDEX AND SAMPLE SIZE. FOR Q VALUES GREATER THAN OR EQUAL TO ZERO, THE PERCENT DEFECTIVE ESTIMATE MAY BE READ DIRECTLY FROM THE TABLE. FOR Q VALUES LESS THAN ZERO, THE TABLE VALUE MUST BE SUBTRACTED FROM 100.

TABLE C Estimation of Lot Percent Defective by Standard Deviation Method (Continued)

VARIABILITY-UNKNOWN PROCEDURE — STANDARD DEVIATION METHOD

QUALITY INDEX (Q)	ESTIMATED LOT PERCENT DEFECTIVE FOR SELECTED SAMPLE SIZES												
	3	4	5	6	7	8	9	10	15	20	30	50	100
0.80	25.64	23.33	22.51	22.11	21.88	21.74	21.64	21.57	21.40	21.33	21.27	21.23	21.21
0.81	25.25	23.00	22.19	21.80	21.58	21.44	21.34	21.27	21.10	21.04	20.98	20.94	20.92
0.82	24.86	22.67	21.87	21.49	21.27	21.14	21.04	20.98	20.81	20.75	20.69	20.66	20.63
0.83	24.47	22.33	21.56	21.18	20.97	20.84	20.75	20.68	20.52	20.46	20.40	20.37	20.35
0.84	24.07	22.00	21.24	20.88	20.67	20.54	20.45	20.39	20.23	20.17	20.12	20.09	20.06
0.85	23.67	21.67	20.93	20.57	20.37	20.24	20.16	20.10	19.94	19.88	19.84	19.80	19.78
0.86	23.26	21.33	20.62	20.27	20.07	19.95	19.87	19.81	19.66	19.60	19.56	19.53	19.51
0.87	22.84	21.00	20.31	19.97	19.78	19.66	19.58	19.52	19.38	19.32	19.28	19.25	19.23
0.88	22.42	20.67	20.00	19.67	19.48	19.37	19.29	19.23	19.10	19.04	19.00	18.98	18.96
0.89	21.99	20.33	19.69	19.37	19.19	19.08	19.00	18.95	18.82	18.77	18.73	18.70	18.69
0.90	21.55	20.00	19.38	19.07	18.90	18.79	18.72	18.67	18.54	18.50	18.46	18.43	18.42
0.91	21.11	19.67	19.07	18.78	18.61	18.51	18.44	18.39	18.27	18.23	18.19	18.17	18.15
0.92	20.66	19.33	18.77	18.49	18.33	18.23	18.16	18.11	18.00	17.96	17.92	17.90	17.89
0.93	20.19	19.00	18.46	18.19	18.04	17.95	17.88	17.84	17.73	17.69	17.66	17.64	17.63
0.94	19.73	18.67	18.16	17.90	17.76	17.67	17.61	17.56	17.46	17.43	17.40	17.38	17.37
0.95	19.25	18.33	17.86	17.61	17.48	17.39	17.33	17.29	17.20	17.16	17.14	17.12	17.11
0.96	18.75	18.00	17.55	17.33	17.20	17.12	17.06	17.03	16.94	16.90	16.88	16.87	16.86
0.97	18.25	17.67	17.25	17.04	16.92	16.85	16.79	16.76	16.68	16.65	16.63	16.61	16.61
0.98	17.74	17.33	16.96	16.76	16.65	16.57	16.53	16.49	16.42	16.39	16.37	16.36	16.36
0.99	17.21	17.00	16.66	16.48	16.37	16.31	16.26	16.23	16.16	16.14	16.12	16.11	16.11
1.00	16.67	16.67	16.36	16.20	16.10	16.04	16.00	15.97	15.91	15.89	15.88	15.87	15.87
1.01	16.11	16.33	16.07	15.92	15.83	15.78	15.74	15.72	15.66	15.64	15.63	15.63	15.62
1.02	15.53	16.00	15.78	15.64	15.56	15.51	15.48	15.46	15.41	15.40	15.39	15.39	15.38
1.03	14.93	15.67	15.48	15.37	15.30	15.25	15.23	15.21	15.17	15.15	15.15	15.15	15.15
1.04	14.31	15.33	15.19	15.09	15.03	15.00	14.97	14.96	14.92	14.91	14.91	14.91	14.91
1.05	13.66	15.00	14.91	14.82	14.77	14.74	14.72	14.71	14.68	14.67	14.67	14.68	14.68
1.06	12.98	14.67	14.62	14.55	14.51	14.49	14.47	14.46	14.44	14.44	14.44	14.45	14.45
1.07	12.27	14.33	14.33	14.29	14.26	14.24	14.22	14.22	14.20	14.20	14.21	14.22	14.22
1.08	11.51	14.00	14.05	14.02	14.00	13.99	13.98	13.97	13.97	13.97	13.98	13.99	14.00
1.09	10.71	13.67	13.76	13.76	13.75	13.74	13.73	13.73	13.74	13.74	13.75	13.77	13.77
1.10	9.84	13.33	13.48	13.50	13.49	13.49	13.49	13.50	13.51	13.52	13.53	13.54	13.55
1.11	8.89	13.00	13.20	13.24	13.25	13.25	13.26	13.26	13.28	13.29	13.31	13.32	13.34
1.12	7.82	12.67	12.93	12.98	13.00	13.01	13.02	13.03	13.05	13.07	13.09	13.11	13.12
1.13	6.60	12.33	12.65	12.72	12.75	12.77	12.79	12.80	12.83	12.85	12.87	12.89	12.91
1.14	5.08	12.00	12.37	12.47	12.51	12.54	12.55	12.57	12.61	12.63	12.66	12.68	12.70
1.15	2.87	11.67	12.10	12.22	12.27	12.30	12.32	12.34	12.39	12.42	12.45	12.47	12.49
1.16	0.00	11.33	11.83	11.97	12.03	12.07	12.10	12.12	12.18	12.21	12.24	12.26	12.28
1.17	0.00	11.00	11.56	11.72	11.79	11.84	11.87	11.90	11.96	12.00	12.03	12.06	12.08
1.18	0.00	10.67	11.29	11.47	11.56	11.61	11.65	11.68	11.75	11.79	11.82	11.85	11.88
1.19	0.00	10.33	11.02	11.23	11.33	11.39	11.43	11.46	11.54	11.58	11.62	11.65	11.68
1.20	0.00	10.00	10.76	10.99	11.10	11.17	11.21	11.24	11.34	11.38	11.42	11.46	11.48
1.21	0.00	9.67	10.50	10.75	10.87	10.94	10.99	11.03	11.13	11.18	11.22	11.26	11.29
1.22	0.00	9.33	10.23	10.51	10.65	10.73	10.78	10.82	10.93	10.98	11.03	11.07	11.09
1.23	0.00	9.00	9.97	10.28	10.42	10.51	10.57	10.61	10.73	10.78	10.84	10.88	10.91
1.24	0.00	8.67	9.72	10.04	10.20	10.30	10.36	10.41	10.53	10.59	10.64	10.69	10.72
1.25	0.00	8.33	9.46	9.81	9.98	10.09	10.15	10.21	10.34	10.40	10.46	10.50	10.53
1.26	0.00	8.00	9.21	9.58	9.77	9.88	9.95	10.00	10.15	10.21	10.27	10.32	10.35
1.27	0.00	7.67	8.96	9.36	9.55	9.67	9.75	9.81	9.96	10.02	10.09	10.13	10.17
1.28	0.00	7.33	8.71	9.13	9.34	9.47	9.55	9.61	9.77	9.84	9.90	9.95	9.99
1.29	0.00	7.00	8.46	8.91	9.13	9.26	9.35	9.42	9.58	9.66	9.72	9.78	9.82
1.30	0.00	6.67	8.21	8.69	8.93	9.06	9.16	9.22	9.40	9.48	9.55	9.60	9.64
1.31	0.00	6.33	7.97	8.48	8.72	8.87	8.96	9.03	9.22	9.30	9.37	9.43	9.47
1.32	0.00	6.00	7.73	8.26	8.52	8.67	8.77	8.85	9.04	9.12	9.20	9.26	9.30
1.33	0.00	5.67	7.49	8.05	8.32	8.48	8.59	8.66	8.86	8.95	9.03	9.09	9.13
1.34	0.00	5.33	7.25	7.84	8.12	8.29	8.40	8.48	8.69	8.78	8.86	8.92	8.97
1.35	0.00	5.00	7.02	7.63	7.92	8.10	8.22	8.30	8.52	8.61	8.69	8.76	8.81
1.36	0.00	4.67	6.79	7.42	7.73	7.91	8.04	8.12	8.35	8.44	8.53	8.60	8.65
1.37	0.00	4.33	6.56	7.22	7.54	7.73	7.86	7.95	8.18	8.28	8.37	8.44	8.49
1.38	0.00	4.00	6.33	7.02	7.35	7.55	7.68	7.77	8.01	8.12	8.21	8.28	8.33
1.39	0.00	3.67	6.10	6.82	7.17	7.37	7.51	7.60	7.85	7.96	8.05	8.12	8.18
1.40	0.00	3.33	5.88	6.63	6.98	7.19	7.33	7.44	7.69	7.80	7.90	7.97	8.02
1.41	0.00	3.00	5.66	6.43	6.80	7.02	7.17	7.27	7.53	7.64	7.74	7.82	7.87
1.42	0.00	2.67	5.44	6.24	6.62	6.85	7.00	7.10	7.37	7.49	7.59	7.67	7.73
1.43	0.00	2.33	5.23	6.05	6.45	6.68	6.83	6.94	7.22	7.34	7.44	7.52	7.58
1.44	0.00	2.00	5.02	5.87	6.27	6.51	6.67	6.78	7.07	7.19	7.30	7.38	7.44
1.45	0.00	1.67	4.81	5.68	6.10	6.35	6.51	6.63	6.92	7.04	7.15	7.24	7.30
1.46	0.00	1.33	4.60	5.50	5.93	6.19	6.35	6.47	6.77	6.90	7.01	7.10	7.16
1.47	0.00	1.00	4.39	5.33	5.77	6.03	6.20	6.32	6.63	6.75	6.87	6.96	7.02
1.48	0.00	0.67	4.19	5.15	5.60	5.87	6.04	6.17	6.48	6.61	6.73	6.82	6.88
1.49	0.00	0.33	3.99	4.98	5.44	5.71	5.89	6.02	6.34	6.48	6.60	6.69	6.75
1.50	0.00	0.00	3.80	4.81	5.28	5.56	5.74	5.87	6.20	6.34	6.46	6.55	6.62
1.51	0.00	0.00	3.61	4.64	5.13	5.41	5.60	5.73	6.06	6.20	6.33	6.42	6.49
1.52	0.00	0.00	3.42	4.47	4.97	5.26	5.45	5.59	5.93	6.07	6.20	6.29	6.36
1.53	0.00	0.00	3.23	4.31	4.82	5.12	5.31	5.45	5.80	5.94	6.07	6.17	6.24
1.54	0.00	0.00	3.05	4.15	4.67	4.97	5.17	5.31	5.67	5.81	5.95	6.04	6.11
1.55	0.00	0.00	2.87	4.00	4.52	4.83	5.03	5.18	5.54	5.69	5.82	5.92	5.99
1.56	0.00	0.00	2.69	3.84	4.38	4.69	4.90	5.05	5.41	5.56	5.70	5.80	5.87
1.57	0.00	0.00	2.52	3.69	4.24	4.56	4.77	4.92	5.29	5.44	5.58	5.68	5.75
1.58	0.00	0.00	2.35	3.54	4.10	4.42	4.64	4.79	5.16	5.32	5.46	5.56	5.64
1.59	0.00	0.00	2.19	3.40	3.96	4.29	4.51	4.66	5.04	5.20	5.34	5.45	5.52

