New Jersey Student Learning Standards  
Mathematics  
(NJSLS—Mathematics)

Office of Standards, Division of Teaching and Learning Services  
New Jersey Department of Education

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## Introduction

N.J.A.C. 6A:8-2.1 directs the Commissioner to engage in the review and readoption of the New Jersey Student Learning Standards (NJSLS) every five years. The English Language Arts (ELA) and Mathematics standards were most recently adopted in 2016; the review process began in July of 2022. After six weeks of intensive discussion, recommendations for revisions are proposed by four external working committees.

The week of July 25, 2022, the expert committees met virtually for three full days to discuss the content and the recent challenges of implementation of the 2016 NJSLS. Members conducted a literature scan of recent research related to English language arts and mathematics teaching and learning, Common Core State Standards implementation, and post pandemic findings related to student performance during and after the COVID-19 global pandemic. The committee members shared their findings and engaged in discussion, considering if and how each study might inform the revision of the NJSLS. The committees deliberated and developed a formal set of recommendations to share with the writing team. The committees also provided the writing and implementation teams with selected, relevant peer-reviewed literature references.

The ELA and Math writing teams met throughout the month of August for six semi-weekly, full day, virtual meetings on August 2, 4, 9, 11, 16, and 18, 2022. The writing committees were grouped into grade band expertise throughout the month for targeted reviews. Each grade band group reviewed the report of recommendations composed by the expert review committee and proposed specific language changes and modifications to the standards. Each proposed change required clear justification from the expert review, based on the research provided. The potential impact on educators, including the need for professional development, was considered heavily. Grade band groups reconvened as an entire writing committee to ensure learning progressions remained intact, developmentally appropriate, and coherent across grade levels K–12.

## Interpretation of the Proposed Revisions

The draft NJSLS are featured in this document, arranged by grade level expectations. The document follows the NJDOE Style Guide with respect to structuring Administrative Code rulemakings and related content. Please note that brackets around text signal deletion of the word or phrase and will appear before insertions. Bolded text signals new language to be included in the NJSLS. Please also note the inclusion of Clarification Statements, which previously existed as footnotes in the NJSLS-Mathematics documents. These statements, featured in bolded red font, have been moved within the document closer to the expectations they support to assist readers in integrating the information.

Deletions are signaled with square brackets [ ] and the text {begin deletion} and {end deletion}.

For example:

Solve systems of two linear equations in two variables {begin deletion} [algebraically,] {end deletion}

New language is signaled with **bolded** text, and the text {begin new} and {end new}.

For example:

{begin new}

**B. Work with money**

**3. Understand that certain objects are coins and dollars, and that coins and dollars represent money. Identify the values of all U.S. coins and the one-dollar bill.**

{end new}

Revisions that are complex or repeat in a specific pattern are signaled with text before the revision.

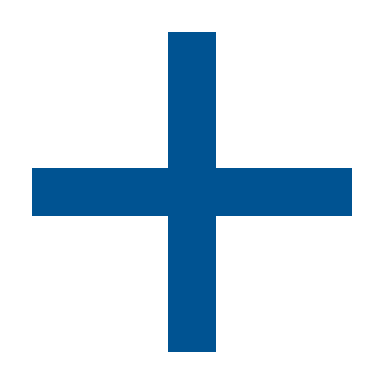
For example:

{Measurement and Data (K.MD) was changed to Measurement (K.M)}

Measurement[and Data] K.M[D]

## Summary of Changes to NJSLS Mathematics

The expert review and writing teams revised the NJSLS Mathematics in the following ways:

1. Decoupled the K–5 ‘Measurement and Data’ domain resulting in a ‘Measurement’ domain and a ‘Data Literacy’ domain. Data literacy is the set of skills needed to ask questions, collect, analyze, interpret and communicate about data. Expanding and enhancing this instruction in early grades lays the foundation for the middle grades’ statistics and probability standards and for high school statistics and/or data science courses.
2. Established new standards in early elementary that establish a progression toward the currently adopted later elementary standards for solving word problems involving money. Students are expected to solve word problems without knowing the comparative values of coins and all dollars, and this revision would develop conceptual understanding of monetary denominations prior to encountering that content.
3. Replaced the term “fluency” throughout the standards. This revision emphasizes that accuracy and efficiency, as opposed to speed, are the most essential aspects of fluency. In all cases, ‘fluently’ was replaced with the phrase ‘with accuracy and efficiency’.
4. Developed middle school foundational work with radicals, which was absent from the standards. Advanced work with radicals is expected in high school advanced math courses, but middle school clarifications were included to ensure that students are prepared to simplify numerical radicals by the end of grade 8 in preparation for Algebra I. This includes enhanced rational and irrational number expectations in grades 6–8 to ensure more comprehensive work with rational and irrational numbers prior to Algebra I.
5. Designated additional standards that may be clustered into courses such as statistics and data science as “plus”, which complements the existing plus standards (****). These newly designated standards complement the focus on data literacy in early grades. This set of knowledge and skills guide students in asking data-based questions, collecting, analyzing, interpreting, and communicating about data.