NUMBERS IN BODY OF TABLE ARE ESTIMATES OF LOT PERCENT DEFECTIVE CORRESPONDING TO SPECIFIC VALUES OF QUALITY INDEX AND SAMPLE SIZE. FOR Q VALUES GREATER THAN OR EQUAL TO ZERO, THE PERCENT DEFECTIVE ESTIMATE MAY BE READ DIRECTLY FROM THE TABLE. FOR Q VALUES LESS THAN ZERO, THE TABLE VALUE MUST BE SUBTRACTED FROM 100.

TABLE C Estimation of Lot Percent Defective by Standard Deviation Method (Continued)

VARIABILITY-UNKNOWN PROCEDURE — STANDARD DEVIATION METHOD

QUALITY INDEX (Q)	ESTIMATED LOT PERCENT DEFECTIVE FOR SELECTED SAMPLE SIZES: 3	4	5	6	7	8	9	10	15	20	30	50	100
1.60	0.00	0.00	2.03	3.25	3.83	4.16	4.38	4.54	4.92	5.08	5.23	5.33	5.41
1.61	0.00	0.00	1.87	3.11	3.69	4.03	4.26	4.41	4.81	4.97	5.12	5.22	5.30
1.62	0.00	0.00	1.72	2.97	3.57	3.91	4.14	4.30	4.69	4.86	5.01	5.11	5.19
1.63	0.00	0.00	1.57	2.84	3.44	3.79	4.02	4.18	4.58	4.75	4.90	5.01	5.08
1.64	0.00	0.00	1.42	2.71	3.31	3.67	3.90	4.06	4.47	4.64	4.79	4.90	4.98
1.65	0.00	0.00	1.28	2.58	3.19	3.55	3.78	3.95	4.36	4.53	4.68	4.79	4.87
1.66	0.00	0.00	1.15	2.45	3.07	3.43	3.67	3.84	4.25	4.43	4.58	4.69	4.77
1.67	0.00	0.00	1.02	2.33	2.95	3.32	3.56	3.73	4.15	4.32	4.48	4.59	4.67
1.68	0.00	0.00	0.89	2.21	2.84	3.21	3.45	3.62	4.05	4.22	4.38	4.49	4.57
1.69	0.00	0.00	0.77	2.09	2.73	3.10	3.34	3.52	3.94	4.12	4.28	4.39	4.47
1.70	0.00	0.00	0.66	1.98	2.62	2.99	3.24	3.41	3.84	4.02	4.18	4.30	4.38
1.71	0.00	0.00	0.55	1.87	2.51	2.89	3.14	3.31	3.75	3.93	4.09	4.20	4.29
1.72	0.00	0.00	0.45	1.76	2.41	2.79	3.03	3.21	3.65	3.83	3.99	4.11	4.19
1.73	0.00	0.00	0.36	1.66	2.30	2.69	2.94	3.11	3.56	3.74	3.90	4.02	4.10
1.74	0.00	0.00	0.27	1.55	2.20	2.59	2.84	3.02	3.46	3.65	3.81	3.93	4.01
1.75	0.00	0.00	0.19	1.45	2.11	2.49	2.75	2.93	3.37	3.56	3.72	3.84	3.93
1.76	0.00	0.00	0.12	1.36	2.01	2.40	2.65	2.83	3.28	3.47	3.63	3.76	3.84
1.77	0.00	0.00	0.06	1.27	1.92	2.31	2.56	2.74	3.20	3.38	3.55	3.67	3.76
1.78	0.00	0.00	0.02	1.18	1.83	2.22	2.47	2.66	3.11	3.30	3.47	3.59	3.67
1.79	0.00	0.00	0.00	1.09	1.74	2.13	2.39	2.57	3.03	3.21	3.38	3.51	3.59
1.80	0.00	0.00	0.00	1.01	1.65	2.04	2.30	2.49	2.94	3.13	3.30	3.43	3.51
1.81	0.00	0.00	0.00	0.93	1.57	1.96	2.22	2.40	2.86	3.05	3.22	3.35	3.43
1.82	0.00	0.00	0.00	0.85	1.49	1.88	2.14	2.32	2.79	2.98	3.15	3.27	3.36
1.83	0.00	0.00	0.00	0.78	1.41	1.80	2.06	2.25	2.71	2.90	3.07	3.19	3.28
1.84	0.00	0.00	0.00	0.71	1.34	1.72	1.98	2.17	2.63	2.82	2.99	3.12	3.21
1.85	0.00	0.00	0.00	0.64	1.26	1.65	1.91	2.09	2.56	2.75	2.92	3.05	3.13
1.86	0.00	0.00	0.00	0.57	1.19	1.58	1.84	2.02	2.48	2.68	2.85	2.97	3.06
1.87	0.00	0.00	0.00	0.51	1.12	1.51	1.76	1.95	2.41	2.61	2.78	2.90	2.99
1.88	0.00	0.00	0.00	0.46	1.06	1.44	1.70	1.88	2.34	2.54	2.71	2.83	2.92
1.89	0.00	0.00	0.00	0.40	0.99	1.37	1.63	1.81	2.28	2.47	2.64	2.77	2.85
1.90	0.00	0.00	0.00	0.35	0.93	1.31	1.56	1.75	2.21	2.40	2.57	2.70	2.79
1.91	0.00	0.00	0.00	0.30	0.87	1.24	1.50	1.68	2.14	2.34	2.51	2.63	2.72
1.92	0.00	0.00	0.00	0.26	0.81	1.18	1.44	1.62	2.08	2.27	2.45	2.57	2.66
1.93	0.00	0.00	0.00	0.22	0.76	1.12	1.37	1.56	2.02	2.21	2.38	2.51	2.60
1.94	0.00	0.00	0.00	0.18	0.70	1.07	1.32	1.50	1.96	2.15	2.32	2.45	2.54
1.95	0.00	0.00	0.00	0.15	0.65	1.01	1.26	1.44	1.90	2.09	2.26	2.39	2.48
1.96	0.00	0.00	0.00	0.12	0.60	0.96	1.20	1.38	1.84	2.03	2.20	2.33	2.42
1.97	0.00	0.00	0.00	0.09	0.56	0.91	1.15	1.33	1.78	1.97	2.15	2.27	2.36