## A Note on Data Literacy

The revisions within the 2023 New Jersey Student Learning Standards—Mathematics reflect the changing ways in which data has increasingly become the lens with which students discuss and respond to data-based questions and make sense of the world. Data literacy integrates and further develops mathematics, statistics, quantitative, technological, and other discipline-specific literacies to allow students to synthesize data in support of decision making and preparing for the future. The proposed standards develop students’ understanding of how to engage with data, while increasing experience with authentic data sets through scaffolded and supported data exploration. Students leverage early work with data literacy to support later work in Statistics where they will formulate developmentally appropriate statistical questions, collect and consider data, analyze the data they collect, and interpret and communicate those findings to others.

## A Note on the Inclusion of Climate Change Opportunities Climate change icon (highlighted as an opportunity for inclusion in an interdisciplinary climate change unit).

With the adoption of the 2020 New Jersey Student Learning Standards (NJSLS), New Jersey became the first state in the nation to include climate change across content areas. The standards are designed to prepare students to understand how and why climate change happens, the impact it has on our local and global communities and to act in informed and sustainable ways.

The ability for students to critically develop data-supported questions and present evidence in order to take action against climate change and other significant issues is paramount. Informed and reasoned discussion about climate change must occur in the public sphere, and New Jersey’s classrooms will support students’ inquiry into new realities, engagement in civilized, data-supported discussion, and the enactment of change. New Jersey is developing a generation of students that can use data to create alternate discourses to change the present and shape the future. The content area of Mathematics develops the ability and responsibility to excite, inspire, and empower students to recognize this potential and become involved in the issues of our age, which includes climate change.

Throughout the 2023 New Jersey Student Learning Standards in English Language Arts and Mathematics, standards that may be leveraged in support of climate change instruction can been identified through the green icon featured above. This icon encourages educators to utilize the specific standard in interdisciplinary units focused on climate change that include authentic learning experiences, integrate a range of perspectives and are action oriented.

## Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

### 1. Make sense of problems and persevere in solving them

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

### 2. Reason abstractly and quantitatively

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

### 3. Construct viable arguments and critique the reasoning of others

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

### 4. Model with mathematics

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

### 5. Use appropriate tools strategically

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

### 6. Attend to precision

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

### 7. Look for and make use of structure

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see   
7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression , older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers *x* and *y*.

### 8. Look for and express regularity in repeated reasoning

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation .  
Noticing the regularity in the way terms cancel when expanding , , and might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Standards for Mathematical Content

### Kindergarten

#### Counting and Cardinality K.CC

##### A. Know number names and the count sequence

1. Count to 100 by ones and by tens.
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

##### B. Count to tell the number of objects

1. Understand the relationship between numbers and quantities; connect counting to cardinality.
   1. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
   2. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
   3. Understand that each successive number name refers to a quantity that is one larger.
2. Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

##### C. Compare numbers

1. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. {begin new} **(Clarification: Include groups with up to ten objects.)** {end new}
2. Compare two numbers between 1 and 10 presented as written numerals.

#### Operations and Algebraic Thinking K.OA

##### A. Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from

1. Represent addition and subtraction up to 10 with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. {begin new} **(Students may monitor and document the daily weather. They can use an icon of the Sun to represent sunny days. They can also use an icon of a cloud to represent cloudy day, and an icon of a raindrop to represent rainy days. They may solve addition and subtraction questions based on their data.) (Clarification: weather is a perquisite to understanding climate change.)** {end new}. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
5. Demonstrate {begin deletion} [fluency] {end deletion} {begin new} **accuracy and efficiency** {end new} for addition and subtraction within 5.

#### Number and Operation in Base Ten K.NBT

##### A. Work with numbers 11–19 to gain foundations for place value

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

{Measurement and Data (K.MD was changed to Measurement (K.M).}

#### Measurement [and Data] K.M[D]

##### A. Describe and compare measurable attributes

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
2. Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.

{begin new}

##### **B. Work with money**

1. **Understand that certain objects are coins and dollars, and that coins and dollars represent money. Identify the values of all U.S. coins and the one-dollar bill.**

#### **Data Literacy K.DL**

{end new}

{K.MD.B was changed to K.DL.A.; K.MD.B.3. was changed to K.DL.A.1.}

##### [B] **A**. Classify objects and count the number of objects in each category

[3] **1**. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. {begin new} **(Clarification: Limit category counts to be less than or equal to 10) (Students may monitor and document the daily weather. They may use an icon of the Sun to represent sunny days. They may use an icon of a cloud to represent cloudy day, and an icon of a raindrop to represent rainy days. The students may classify, count and sort the categories by count.)** **(Clarification Statement: weather is a perquisite to understanding climate change.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit. {end new}

#### Geometry K.G

##### A. Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres)

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.
2. Correctly name shapes regardless of their orientations or overall size.
3. Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).

##### B. Analyze, compare, create, and compose shapes

1. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length). {begin new} (Students may **design and build a structure, using common objects found in the classroom, to investigate how sunlight** **warms the Earth’s surface.** **Throughout the design and building, students may compare two- and three-dimensional objects.)** **(Clarification Statement: Examples of Earth’s surface could include sand, soil, rocks, and water.]**. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit. {end new}
2. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. {begin new} (Students may **design and build a structure to investigate how sunlight** **warms the Earth’s surface, they may build shapes from components and drawing shapes. Throughout the design and building, students may compare two- and three-dimensional objects.)** **(Clarification Statement: Examples of structures could include umbrellas, canopies, and tents that minimize the warming effect of the sun. Understanding how the Sun provides thermal energy to the Earth’s surface is a perquisite to understanding climate change.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}
3. Compose simple shapes to form larger shapes. For example, “Can you join these two triangles with full sides touching to make a rectangle?”.

### Grade 1

#### Operations and Algebraic Thinking 1.OA

##### A. Represent and solve problems involving addition and subtraction

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. {begin new} **(Students may investigate how global interconnections occur between human and physical systems across different regions of the world. Students may collect data and consider sources from multiple perspectives to become informed about a climate change issue and identify possible solutions. The data may be used to create addition and subtraction problems.)** [[1]](#footnote-1) highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}
2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. {begin new} **(Students may collect data and consider sources from multiple perspectives and become informed about a climate change issue and identify possible solutions. The data may be used to create word problems that call for addition of three whole numbers sum is less than or equal to 20.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}

##### B. Understand and apply properties of operations and the relationship between addition and subtraction

1. Apply properties of operations as strategies to add and subtract. Examples: If 8+3=11 is known, then 3+8=11 is also known. (Commutative property of addition.) To add 2+6+4 the second two numbers can be added to make a ten, so 2+6+4=2+10=12. (Associative property of addition.) {begin new} **(Clarification: Students need not use formal terms for these properties.)** {end new}
2. Understand subtraction as an unknown-addend problem. For example, subtract 10−8 by finding the number that makes 10 when added to 8.

##### C. Add and subtract within 20

1. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
2. Add and subtract within 20, demonstrating {begin deletion} [fluency] {end deletion} {begin new} **accuracy and efficiency** {end new} for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8+6=8+2+4=10+4=14); decomposing a number leading to a ten (e.g., 13−4=13−3−1=10−1=9); using the relationship between addition and subtraction (e.g., knowing that 8+4=12, one knows 12−8=4); and creating equivalent but easier or known sums (e.g., adding 6+7 by creating the known equivalent 6+6+1=12+1=13).

##### D. Work with addition and subtraction equations

1. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6=6, 7=8−1, 5+2=2+5, 4+1=5+2.
2. Determine the unknown whole number in an addition or subtraction equation relating to three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + � = 11, 5 = □ − 3 , 6 + 6 = �.

#### Number and Operation in Base Ten 1.NBT

##### A. Extend the counting sequence

1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

##### B. Understand place value

1. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
   1. 10 can be thought of as a bundle of ten ones — called a “ten.”
   2. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
   3. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
2. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

##### C. Use place value understanding and properties of operations to add and subtract

1. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models (e.g., base ten blocks) or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
2. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
3. Subtract multiples of 10 in the range 10–90 from multiples of 10 in the range 10–90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

{Measurement and Data (1.MD) was changed to Measurement (1.M).}

#### Measurement [and Data] 1.M[D]

##### A. Measure lengths indirectly and by iterating length units

1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.
2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.

##### B. Tell and write time

1. Tell and write time in hours and half-hours using analog and digital clocks.

{begin new}

##### **C. Work with money**

1. **Know the comparative values of coins and all dollars (e.g., a dime is of greater value than a nickel). Use appropriate notation (e.g., 69¢, $10).**
2. **Use dollars in the solutions of problems up to $20. Find equivalent monetary values (e.g., a nickel is equivalent in value to five pennies). Show monetary values in multiple ways. For example, show 25¢ as two dimes and one nickel, and as five nickels. Show $20 as two tens and as 20 ones.**

#### **Data Literacy 1.DL**

{end new}

{1.MD.C was changed to 1.DL.A.; 1.MD.C.4 was changed to 1.DL.A.1.}

##### [C] **A**. Represent and interpret data

[4] **1**. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another. {begin new} **(Students may** **organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.**)highlighted as an opportunity for inclusion in an interdisciplinary climate change unit. {end new}

#### Geometry 1.G

##### A. Reason with shapes and their attributes

1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.{begin new} **(Clarification: Students do not need to learn formal names such as “right rectangular prism.”)** {end new}
3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

### Grade 2

#### Operations and Algebraic Thinking 2.OA

##### A. Represent and solve problems involving addition and subtraction

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions*,* e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. [[2]](#footnote-2) {begin new} **(Students may make sense of the ideas that plants need water and light to grow and that climate change affects the health of plants, animals and people. In this unit there would an opportunity to measure variables and use the date to add and subtract within 100.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit. {end new}

##### B. Add and subtract within 20

1. {begin deletion} [Fluently] {end deletion} {begin new} **With accuracy and efficiency** {end new}, add and subtract within 20 using mental strategies.[[3]](#footnote-3) By end of Grade 2, know from memory all sums of two one-digit numbers.