1.98	0.00	0.00	0.00	0.07	0.51	0.86	1.10	1.27	1.73	1.92	2.09	2.21	2.30
1.99	0.00	0.00	0.00	0.05	0.47	0.81	1.05	1.22	1.67	1.86	2.03	2.16	2.25
2.00	0.00	0.00	0.00	0.03	0.43	0.76	1.00	1.17	1.62	1.81	1.98	2.10	2.19
2.01	0.00	0.00	0.00	0.02	0.39	0.72	0.95	1.12	1.57	1.76	1.93	2.05	2.14
2.02	0.00	0.00	0.00	0.01	0.36	0.67	0.90	1.07	1.52	1.71	1.87	2.00	2.09
2.03	0.00	0.00	0.00	0.00	0.32	0.63	0.86	1.03	1.47	1.66	1.82	1.95	2.04
2.04	0.00	0.00	0.00	0.00	0.29	0.59	0.82	0.98	1.42	1.61	1.77	1.90	1.99
2.05	0.00	0.00	0.00	0.00	0.26	0.55	0.77	0.94	1.37	1.56	1.73	1.85	1.94
2.06	0.00	0.00	0.00	0.00	0.23	0.52	0.73	0.90	1.33	1.51	1.68	1.80	1.89
2.07	0.00	0.00	0.00	0.00	0.21	0.48	0.70	0.86	1.28	1.47	1.63	1.76	1.84
2.08	0.00	0.00	0.00	0.00	0.18	0.45	0.66	0.82	1.24	1.42	1.59	1.71	1.79
2.09	0.00	0.00	0.00	0.00	0.16	0.42	0.62	0.78	1.20	1.38	1.54	1.66	1.75
2.10	0.00	0.00	0.00	0.00	0.14	0.39	0.59	0.74	1.16	1.34	1.50	1.62	1.71
2.11	0.00	0.00	0.00	0.00	0.12	0.36	0.55	0.71	1.12	1.30	1.46	1.58	1.66
2.12	0.00	0.00	0.00	0.00	0.10	0.33	0.52	0.67	1.08	1.26	1.42	1.54	1.62
2.13	0.00	0.00	0.00	0.00	0.08	0.30	0.49	0.64	1.04	1.22	1.38	1.50	1.58
2.14	0.00	0.00	0.00	0.00	0.07	0.28	0.46	0.61	1.00	1.18	1.34	1.46	1.54
2.15	0.00	0.00	0.00	0.00	0.06	0.26	0.43	0.58	0.97	1.14	1.30	1.42	1.50
2.16	0.00	0.00	0.00	0.00	0.05	0.23	0.41	0.55	0.93	1.10	1.26	1.38	1.46
2.17	0.00	0.00	0.00	0.00	0.04	0.21	0.38	0.52	0.90	1.07	1.22	1.34	1.42
2.18	0.00	0.00	0.00	0.00	0.03	0.19	0.36	0.49	0.87	1.03	1.19	1.30	1.39
2.19	0.00	0.00	0.00	0.00	0.02	0.17	0.33	0.46	0.83	1.00	1.15	1.27	1.35
2.20	0.00	0.00	0.00	0.00	0.01	0.16	0.31	0.44	0.80	0.97	1.12	1.23	1.31
2.21	0.00	0.00	0.00	0.00	0.01	0.14	0.29	0.41	0.77	0.94	1.09	1.20	1.28
2.22	0.00	0.00	0.00	0.00	0.01	0.13	0.27	0.39	0.74	0.90	1.05	1.17	1.25
2.23	0.00	0.00	0.00	0.00	0.00	0.11	0.25	0.37	0.71	0.87	1.02	1.13	1.21
2.24	0.00	0.00	0.00	0.00	0.00	0.10	0.23	0.34	0.69	0.85	0.99	1.10	1.18
2.25	0.00	0.00	0.00	0.00	0.00	0.09	0.21	0.32	0.66	0.82	0.96	1.07	1.15
2.26	0.00	0.00	0.00	0.00	0.00	0.08	0.20	0.30	0.63	0.79	0.93	1.04	1.12
2.27	0.00	0.00	0.00	0.00	0.00	0.07	0.18	0.29	0.61	0.76	0.90	1.01	1.09
2.28	0.00	0.00	0.00	0.00	0.00	0.06	0.17	0.27	0.58	0.74	0.88	0.98	1.06
2.29	0.00	0.00	0.00	0.00	0.00	0.05	0.15	0.25	0.56	0.71	0.85	0.95	1.03
2.30	0.00	0.00	0.00	0.00	0.00	0.04	0.14	0.23	0.54	0.68	0.82	0.93	1.00
2.31	0.00	0.00	0.00	0.00	0.00	0.04	0.13	0.22	0.52	0.66	0.80	0.90	0.97
2.32	0.00	0.00	0.00	0.00	0.00	0.03	0.11	0.20	0.49	0.64	0.77	0.87	0.95
2.33	0.00	0.00	0.00	0.00	0.00	0.02	0.10	0.19	0.47	0.61	0.75	0.85	0.92
2.34	0.00	0.00	0.00	0.00	0.00	0.02	0.09	0.18	0.45	0.59	0.72	0.82	0.90
2.35	0.00	0.00	0.00	0.00	0.00	0.02	0.08	0.16	0.43	0.57	0.70	0.80	0.87
2.36	0.00	0.00	0.00	0.00	0.00	0.01	0.08	0.15	0.42	0.55	0.68	0.78	0.85
2.37	0.00	0.00	0.00	0.00	0.00	0.01	0.07	0.14	0.40	0.53	0.66	0.75	0.82
2.38	0.00	0.00	0.00	0.00	0.00	0.01	0.06	0.13	0.38	0.51	0.63	0.73	0.80
2.39	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.12	0.36	0.49	0.61	0.71	0.78

NUMBERS IN BODY OF TABLE ARE ESTIMATES OF LOT PERCENT DEFECTIVE CORRESPONDING TO SPECIFIC VALUES OF QUALITY INDEX AND SAMPLE SIZE. FOR Q VALUES GREATER THAN OR EQUAL TO ZERO, THE PERCENT DEFECTIVE ESTIMATE MAY BE READ DIRECTLY FROM THE TABLE. FOR Q VALUES LESS THAN ZERO, THE TABLE VALUE MUST BE SUBTRACTED FROM 100.