##### C. Work with equal groups of objects to gain foundations for multiplication

1. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
2. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal add

#### Number and Operation in Base Ten 2.NBT

##### A. Understand place value

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
   1. 100 can be thought of as a bundle of ten tens — called a “hundred.”
   2. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. Count within 1000; skip-count by 5s, 10s, and 100s.
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

##### B. Use place value understanding and properties of operations to add and subtract

1. {begin deletion} [Fluently] {end deletion} {begin new} **With accuracy and efficiency** {end new}, add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
2. Add up to four two-digit numbers using strategies based on place value and properties of operations.
3. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
4. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
5. Explain why addition and subtraction strategies work, using place value and the properties of operations. {begin new} **(Clarification: Explanations should be supported by drawings or objects.)** {end new}

{Measurement and Data (2.MD) was changed to Measurement (2.M).}

#### Measurement [and Data] 2.M[D]

##### A. Measure and estimate lengths in standard units

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
3. Estimate lengths using units of inches, feet, centimeters, and meters.
4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

##### B. Relate addition and subtraction to length

1. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. {begin new} **(Students may measure the lengths of different plants to solve a word problem that is based on an investigation where they make sense of the idea that plants need water and light to grow and how climate change affects the health of plants, animals and people.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}
2. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

##### C. Work with time and money

1. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
2. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?

{begin new}

#### **Data Literacy 2.DL**

##### **A. Understand concepts of data**

1. **Understand that people collect data to answer questions. Understand that data can vary.** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
2. **Identify what could count as data (e.g., visuals, sounds, numbers).** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.

{end new}

{2.MD.D was changed to 2.DL.B.}

##### [D] **B**. Represent and interpret data

{2.MD.D.9 was changed to 2.DL.B.3.}

[9] **3**. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.

{2.MD.D.10 was changed to 2.DL.B.3.}

[10] **4**. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put together, take-apart, and compare problems[[4]](#footnote-4) using information presented in a bar graph. {begin new} **(Students may make sense of the idea that climate change affects the health of plants, animals and people, and may draw a picture graph and a bar graph (with single-unit scale) to represent a data set using information presented in a bar graph.**) highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}

#### Geometry 2.G

##### A. Reason with shapes and their attributes

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. {begin new} **(Clarification: sizes are compared directly or visually, not compared by measuring)** {end new}
2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

### Grade 3

#### Operations and Algebraic Thinking 3.OA

##### A. Represent and solve problems involving multiplication and division

1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe and/or represent a context in which a total number of objects can be expressed as 5 × 7.
2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe and/or represent a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[[5]](#footnote-5) {begin new} **(Students may make sense of the idea that climate change affects the health of plants, animals and people. During an investigation they would record and represent weather and climate data which can be used as the basis for multiplication and division word problems.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × � = 48, 5 = □÷ 3, 6 × 6 = �.

##### B. Understand properties of multiplication and the relationship between multiplication and division

1. Apply properties of operations as strategies to multiply and divide.Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10= 30. (Associative property of multiplication.) Knowing that 8 × 5= 40 and 8 × 2= 16, one can find 8 × 7 as . (Distributive property.) {begin new} **{Clarification: Students need not use formal terms for these properties}** {end new}
2. Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

##### C. Multiply and divide within 100

1. {begin deletion} [Fluently] {end deletion} {begin new} **With accuracy and efficiency,** {end new} multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

##### D. Solve problems involving the four operations, and identify and explain patterns in arithmetic

1. Solve two-step word problems {begin new}**, including problems involving money,** {end new} using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. {begin new} **(Clarification: This standard is limited to problems posed with whole numbers and having whole number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order) (Order of Operations) (Students may record and represent weather and climate data and use it as the basis two-step word problems using the four operations.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}
2. Identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

#### Number and Operations in Base Ten 3.NBT

A. Use place value understanding and properties of operations to perform multi-digit arithmetic

{begin new} **(Clarification: A range of algorithms may be used)** {end new}

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. {begin deletion} [Fluently] {end deletion} {end new} **With accuracy and efficiency,** {end new} add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

#### Number and Operations - Fractions[[6]](#footnote-6) 3.NF

##### A. Develop understanding of fractions as numbers

1. Understand a fraction as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction as the quantity formed by *a* parts of size .
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
   1. Represent a fraction on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size and that the endpoint of the part based at 0 locates the number on the number line.
   2. Represent a fraction on a number line diagram by marking off *a* lengths from 0. Recognize that the resulting interval has size and that its endpoint locates the number on the number line.
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
   1. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
   2. Recognize and generate simple equivalent fractions, (e.g., , ). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
   3. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form ; recognize that ; locate and 1 at the same point of a number line diagram.
   4. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