TABLE C Estimation of Lot Percent Defective by Standard Deviation Method (Continued)

VARIABILITY-UNKNOWN PROCEDURE — STANDARD DEVIATION METHOD

QUALITY INDEX (Q)	ESTIMATED LOT PERCENT DEFECTIVE FOR SELECTED SAMPLE SIZES												
	3	4	5	6	7	8	9	10	15	20	30	50	100
2.40	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.11	0.35	0.47	0.59	0.69	0.75
2.41	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.10	0.33	0.45	0.57	0.67	0.73
2.42	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.09	0.32	0.44	0.56	0.65	0.71
2.43	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.08	0.30	0.42	0.54	0.63	0.69
2.44	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.08	0.29	0.40	0.52	0.61	0.67
2.45	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.07	0.27	0.39	0.50	0.59	0.65
2.46	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.26	0.37	0.48	0.57	0.63
2.47	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.25	0.36	0.47	0.55	0.62
2.48	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.05	0.24	0.34	0.45	0.54	0.60
2.49	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.05	0.23	0.33	0.44	0.52	0.58
2.50	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.21	0.32	0.42	0.50	0.56
2.51	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.20	0.30	0.41	0.49	0.55
2.52	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.19	0.29	0.39	0.47	0.53
2.53	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.18	0.28	0.38	0.46	0.51
2.54	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.17	0.27	0.37	0.44	0.50
2.55	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.17	0.26	0.35	0.43	0.48
2.56	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.16	0.25	0.34	0.41	0.47
2.57	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.15	0.24	0.33	0.40	0.46
2.58	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.14	0.23	0.32	0.39	0.44
2.59	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.13	0.22	0.30	0.38	0.43
2.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.13	0.21	0.29	0.36	0.41
2.61	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.12	0.20	0.28	0.35	0.40
2.62	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.11	0.19	0.27	0.34	0.39
2.63	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.11	0.18	0.26	0.33	0.38
2.64	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.10	0.17	0.25	0.32	0.37
2.65	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.16	0.24	0.31	0.35
2.66	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.16	0.23	0.30	0.34
2.67	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.15	0.22	0.29	0.33
2.68	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.14	0.22	0.28	0.32
2.69	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.14	0.21	0.27	0.31
2.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	0.13	0.20	0.26	0.30
2.71	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.12	0.19	0.25	0.29
2.72	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.12	0.18	0.24	0.28
2.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06	0.11	0.18	0.23	0.27
2.74	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.11	0.17	0.22	0.27
2.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.10	0.16	0.22	0.26
2.76	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.10	0.16	0.21	0.25
2.77	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.09	0.15	0.20	0.24
2.78	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.09	0.14	0.19	0.23
2.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.08	0.14	0.19	0.23
2.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.08	0.13	0.18	0.22
2.81	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.07	0.13	0.17	0.21
2.82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.07	0.12	0.17	0.20
2.83	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.07	0.12	0.16	0.20
2.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.06	0.11	0.16	0.19
2.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.11	0.15	0.18
2.86	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.10	0.14	0.18
2.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.05	0.10	0.14	0.17
2.88	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.05	0.09	0.13	0.17
2.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.05	0.09	0.13	0.16
2.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.05	0.09	0.12	0.16
2.91	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.08	0.12	0.15
2.92	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.08	0.12	0.14
2.93	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.08	0.11	0.14
2.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.07	0.11	0.13
2.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.07	0.10	0.13
2.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.07	0.10	0.13
2.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.06	0.10	0.12
2.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.06	0.09	0.12
2.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.06	0.09	0.11
3.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.05	0.08	0.11
3.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.05	0.08	0.11
3.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.05	0.08	0.10
3.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.05	0.07	0.10
3.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.05	0.07	0.09
3.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.04	0.07	0.09
3.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.04	0.07	0.09
3.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.04	0.06	0.08
3.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.06	0.08
3.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.06	0.08
3.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.06	0.08
3.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.05	0.07
3.12	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.05	0.07
3.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.05	0.07
3.14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.05	0.07
3.15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.05	0.06
3.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.06
3.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.06
3.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.06
3.19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.05

NUMBERS IN BODY OF TABLE ARE ESTIMATES OF LOT PERCENT DEFECTIVE CORRESPONDING TO SPECIFIC VALUES OF QUALITY INDEX AND SAMPLE SIZE. FOR Q VALUES GREATER THAN OR EQUAL TO ZERO, THE PERCENT DEFECTIVE ESTIMATE MAY BE READ DIRECTLY FROM THE TABLE. FOR Q VALUES LESS THAN ZERO, THE TABLE VALUE MUST BE SUBTRACTED FROM 100.

TABLE C Estimation of Lot Percent Defective by Standard Deviation Method (Continued)

VARIABILITY-UNKNOWN PROCEDURE — STANDARD DEVIATION METHOD

QUALITY INDEX (Q)	ESTIMATED LOT PERCENT DEFECTIVE FOR SELECTED SAMPLE SIZES												
	3	4	5	6	7	8	9	10	15	20	30	50	100
3.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.05
3.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.05
3.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.05
3.23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.05
3.24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.03	0.04
3.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.03	0.04
3.26	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.04
3.27	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.04
3.28	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.03	0.04
3.29	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04
3.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04
3.31	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03
3.32	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03
3.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03
3.34	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03
3.35	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03
3.36	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03
3.37	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03
3.38	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03
3.39	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.02
3.40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02
3.41	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02
3.42	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02
3.43	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02
3.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.02
3.45	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02
3.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02
3.47	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02
3.48	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02
3.49	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02
3.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02
3.51	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
3.52	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
3.53	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
3.54	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
3.55	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
3.56	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
3.57	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
3.58	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
3.59	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
3.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
3.61	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
3.62	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
3.63	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
3.64	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
3.65	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
3.66	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
3.67	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
3.68	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
3.69	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
3.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
3.71	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
3.72	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
3.73	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
3.74	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
3.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
3.76	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.77	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.78	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

NUMBERS IN BODY OF TABLE ARE ESTIMATES OF LOT PERCENT DEFECTIVE CORRESPONDING TO SPECIFIC VALUES OF QUALITY INDEX AND SAMPLE SIZE. FOR Q VALUES GREATER THAN OR EQUAL TO ZERO, THE PERCENT DEFECTIVE ESTIMATE MAY BE READ DIRECTLY FROM THE TABLE. FOR Q VALUES LESS THAN ZERO, THE TABLE VALUE MUST BE SUBTRACTED FROM 100.

A P P E N D I X D

VARIABLES PLANS BASED ON THE RANGE

APPENDIX D

VARIABLES PLANS BASED ON THE RANGE

D.1 General Information

D.1.1 Acceptance plans based on the standard deviation (S) are advocated in favor of those based on the range (R). This appendix is included primarily to provide the capability of analyzing range plans already in existence.

D.1.2 In the past, range plans were regarded as simpler than standard deviation plans because the range was easier to understand and compute. This is less true today with the wide use of statistical methods and the advent of scientific pocket calculators that compute the standard deviation directly.

D.1.3 The range method is not as statistically efficient as the standard deviation method, especially for larger sample sizes. For example, a range plan using a sample size of n = 12 has essentially the same discriminating power (OC curve) as a standard deviation plan using a sample size of n = 10.

D.1.4 Like the variables plans based on the standard deviation, range plans assume a normally distributed population of unknown variability.

D.2 Analysis of Range Plans

D.2.1 The steps to be followed in the analysis or development of acceptance plans based on the range are identically the same as those for standard deviation plans. The only differences are that the range replaces the standard deviation in the quality index (Q) equations and different tables are used.

$$Q_L = \frac{\bar{X} - L}{R} \qquad (D.1)$$

$$Q_U = \frac{U - \bar{X}}{R} \qquad (D.2)$$

in which

Q = quality index

$\bar{X}$ = sample mean

R = sample range

L, U = lower and upper limits outside of which the material or work is defined to be defective

D.2.2 For simplicity, no subgrouping is required for the method presented in this appendix. The range (R) used in Equations D.1 and D.2 is computed directly from the complete set of values obtained from the sample.