{Measurement and Data (3.MD) was changed to Measurement (3.M).}

#### Measurement[and Data] 3.M[D]

##### A. Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.{begin new} **(Clarification: Measure and estimate liquid volumes and masses excludes compound units such as cm3 and finding the geometric volume of a container. Multiplying to solve one-step word problems excludes multiplicative comparison problems) (problems involving “times as much”; See Glossary, Table 2)** {end new}

{3.MD.C. was changed to 3.M.B.}

##### [C] **B**. Geometric measurement: understand concepts of area and relate area to multiplication and to addition

{3.MD.C.5 was changed to 3.M.B.3.}

[5] **3**. Recognize area as an attribute of plane figures and understand concepts of area measurement.

1. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
2. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

{3.MD.C.6 was changed to 3.M.B.4.}

[6] **4**. Measure areas by counting unit squares (square cm, square m, square in, square ft, and non-standard units).

{3.MD.C.7 was changed to 3.M.B.5.}

[7] **5**. Relate area to the operations of multiplication and addition.

1. Find the area of a rectangle with whole-number side lengths by tiling it and show that the area is the same as would be found by multiplying the side lengths.
2. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
3. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning.
4. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

{3.MD.D. was changed to 3.M.C.}

##### [D] **C**. Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures

{3.MD.D.8 was changed to 3.M.C.6.}

[8] **6**. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. {begin new} **(Students may make sense of the how long-term patterns of weather are used to describe climate in different regions of the world. During this investigation students would have the opportunity to recognize perimeter as an attribute of plane figures (e.g. maps of cities, regions and countries.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.

{begin new}

#### **Data Literacy 3.DL**

##### **A. Understand data-based questions and data collection.**

1. **Develop data-based questions and decide what data will answer the question. (e.g. “What size shoe does a 3rd grader wear?”, “How many books does a 3rd grader read?”)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
2. **Collect student-centered data (e.g. collect data on students’ favorite ice cream flavor) or use existing data to answer data-based questions.** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.

{end new}

##### B. Represent and interpret data

1. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
2. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.

#### Geometry 3.G

##### A. Reason with shapes and their attributes

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as ¼ of the area of the shape.

### Grade 4

#### Operations and Algebraic Thinking 4.OA

##### A. Use the four operations with whole numbers to solve problems

1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[[7]](#footnote-7)
3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. {begin new} **(Students may obtain data and information to describe that energy and fuels are derived from natural resources and their uses affect the climate and solve multistep word problem based on the data collected.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit. {end new}

##### B. Gain familiarity with factors and multiples

1. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range   
   1–100 is prime or composite.

##### C. Generate and analyze patterns

1. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.

#### Number and Operations in Base Ten[[8]](#footnote-8) 4.NBT

##### A. Generalize place value understanding for multi-digit whole numbers

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

##### B. Use place value understanding and properties of operations to perform multi-digit arithmetic

1. {begin deletion} [Fluently] {end deletion} {begin new} **With accuracy and efficiency,** {begin new}add and subtract multi-digit whole numbers using the standard algorithm.
2. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
3. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area model.

#### Number and Operations - Fractions[[9]](#footnote-9) 4.NF

##### A. Extend understanding of fractions equivalence and ordering

1. Explain why a fraction is equivalent to a fraction by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

##### B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers

1. Understand a fraction with a > 1 as a sum of fractions .
   1. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
   2. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.   
      Examples: ; ; .
   3. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
   4. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
2. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   1. Understand a fraction as a multiple of .   
      For example, use a visual fraction model to represent as the product 5 × (¼), recording the conclusion by the equation .
   2. Understand a multiple of as a multiple of , and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express as , recognizing this product as . (In general, .)
   3. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

##### C. Understand decimal notation for fractions and compare decimal fractions

1. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.For example, express as , and add .{begin new}  
   **(Clarification: Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)** {end new}
2. Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as ; describe a length as 0.62 meters; locate 0.62 on a number line diagram.
3. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

{Measurement and Data (4.MD) was changed to Measurement (4.M).}

#### Measurement[and Data] 4.M[D]

##### A. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit

1. Know relative sizes of measurement units within one system of units including km, m, cm. mm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...
2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. {begin new} **(Students may perform bench science investigations that model global systems. These investigations may involve distances, time, liquid volumes, and the mass of objects.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}
3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

{4.MD.C was changed to 4.M.B.}

##### [C] **B**. Geometric measurement: understand concepts of angle and measure angles

{4.MD.C.5 was changed to 4.M.B.4.}

[5] **4**. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

* 1. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through th of a circle is called a “one-degree angle,” and can be used to measure angles.
  2. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

{4.MD.C.6 was changed to 4.M.B.5.}

[6] **5**. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

{4.MD.C.7 was changed to 4.M.B.6.}

[7] **6**. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

{begin new}

#### **Data Literacy 4.DL**

##### **A. Organize data and understand data visualizations**

1. **Create data-based questions, generate ideas based on the questions, and then refine the questions.** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
2. **Develop strategies to collect various types of data and organize data digitally.** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
3. **Understand that subsets of data can be selected and analyzed for a particular purpose.** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
4. **Analyze visualizations of a single data set, share explanations and draw conclusions that the data supports.** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.