D.3 Explanation of Tables

D.3.1 The tables in this appendix were prepared with the same computer algorithms used to generate the tables in Appendix B and Appendix C by substituting (or interpolating for) the appropriate noninteger degrees of freedom associated with range estimates of variability presented in Section E.2.7 of Appendix E. They are formatted the same and are used in the same manner as the tables in Appendix B and Appendix C.

D.3.2 The tables provide information for sample sizes from $n = 3$ to $n = 15$, inclusive. Above $n = 15$, range plans are considerably less efficient than standard deviation plans.

D.3.3 Table D.1 provides the operating characteristics for numerous different range plans. Like the tables in Appendix B, it may occasionally be necessary to interpolate between the plans listed in this table.

D.3.4 Table D.2 gives the estimate of percent defective (PD) associated with the quality index (Q) computed from the sample results. Because the range tends to be larger than the standard deviation, the Q values themselves tend to be smaller, and the table is more compact than its counterpart for the standard deviation method in Appendix C. Also, because it is believed that some precision is lost in adapting the standard deviation method algorithms to construct the range table, the percent defective estimates in the body of the table have been printed to only the first decimal place.

TABLE D.1 Operating Characteristics of Variables Acceptance Plans (Range Method)

VARIABLES ACCEPTANCE PLANS *** VARIABILITY-UNKNOWN PROCEDURE *** RANGE METHOD

SAMPLE SIZE (n)	MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M)	MINIMUM ALLOWABLE QUALITY INDEX (k)	PROBABILITY OF ACCEPTANCE FOR SELECTED LEVELS OF LOT PERCENT DEFECTIVE						
			10	20	30	40	50	60	70
3	34	0.293	0.89	0.71	0.52	0.35	0.22	0.12	0.05
3	36	0.259	0.91	0.74	0.54	0.39	0.24	0.13	0.06
3	38	0.224	0.93	0.78	0.60	0.42	0.27	0.15	0.07
3	40	0.188	0.94	0.81	0.63	0.46	0.30	0.17	0.08
3	42	0.151	0.96	0.84	0.67	0.50	0.33	0.20	0.09
3	44	0.114	0.97	0.87	0.71	0.54	0.37	0.22	0.11
3	46	0.076	0.98	0.89	0.75	0.58	0.41	0.26	0.13
3	48	0.038	0.98	0.91	0.79	0.63	0.46	0.29	0.14
4	30	0.269	0.90	0.70	0.48	0.30	0.16	0.07	0.03
4	32	0.242	0.92	0.73	0.52	0.33	0.18	0.08	0.03
4	34	0.216	0.94	0.77	0.56	0.36	0.21	0.10	0.04
4	36	0.189	0.95	0.80	0.60	0.40	0.23	0.11	0.04
4	38	0.162	0.96	0.83	0.64	0.44	0.26	0.13	0.05
4	40	0.135	0.97	0.86	0.68	0.48	0.30	0.15	0.06
4	42	0.108	0.98	0.89	0.72	0.52	0.33	0.18	0.07
4	44	0.081	0.99	0.91	0.76	0.57	0.37	0.21	0.09
5	26	0.280	0.89	0.65	0.40	0.22	0.10	0.04	0.01
5	28	0.256	0.91	0.69	0.45	0.25	0.12	0.05	0.01
5	30	0.232	0.93	0.73	0.49	0.28	0.14	0.06	0.02
5	32	0.208	0.95	0.77	0.53	0.32	0.16	0.07	0.02
5	34	0.184	0.96	0.80	0.58	0.36	0.19	0.08	0.02
5	36	0.161	0.97	0.84	0.62	0.40	0.22	0.09	0.03
5	38	0.138	0.98	0.87	0.67	0.44	0.25	0.11	0.04
5	40	0.115	0.98	0.89	0.71	0.49	0.28	0.13	0.05
5	42	0.092	0.99	0.91	0.75	0.53	0.32	0.16	0.06
6	24	0.278	0.88	0.61	0.36	0.17	0.07	0.02	0.01
6	26	0.255	0.91	0.66	0.40	0.20	0.09	0.03	0.01
6	28	0.233	0.93	0.71	0.45	0.24	0.10	0.03	0.01
6	30	0.210	0.95	0.75	0.49	0.27	0.12	0.04	0.01
6	32	0.188	0.96	0.79	0.54	0.31	0.14	0.05	0.01
6	34	0.167	0.97	0.83	0.59	0.35	0.17	0.06	0.02
6	36	0.145	0.98	0.86	0.64	0.40	0.20	0.08	0.02
6	38	0.124	0.99	0.89	0.69	0.44	0.23	0.10	0.03
7	24	0.260	0.90	0.62	0.35	0.16	0.06	0.02	0.00
7	26	0.238	0.93	0.67	0.39	0.19	0.07	0.02	0.00
7	28	0.217	0.95	0.72	0.44	0.22	0.09	0.03	0.01
7	30	0.196	0.96	0.77	0.50	0.26	0.11	0.03	0.01
7	32	0.175	0.97	0.81	0.55	0.30	0.13	0.04	0.01
7	34	0.155	0.98	0.85	0.60	0.34	0.16	0.05	0.01
7	36	0.135	0.99	0.88	0.65	0.39	0.19	0.07	0.01
8	22	0.269	0.88	0.57	0.29	0.12	0.04	0.01	0.00
8	24	0.247	0.91	0.63	0.34	0.14	0.05	0.01	0.00
8	26	0.226	0.94	0.69	0.39	0.17	0.06	0.02	0.00
8	28	0.205	0.96	0.74	0.44	0.21	0.08	0.02	0.00
8	30	0.185	0.97	0.78	0.50	0.25	0.09	0.03	0.00
8	32	0.164	0.98	0.82	0.55	0.29	0.12	0.03	0.01
8	34	0.147	0.99	0.86	0.61	0.34	0.14	0.04	0.01
9	22	0.257	0.90	0.58	0.28	0.11	0.03	0.01	0.00
9	24	0.236	0.92	0.64	0.33	0.13	0.04	0.01	0.00
9	26	0.216	0.95	0.69	0.38	0.16	0.05	0.01	0.00
9	28	0.196	0.96	0.75	0.44	0.20	0.07	0.02	0.00
9	30	0.177	0.98	0.80	0.50	0.24	0.08	0.02	0.00
9	32	0.158	0.98	0.84	0.56	0.28	0.11	0.03	0.00
9	34	0.140	0.99	0.87	0.61	0.33	0.13	0.04	0.01
10	20	0.249	0.87	0.52	0.23	0.07	0.02	0.00	0.00
10	22	0.248	0.91	0.58	0.27	0.10	0.03	0.00	0.00
10	24	0.228	0.93	0.64	0.32	0.12	0.03	0.01	0.00
10	26	0.208	0.95	0.70	0.38	0.15	0.04	0.01	0.00
10	28	0.189	0.97	0.76	0.44	0.19	0.06	0.01	0.00
10	30	0.171	0.98	0.81	0.50	0.23	0.08	0.02	0.00
10	32	0.153	0.99	0.85	0.56	0.27	0.10	0.02	0.00
11	20	0.241	0.88	0.52	0.21	0.07	0.01	0.00	0.00
11	22	0.241	0.91	0.58	0.26	0.09	0.02	0.00	0.00
11	24	0.221	0.94	0.65	0.32	0.11	0.03	0.00	0.00
11	26	0.202	0.96	0.71	0.38	0.14	0.04	0.01	0.00
11	28	0.183	0.97	0.77	0.44	0.18	0.05	0.01	0.00
11	30	0.165	0.98	0.82	0.50	0.22	0.07	0.01	0.00
11	32	0.148	0.99	0.86	0.56	0.27	0.09	0.02	0.00
12	20	0.255	0.89	0.51	0.21	0.06	0.01	0.00	0.00
12	22	0.235	0.92	0.59	0.26	0.08	0.02	0.00	0.00
12	24	0.215	0.95	0.65	0.31	0.10	0.02	0.00	0.00
12	26	0.197	0.97	0.72	0.37	0.13	0.03	0.01	0.00
12	28	0.179	0.98	0.77	0.43	0.17	0.05	0.01	0.00
12	30	0.161	0.99	0.82	0.50	0.21	0.06	0.01	0.00
13	20	0.249	0.89	0.51	0.20	0.05	0.01	0.00	0.00
13	22	0.229	0.93	0.59	0.25	0.07	0.01	0.00	0.00
13	24	0.210	0.95	0.66	0.31	0.10	0.02	0.00	0.00
13	26	0.192	0.97	0.72	0.37	0.13	0.03	0.00	0.00
13	28	0.175	0.98	0.78	0.43	0.16	0.04	0.01	0.00
13	30	0.157	0.99	0.83	0.50	0.21	0.06	0.01	0.00
14	20	0.244	0.90	0.51	0.19	0.05	0.01	0.00	0.00
14	22	0.225	0.93	0.59	0.24	0.07	0.01	0.00	0.00
14	24	0.206	0.96	0.66	0.30	0.09	0.02	0.00	0.00
14	26	0.188	0.97	0.73	0.36	0.12	0.03	0.00	0.00
14	28	0.171	0.98	0.79	0.43	0.16	0.04	0.01	0.00
14	30	0.154	0.99	0.84	0.50	0.20	0.05	0.01	0.00
15	20	0.240	0.90	0.51	0.19	0.04	0.01	0.00	0.00
15	22	0.221	0.94	0.59	0.24	0.06	0.01	0.00	0.00
15	24	0.202	0.96	0.67	0.30	0.09	0.02	0.00	0.00
15	26	0.185	0.98	0.73	0.36	0.11	0.02	0.00	0.00
15	28	0.168	0.99	0.79	0.43	0.15	0.03	0.00	0.00

THE ACCEPTANCE PROBABILITIES IN THIS TABLE HAVE BEEN COMPUTED BY INTERPOLATION IN THE NONCENTRAL T DISTRIBUTION USING NONINTEGER DEGREES OF FREEDOM ASSOCIATED WITH RANGE ESTIMATES OF VARIABILITY. THESE PROBABILITY VALUES ARE QUITE ACCURATE FOR SINGLE-LIMIT PLANS AND APPROXIMATELY CORRECT FOR DOUBLE-LIMIT PLANS. FOR SINGLE-LIMIT PLANS, EITHER THE MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE (M) OR THE MINIMUM ALLOWABLE QUALITY INDEX (k) MAY BE SPECIFIED. FOR DOUBLE-LIMIT PLANS, ONLY THE MAXIMUM ALLOWABLE ESTIMATED PERCENT DEFECTIVE SHOULD BE USED.

TABLE D.2 Estimation of Lot Percent Defective (Range Method)

VARIABILITY-UNKNOWN PROCEDURE — RANGE METHOD

QUALITY INDEX (Q)	ESTIMATED LOT PERCENT DEFECTIVE FOR SELECTED SAMPLE SIZES												
	3	4	5	6	7	8	9	10	11	12	13	14	15
0.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
0.01	49.5	49.3	49.1	49.0	48.9	48.9	48.8	48.8	48.7	48.7	48.7	48.7	48.6
0.02	49.0	48.5	48.2	48.1	47.9	47.8	47.7	47.6	47.5	47.4	47.4	47.3	47.3
0.03	48.4	47.8	47.4	47.1	46.9	46.7	46.5	46.4	46.2	46.1	46.1	46.0	45.9
0.04	47.9	47.0	46.5	46.1	45.8	45.6	45.3	45.2	45.0	44.9	44.7	44.6	44.5
0.05	47.4	46.3	45.6	45.1	44.8	44.5	44.2	44.0	43.8	43.6	43.4	43.3	43.2
0.06	46.9	45.6	44.8	44.2	43.7	43.4	43.0	42.8	42.5	42.3	42.1	42.0	41.8
0.07	46.3	44.8	43.9	43.2	42.7	42.3	41.9	41.6	41.3	41.0	40.8	40.6	40.5
0.08	45.8	44.1	43.0	42.2	41.6	41.2	40.7	40.4	40.1	39.8	39.6	39.3	39.1
0.09	45.3	43.3	42.1	41.3	40.6	40.1	39.6	39.2	38.9	38.5	38.3	38.0	37.8
0.10	44.7	42.6	41.3	40.3	39.6	39.0	38.5	38.0	37.6	37.3	37.0	36.7	36.5
0.11	44.2	41.8	40.4	39.4	38.5	37.9	37.3	36.9	36.4	36.1	35.8	35.4	35.2
0.12	43.7	41.1	39.5	38.4	37.5	36.8	36.2	35.7	35.2	34.9	34.5	34.2	33.9
0.13	43.1	40.4	38.7	37.5	36.5	35.8	35.1	34.6	34.1	33.6	33.3	32.9	32.6
0.14	42.6	39.6	37.8	36.5	35.5	34.7	34.0	33.4	32.9	32.5	32.1	31.7	31.4
0.15	42.1	38.9	36.9	35.6	34.5	33.7	32.9	32.3	31.7	31.3	30.9	30.5	30.1
0.16	41.5	38.1	36.1	34.6	33.5	32.6	31.8	31.2	30.6	30.1	29.7	29.3	28.9
0.17	41.0	37.4	35.2	33.7	32.5	31.6	30.8	30.1	29.5	29.0	28.5	28.1	27.7
0.18	40.4	36.6	34.4	32.8	31.5	30.6	29.7	29.0	28.4	27.8	27.4	26.9	26.5
0.19	39.9	35.9	33.5	31.9	30.6	29.5	28.6	27.9	27.3	26.7	26.2	25.8	25.4
0.20	39.3	35.2	32.7	30.9	29.6	28.5	27.6	26.9	26.2	25.6	25.1	24.7	24.2
0.21	38.8	34.4	31.8	30.0	28.6	27.5	26.6	25.8	25.1	24.6	24.1	23.6	23.1
0.22	38.2	33.7	31.0	29.1	27.7	26.6	25.6	24.8	24.1	23.5	23.0	22.5	22.1
0.23	37.7	32.9	30.2	28.2	26.8	25.6	24.6	23.8	23.1	22.5	21.9	21.4	21.0
0.24	37.1	32.2	29.3	27.3	25.8	24.6	23.6	22.8	22.1	21.5	20.9	20.4	20.0
0.25	36.5	31.4	28.5	26.5	24.9	23.7	22.7	21.8	21.1	20.5	19.9	19.4	19.0
0.26	35.9	30.7	27.7	25.6	24.0	22.8	21.7	20.9	20.1	19.5	18.9	18.4	18.0
0.27	35.4	29.9	26.9	24.7	23.1	21.9	20.8	19.9	19.2	18.5	18.0	17.5	17.0
0.28	34.8	29.2	26.0	23.9	22.2	21.0	19.9	19.0	18.3	17.6	17.1	16.5	16.1
0.29	34.2	28.4	25.2	23.0	21.4	20.1	19.0	18.1	17.4	16.7	16.2	15.6	15.2
0.30	33.6	27.7	24.4	22.2	20.5	19.2	18.1	17.2	16.5	15.8	15.3	14.8	14.3
0.31	33.0	27.0	23.6	21.4	19.6	18.4	17.3	16.4	15.6	15.0	14.5	13.9	13.5
0.32	32.3	26.2	22.8	20.5	18.8	17.5	16.4	15.6	14.9	14.2	13.6	13.1	12.7
0.33	31.7	25.5	22.0	19.7	18.0	16.7	15.6	14.7	14.0	13.4	12.8	12.3	11.9
0.34	31.1	24.7	21.2	18.9	17.2	15.9	14.8	14.0	13.2	12.6	12.1	11.6	11.1
0.35	30.4	24.0	20.5	18.1	16.4	15.1	14.0	13.2	12.5	11.8	11.3	10.8	10.4
0.36	29.8	23.2	19.7	17.4	15.6	14.4	13.3	12.4	11.7	11.1	10.6	10.1	9.7
0.37	29.1	22.5	18.9	16.6	14.9	13.6	12.5	11.7	11.0	10.4	9.9	9.4	9.1
0.38	28.5	21.7	18.1	15.8	14.1	12.9	11.8	11.0	10.3	9.7	9.3	8.8	8.4
0.39	27.8	21.0	17.4	15.1	13.4	12.2	11.1	10.3	9.7	9.1	8.6	8.2	7.8
0.40	27.1	20.2	16.6	14.4	12.7	11.5	10.5	9.7	9.0	8.5	8.0	7.6	7.2
0.41	26.4	19.4	15.9	13.6	12.0	10.8	9.8	9.1	8.4	7.9	7.4	7.0	6.7
0.42	25.6	18.7	15.2	12.9	11.3	10.2	9.2	8.5	7.8	7.3	6.9	6.5	6.2
0.43	24.9	17.9	14.4	12.2	10.7	9.5	8.6	7.9	7.3	6.8	6.4	6.0	5.7
0.44	24.1	17.2	13.7	11.6	10.0	8.9	8.0	7.3	6.7	6.3	5.9	5.5	5.2
0.45	23.3	16.4	13.0	10.9	9.4	8.3	7.4	6.8	6.2	5.8	5.4	5.0	4.8
0.46	22.5	15.7	12.3	10.2	8.8	7.8	6.9	6.3	5.7	5.3	4.9	4.6	4.3
0.47	21.7	14.9	11.6	9.6	8.2	7.2	6.4	5.8	5.3	4.9	4.5	4.2	4.0
0.48	20.8	14.1	11.0	9.0	7.6	6.7	5.9	5.3	4.8	4.4	4.1	3.8	3.6
0.49	19.9	13.4	10.3	8.4	7.1	6.2	5.4	4.9	4.4	4.0	3.7	3.5	3.2
0.50	19.0	12.6	9.6	7.8	6.5	5.7	5.0	4.4	4.0	3.7	3.4	3.1	2.9
0.51	18.0	11.9	9.0	7.2	6.0	5.2	4.5	4.0	3.7	3.3	3.1	2.8	2.6
0.52	17.0	11.1	8.3	6.7	5.5	4.8	4.1	3.7	3.3	3.0	2.8	2.5	2.4
0.53	15.9	10.3	7.7	6.2	5.1	4.3	3.8	3.3	3.0	2.7	2.5	2.3	2.1
0.54	14.7	9.6	7.1	5.6	4.6	3.9	3.4	3.0	2.7	2.4	2.2	2.0	1.9
0.55	13.5	8.8	6.5	5.1	4.2	3.6	3.0	2.7	2.4	2.2	2.0	1.8	1.6
0.56	12.1	8.0	5.9	4.7	3.8	3.2	2.7	2.4	2.1	1.9	1.7	1.6	1.5
0.57	10.5	7.2	5.4	4.2	3.4	2.9	2.4	2.1	1.9	1.7	1.5	1.4	1.3
0.58	8.6	6.5	4.8	3.8	3.0	2.6	2.2	1.9	1.7	1.5	1.3	1.2	1.1
0.59	6.2	5.7	4.3	3.3	2.7	2.3	1.9	1.6	1.5	1.3	1.2	1.0	1.0
0.60	1.4	4.9	3.7	2.9	2.4	2.0	1.7	1.4	1.2	1.1	1.0	0.9	0.8
0.61	0.0	4.1	3.2	2.6	2.1	1.7	1.4	1.2	1.1	1.0	0.9	0.8	0.7
0.62	0.0	3.3	2.6	2.2	1.8	1.5	1.2	1.1	0.9	0.8	0.7	0.7	0.6
0.63	0.0	2.5	2.0	1.9	1.5	1.2	1.1	0.9	0.8	0.7	0.6	0.6	0.5
0.64	0.0	1.7	1.9	1.6	1.3	1.1	0.9	0.8	0.7	0.6	0.5	0.5	0.4
0.65	0.0	0.9	1.5	1.3	1.1	0.9	0.7	0.6	0.6	0.5	0.4	0.4	0.4
0.66	0.0	0.0	1.1	1.0	0.9	0.7	0.6	0.5	0.5	0.4	0.4	0.3	0.3
0.67	0.0	0.0	0.8	0.8	0.7	0.6	0.5	0.4	0.4	0.3	0.3	0.3	0.2
0.68	0.0	0.0	0.4	0.6	0.5	0.5	0.4	0.3	0.3	0.3	0.2	0.2	0.2
0.69	0.0	0.0	0.2	0.4	0.4	0.4	0.3	0.3	0.2	0.2	0.2	0.2	0.2
0.70	0.0	0.0	0.0	0.3	0.3	0.3	0.2	0.2	0.2	0.2	0.2	0.1	0.1
0.71	0.0	0.0	0.0	0.1	0.2	0.2	0.2	0.2	0.1	0.1	0.1	0.1	0.1
0.72	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.73	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
0.74	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.0	0.0	0.0
0.75	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0
0.76	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.77	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.78	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.79	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