{end new}

{The word "measurement" was added. New cluster is "B. Represent and interpret measurement data."}

##### B. Represent and interpret **measurement** data

{4.MD.B.4 was changed to 4.DL.B.5.}

[4] **5**. Make a line plot to display a data set of measurements in fractions of a unit (½, ¼, ⅛). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection. {begin new} **(Students may perform bench science investigations that model global systems. These investigations may involve distances, time, liquid volumes, and the mass of objects. This data can be used to create line plot to display a data set of measurements in fractions of a unit (½, ¼, ⅛). Solve problems involving addition and subtraction of fractions by using information presented in line plots.)**.highlighted as an opportunity for inclusion in an interdisciplinary climate change unit. {end new}

#### Geometry 4.G

##### A. Draw and identify lines and angles, and classify shapes by properties of their lines and angles

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category and identify right triangles.
3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

### Grade 5

#### Operations and Algebraic Thinking 5.OA

##### A. Write and interpret numerical expressions

1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as . Recognize that is three times as large as 18932 + 921, without having to calculate the indicated sum or product.

##### B. Analyze patterns and relationships

1. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

#### Number and Operations in Base Ten 5.NBT

##### A. Understand the place value system

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and ⅒ of what it represents in the place to its left.
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
3. Read, write, and compare decimals to thousandths.
   1. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., .
   2. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
4. Use place value understanding to round decimals to any place.

##### B. Perform operations with multi-digit whole numbers & with decimals to hundredths

1. {begin deletion} [Fluently] {end deletion} {begin new} **With accuracy and efficiency,** {begin new}multiply multi-digit whole numbers using the standard algorithm.
2. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
3. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

#### Number and Operations — Fractions 5.NF

##### A. Use equivalent fractions as a strategy to add and subtract fractions

1. Explain why a fraction is equivalent to a fraction by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as ½. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

##### B. Apply and extend previous understandings of multiplication and division to multiply and divide fractions

1. Interpret a fraction as division of the numerator by the denominator . Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret ¾ as the result of dividing 3 by 4, noting that ¾ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size ¾. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? {begin new}, **(Students may obtain and combine information to examine the impact climate change has on agriculture there are opportunities to solve word problems about the reduced yields of staple crops and their distribution.** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}
2. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
   1. Interpret the product as *a* parts of a partition of *q* into *b* equal parts; equivalently, as the result of a sequence of operations For example, use a visual fraction model to show , and create a story context for this equation. Do the same with . (In general, ).
   2. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
3. Interpret multiplication as scaling (resizing), by:
4. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
5. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence to the effect of multiplying by 1.
6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.1
8. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (⅓) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that because .
9. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (⅕), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (⅕) = 20 because 20 × (⅕) = 4.
10. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share ½ lb of chocolate equally? How many ⅓-cup servings are in 2 cups of raisins? {begin new} **(Students may obtain and combine information to examine the impact climate change has on agriculture there are opportunities to solve word problems involving the division of whole numbers and division of whole numbers by unit fractions.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}

{Measurement and Data (5.MD) was changed to Measurement (5.M).}

#### Measurement[and Data] 5.M[D]

##### A. Convert like measurement units within a given measurement system

1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

{5.MD.C was changed to 5.M.B.}

##### [C] **B**. Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition

{5.MD.C .3. was changed to 5.M.B.2.}

[3] **2**. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

1. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
2. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

{5.MD.C .4. was changed to 5.M.B.3.}

[4] **3**. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and non-standard units.

{5.MD.C .5. was changed to 5.M.B.4.}

[5] **4**. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

1. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
2. Apply the formulas and for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems.
3. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

{begin new}

#### **Data Literacy 5.DL**

##### **A. Understand and analyze data visualizations**

1. **Understand how different visualizations can highlight different aspects of data; Ask questions and interpret data visualizations to describe and analyze patterns.** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
2. **Develop strategies to collect, organize and represent data of various types and from various sources. Communicate results digitally through a data visual (e.g. chart, storyboard, video presentation).** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
3. **Collect and clean data to be analyzable (e.g., make sure each entry is formatted correctly, deal with missing or incomplete data).** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
4. **Using appropriate visualizations (i.e. double line plot), analyze data across samples.** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.

{end new}

##### B. Represent and interpret data

{5.MD.B.2. was changed to 5.DL.B.5}

[2] **5**. Make a line plot to display a data set of measurements in fractions of a unit (½, ¼, ⅛). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

#### Geometry 5.G

##### A. Graph points on the coordinate plane to solve real-world and mathematical problems

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).
2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation. {begin new} **(Students may represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}

##### B. Classify two-dimensional figures into categories based on their properties

1. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.
2. Classify two-dimensional figures in a hierarchy based on properties.

### Grade 6

#### Ratios and Proportional Relationships 6.RP

##### A. Understand ratio concepts and use ratio reasoning to solve problems

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”
2. Understand the concept of a unit rate *a*/*b* associated with a ratio *a*:*b* with *b* ≠0, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾up of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.” {begin new} **(Clarification: Expectations for unit rates in this grade are limited to non-complex fractions.}** {end new}
3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
   1. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
   2. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
   3. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means times the quantity); solve problems involving finding the whole, given a part and the percent.
   4. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

#### The Number System 6.NS

##### A. Apply and extend previous understandings of multiplication and division to divide fractions by fractions

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context forand use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that because of is . (In general, How much chocolate will each person get if 3 people share ½ lb of chocolate equally? How many ¾-cup servings are in ⅔ of a cup of yogurt? How wide is a rectangular strip of land with length ¾ mi and area ½ square mi?

##### B. Compute fluently with multi-digit numbers & find common factors & multiples

1. {begin deletion} [Fluently] {end deletion} {begin new} **With accuracy and efficiency,** {begin new}divide multi-digit numbers using the standard algorithm.
2. {begin deletion} [Fluently] {end deletion} {begin new} **With accuracy and efficiency,** {begin new}add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
3. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as .

##### C. Apply and extend previous understandings of numbers to the system of rational numbers

1. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
2. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
   1. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., −(−3) = 3, and that 0 is its own opposite.
   2. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
   3. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
3. Understand ordering and absolute value of rational numbers.
   1. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret −3 >−7 as a statement that   
      −3 is located to the right of −7 on a number line oriented from left to right.
   2. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write −3°C > −7°C to express the fact that−3°C is warmer than −7° C.
   3. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of −30 dollars, write |–30| = 30 to describe the size of the debt in dollars.
   4. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than −30 dollars represents a debt greater than 30 dollars.
4. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate

#### Expressions and Equations 6.EE

##### A. Apply and extend previous understandings of arithmetic to algebraic expressions

1. Write and evaluate numerical expressions involving whole-number exponents.
2. Write, read, and evaluate expressions in which letters stand for numbers.
   1. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract *y* from 5” as 5 − *y*.
   2. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.
   3. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas and to find the volume and surface area of a cube with sides of length s = ½.
3. Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression to produce the equivalent expression   
   ; apply the distributive property to the expression to produce the equivalent expression ; apply properties of operations to to produce the equivalent expression .
4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions and are equivalent because they name the same number regardless of which number *y* stands for

##### B. Reason about and solve one-variable equations and inequalities

1. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
2. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
3. Solve real-world and mathematical problems by writing and solving equations of the form   
   and for cases in which *p*, *q* and *x* are all nonnegative rational numbers. {begin new} **(Students may reason and solve one-variable equations and inequalities. Sample question, if the temperature at sea level is 20oC, what is the temperature at 100 m above sea level?) (Clarification Statements: As altitude increases, temperature decreases. With every 100 meters, the temperature drops by an average of 1oC.** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}
4. Write an inequality of the form *x* > *c* or *x* < *c* to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form *x* > *c* or *x* < *c* have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

##### C. Represent and analyze quantitative relationships between dependent and independent variables

1. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation to represent the relationship between distance and time. {begin new} **(Students analyze climate change computational models and propose refinements. These models would require students analyze the relationship between the dependent and independent variables using graphs and tables and relate these to the equation.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}

#### Geometry 6.G

##### A. Solve real-world and mathematical problems involving area, surface area, and volume

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas and to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

#### Statistics and Probability 6.SP

##### A. Develop understanding of statistical variability

1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.
2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

##### B. Summarize and describe distributions

1. Display numerical data in plots on a number line, including dot plots, histograms, and box plots. {begin new} **(Students may develop and use a model to describe how unequal heating and rotation of the Earth cause patterns of atmospheric and oceanic circulation that determine regional climates students may display numerical data in plots on a number line, including dot plots, histograms, and box plots.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}
2. Summarize numerical data sets in relation to their context, such as by:
   1. Reporting the number of observations. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
   2. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
   3. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
   4. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.