* VALUES IN BODY OF TABLE ARE ESTIMATES OF LOT PERCENT DEFECTIVE CORRESPONDING TO SPECIFIC VALUES OF QUALITY INDEX AND SAMPLE SIZE. FOR Q VALUES GREATER THAN OR EQUAL TO ZERO, THE PERCENT DEFECTIVE ESTIMATE MAY BE READ DIRECTLY FROM THE TABLE. FOR VALUES LESS THAN ZERO, THE TABLE VALUE MUST BE SUBTRACTED FROM 100.

A P P E N D I X E

UNDERLYING THEORETICAL PRINCIPLES

APPENDIX E

UNDERLYING THEORETICAL PRINCIPLES

E.1 Development of Operating Characteristic Tables for Attributes Plans

E.1.1 The cumulative hypergeometric distribution function gives the probability of acceptance for an attributes plan.

$$P = \frac{\sum_{x=0}^{x=c} C_{d,x} C_{N-d,n-x}}{C_{N,n}} \qquad (E.1)$$

in which

P = probability of acceptance

N = population (lot) size

n = sample size

d = number of defectives in the population

c = acceptance number, maximum allowable number of defective items in the sample

$C_{m,n}$ = number of possible combinations of m items taken n at a time = m!/(n!(m-n)!)

x = summation variable

E.1.2 For the hypergeometric distribution, the lot percent defective would be expressed as 100d/N.

E.1.3 As the population size increases, the hypergeometric distribution approaches the binomial distribution as a limit. When attributes procedures are

applied to continuous data, the lot size is considered to be infinite and the acceptance probabilities are computed as follows:

$$P = \sum_{x=0}^{x=c} C_{n,x} p^x (1 - p)^{n-x} \qquad (E.2)$$

in which

P = probability of acceptance

n = sample size

p = fraction defective of the population

c = acceptance number, maximum allowable number of defective items in the sample

$C_{m,n}$ = number of possible combinations of m items taken n at a time = m!/(n!(m-n)!)

x = summation variable

E.1.4 For the binomial distribution, the lot percent defective would be expressed as 100p.

E.1.5 Both the hypergeometric distribution and the binomial distribution were used to construct the tables in Appendix A.

E.2 Development of Operating Characteristic Tables for Variables Plans

E.2.1 The estimates of lot percent defective provided in the tables in Appendix C are obtained by numerically integrating the beta distribution function:

$$p = \int_{x=0}^{x = \mathrm{Max}(0,\ 1/2 - Q\sqrt{n}/(2(n-1)))} \beta(a,b,x)\, dx \qquad \text{(E.3)}$$

in which

p	=	fraction defective of the population for single-limit applications (for double-limit applications, two separate integration steps must be performed and the results added to obtain the total fraction defective)
$\beta(a,b,x)$	=	beta distribution function
a, b	=	parameters of the beta distribution = n/2 - 1
n	=	sample size
Q	=	quality index, $(\overline{X} - L)/S$ or $(U - \overline{X})/S$ for single-limit applications, both required for double-limit applications
$\overline{X}$	=	sample mean
S	=	sample standard deviation
L, U	=	lower and upper specification limits
x	=	integration variable

E.2.2 The area under the beta distribution obtained in this manner is the fraction defective which must be multiplied by 100 to yield the estimate of percent defective.

E.2.3 Although tables of the beta distribution may be used manually, it is far more practical to use computer assistance to perform the numerical integration.

E.2.4 The probabilities of acceptance provided in Appendix B are obtained by numerically integrating the noncentral t distribution function:

$$P = 1 - \int_{x=-\infty}^{x=k\sqrt{n}} t(v,\delta,x)\,dx \qquad (E.4)$$

in which

P	=	probability of acceptance
$t(v,\delta,x)$	=	noncentral t distribution function
v	=	degrees of freedom = $n - 1$
n	=	sample size
δ	=	noncentrality parameter = $K_p\sqrt{n}$
K_p	=	normal z-score associated with each level of population percent defective for which the computation is to be made
k	=	acceptance constant
x	=	integration variable

E.2.5 If the acceptance procedure is stated in terms of the maximum allowable estimated percent defective (M) rather than the minimum allowable value (k) of the quality index (Q), this must first be converted to a k value using the tables in Appendix C (or Table D.2 in Appendix D for the range method).

E.2.6 Tables of the noncentral t distribution are available but, like the procedure involving the beta distribution, these computations can best be performed by computer.

E.2.7 When these same operations are to be performed for acceptance plans based on the range (R) method, minor modifications must be made to account for the reduced degrees of freedom associated with range estimates of variability.

E.2.7.1 To obtain estimates of lot percent defective using the range method, the upper integration limit in Equation E.3 must be changed to

$$x = \text{Max}(0, 1/2 - d_2^* Q\sqrt{(v + 1)}/2v) \qquad \text{(E.5)}$$

in which

x = integration variable

Q = quality index computed by the range method, $(\bar{X} - L)/R$ or $(U - \bar{X})/R$ for single-limit applications, both required for double-limit applications

d_2^* = factor that, when divided into the range computed from the sample, converts it into an estimate of the standard deviation

v = degrees of freedom (the appropriate noninteger values listed in Section E.2.7.3 must be used)

E.2.7.2 To develop operating characteristic curves for acceptance plans based on the range, Equation E.4 may be used except that it will be necessary to compute two values for integral degrees of freedom in order to interpolate for the appropriate noninteger degrees of freedom given in Section E.2.7.3.

E.2.7.3 The following values may be found in Table D3 of Quality Control and Industrial Statistics by A. J. Duncan, published by Richard D. Irwin, Inc., 1965:

SAMPLE SIZE	CONVERSION FACTOR (d_2^*)	DEGREES OF FREEDOM (RANGE METHOD)
3	1.91	2.0
4	2.24	2.9
5	2.48	3.8
6	2.67	4.7
7	2.83	5.5
8	2.96	6.3
9	3.08	7.0
10	3.18	7.7
11	3.27	8.4
12	3.35	9.0
13	3.42	9.6
14	3.49	10.2
15	3.55	10.8

A P P E N D I X F

DEFINITIONS OF TERMS AND SYMBOLS

APPENDIX F

DEFINITIONS OF TERMS AND SYMBOLS

F.1 Definitions of Terms

ACCEPTABLE QUALITY LEVEL (AQL) — that level of lot percent defective at or below which the work is considered to be satisfactory.

ACCEPTANCE CONSTANT (k) — the minimum allowable quality index (Q) for a variables acceptance procedure.

ACCEPTANCE NUMBER (c) — the maximum allowable number of defective items in a sample for an attributes acceptance procedure.

ACCURACY — the degree to which the mean of a distribution of measurements tends to coincide with the true population value.

ATTRIBUTES ACCEPTANCE PLAN — a statistical acceptance procedure based on qualities that are counted rather than measured.

BETA DISTRIBUTION — a statistical distribution that underlies the percent defective estimation process used with variables acceptance procedures.

BINOMIAL DISTRIBUTION — a statistical distribution used to develop operating characteristic curves for attributes acceptance plans applied to continuous data.

BUYER'S RISK (β) — the probability that an acceptance plan will erroneously accept a lot that is truly rejectable.

CRITICALITY — the relative importance of various factors affecting the safety, serviceability, cost, and/or contractual requirements pertaining to a particular item of work.

HYPERGEOMETRIC DISTRIBUTION -- a statistical distribution used to develop operating characteristic curves for attributes acceptance procedures.

LIMIT (L, U) — value below or above which it is desired that the percent defective be maintained at a low level.

LOT -- a discrete quantity of material or work to which an acceptance procedure is applied.

LOT PERCENT DEFECTIVE -- see PERCENT DEFECTIVE.

MEAN ($\overline{X}$) -- a statistical measure of central tendency, the arithmetic average of a group of measurements or test results, used with variables acceptance procedures.

NONCENTRAL t DISTRIBUTION -- a statistical distribution used to develop operating characteristic curves for variables acceptance procedures.

NORMAL DISTRIBUTION -- a statistical distribution that closely approximates the distributions of many types of construction data. The variables

acceptance procedures described in this standard assume that the population being sampled is normally distributed.

OPERATING CHARACTERISTIC (OC) CURVE -- a graphical representation of an acceptance plan's capability to discriminate between satisfactory and unsatisfactory work.

PERCENT DEFECTIVE (PD) — percentage of the lot falling outside specification limits, may refer to either the population value or the sample estimate of the population value.

POPULATION -- any group having some characteristic in common, each lot is regarded as a population for attributes and variables acceptance procedures.

PRECISION — connotes the degree to which tests or measurements on identical samples tend to produce the same results.

QUALITY INDEX (Q) -- a statistic computed when applying the variables acceptance procedures described in this standard.

RANDOM SAMPLE -- a sample selected in such a way that every element of the population has an equally likely opportunity to be included in the sample.

RANGE (R) -- a statistical measure of dispersion, the difference between the largest and smallest value of a group, used with one type of variables acceptance procedure.

REJECTABLE QUALITY LEVEL (RQL) — that level of lot percent defective at or above which the work is considered to be unacceptable.

RISK — see BUYER'S RISK and SELLER'S RISK

SAMPLE -- a subset of the population (lot), tests or measurements on which are used to assess the quality of the lot.

SELLER'S RISK (α) -- the probability that an acceptance plan will erroneously reject a lot that is truly acceptable.

SIGNIFICANT CHARACTERISTIC -- a property of a material or item of work that directly affects its performance.

SPECIFICATION LIMIT -- see LIMIT.

STANDARD DEVIATION (S, σ) — a statistical measure of dispersion used with variables acceptance procedures.

STRATIFIED RANDOM SAMPLING — a method of random sampling that causes the samples to be spread more uniformly throughout the lot.

VARIABLES ACCEPTANCE PLAN -- a statistical acceptance procedure based on characteristics that are measured rather than counted and which involves the computation of statistical parameters.

F.2 Definitions of Symbols

a, b — parameters of the beta distribution

AQL — acceptable quality level

α — alpha, seller's risk

β — beta, buyer's risk, also refers to beta distribution

$C_{m,n}$ — number of combinations of m items taken n at a time

c — acceptance number in an attributes acceptance procedure

d — number of defective items in a lot being evaluated by an attributes acceptance procedure

d_2^* — factor that, when divided into the range computed from a sample, converts it into an estimate of the standard deviation

δ — delta, the noncentrality parameter of the noncentral t distribution

K_p — normal z-score associated with each particular level of percent defective when using the noncentral t distribution to develop operating characteristic curves for variables acceptance plans

k — the acceptance constant of a variables acceptance procedure (either k or M must be specified)

L — lower specification limit

M — maximum allowable estimated percent defective of a variables acceptance procedure (either M or k must be specified)

m — variable used in definition of $C_{m,n}$

N — size of a finite population, number of items in a lot being evaluated by an attributes acceptance procedure

n — sample size, variable used in definition of $C_{m,n}$

v — nu, degrees of freedom

OC — operating characteristic

P — probability of acceptance

PD — percent defective

p — fraction defective in expressions for binomial and beta distributions

Q — quality index computed when applying the variables acceptance procedures described in this standard

R — sample range

RQL — rejectable quality level

S — sample standard deviation

σ — population standard deviation

t — refers to noncentral t distribution

U — upper specification limit

$\bar{X}$ — sample mean

x — summation variable used in computation of hypergeometric and binomial probabilities, integration variable used for computations involving beta and noncentral t distributions