### Grade 7

#### Ratios and Proportional Relationships 7.RP

##### A. Analyze proportional relationships and use them to solve real-world and mathematical problems

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction (½)/(¼) miles per hour, equivalently 2 miles per hour.
2. Recognize and represent proportional relationships between quantities.
   1. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
   2. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
   3. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as
   4. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.
3. Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

#### The Number System 7.NS

##### A. Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   1. Describe situations in which opposite quantities combine to make 0. For example, in the first round of a game, Maria scored 20 points. In the second round of the same game, she lost 20 points. What is her score at the end of the second round?
   2. Understand *p* + *q* as the number located a distance |*q*| from *p*, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
   3. Understand subtraction of rational numbers as adding the additive inverse, . Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
   4. Apply properties of operations as strategies to add and subtract rational numbers.
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
   1. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (−1)(−1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
   2. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then . Interpret quotients of rational numbers by describing real world contexts.
   3. Apply properties of operations as strategies to multiply and divide rational numbers.
   4. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
3. Solve real-world and mathematical problems involving the four operations with rational numbers. {begin new} (**Clarification: Computations with rational numbers extend the rules for manipulating fractions to complex fractions.) (Students may solve mathematical problems based on quantitative data related to the five main contributors to climate change:**

* **Burning coal, oil and gas produces carbon dioxide and nitrous oxide**
* **Cutting down forests (deforestation)**
* **Increasing livestock farming**
* **Fertilizers containing nitrogen produce nitrous oxide emissions, and**
* **Fluorinated gases are emitted from equipment and products that use these gases.** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}

#### Expressions and Equations 7.EE

##### A. Use properties of operations to generate equivalent expressions

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example,   
   means that “increase by 5%” is the same as “multiply by 1.05.”

##### B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations

1. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional ⅒ of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 ¾ inches long in the center of a door that is 27 ½ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation. {begin new} **(Students may solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form, based on quantitative data related to the five main contributors to climate change:** 
   * **Burning coal, oil and gas produces carbon dioxide and nitrous oxide**
   * **Cutting down forests (deforestation)**
   * **Increasing livestock farming**
   * **Fertilizers containing nitrogen produce nitrous oxide emissions, and**
   * **Fluorinated gases are emitted from equipment and products that use these gases.) {end new} highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.** {end new}
2. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
   1. Solve word problems leading to equations of the form and , where *p*, *q*, and *r* are specific rational numbers. Solve equations of these forms {begin deletion} [fluently] {end deletion} {begin new} **with accuracy and efficiency**{begin new}. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?
   2. Solve word problems leading to inequalities of the form or , where *p*, *q*, and *r* are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.

#### Geometry 7.G

##### A. Draw, construct, and describe geometrical figures and describe the relationships between them

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
2. Draw (with technology, with ruler and protractor, as well as freehand) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

##### B. Solve real-life and mathematical problems involving angle measure, area, surface area, and volume

1. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
2. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
3. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. {begin new} **(Students may solve multi-step real-life and mathematical problems involving angle measure, area, surface area, and volume quantitative data related to the five main contributors to climate change**:
   * **Burning coal, oil and gas produces carbon dioxide and nitrous oxide**
   * **Cutting down forests (deforestation)**
   * **Increasing livestock farming**
   * **Fertilizers containing nitrogen produce nitrous oxide emissions, and**
   * **Fluorinated gases are emitted from equipment and products that use these gases.).** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}

#### Statistics and Probability 7.SP

##### A. Use random sampling to draw inferences about a population

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.

##### B. Draw informal comparative inferences about two populations

1. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
2. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.

##### C. Investigate chance processes and develop, use, & evaluate probability models

1. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around ½ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
2. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
3. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
   1. Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
   2. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
4. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
   1. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
   2. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
   3. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

### Grade 8

#### The Number System 8.NS

##### A. Know that there are numbers that are not rational and approximate them by rational numbers

{8.NA.A.3. is new}

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number.
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., ). For example, by truncating the decimal expansion of Ö2, show that Ö2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

{begin new to grade 8}

**3. Understand that the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.**

{end new}

#### Expressions and Equations 8.EE

##### A. Work with radicals and integer exponents

{Part of 8.EE.A.2 was moved to 8.EE.A.2.a. 8.EE.A.2.b. is new}

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, .
2. Use square root and cube root symbols to represent solutions to equations of the form and where *p* is a positive rational number.
   1. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that Ö2 is irrational.
   2. {begin new} **Simplify numerical radicals, limiting to square roots (i.e. nonperfect squares). For example, simplify to .** {end new}
3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as and the population of the world as, and determine that the world population is more than 20 times larger.
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

##### B. Understand the connections between proportional relationships, lines, and linear equations

1. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
2. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation for a line through the origin and the equation for a line intercepting the vertical axis at *b*.

##### C. Analyze and solve linear equations and pairs of simultaneous linear equations

1. Solve linear equations in one variable.
   1. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form *x* = *a*, *a* = *a*, or *a* = *b* results (where *a* and *b* are different numbers).
   2. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.
2. Analyze and solve pairs of simultaneous linear equations.
3. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
4. Solve systems of two linear equations in two variables {begin deletion} [algebraically,] {end deletion} {begin new} **using the substitution method** {end new} and estimate solutions by graphing the equations. Solve simple cases by inspection. For example: {begin new} by inspection, conclude that {end new} and have no solution because cannot simultaneously be 5 and 6. {begin new} Solve and using the substitution method; Solve and using the substitution method. {end new}
5. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

#### Functions 8.F

##### A. Define, evaluate and compare functions

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.   
   {begin new} **(Clarification: Function notation is not required in Grade 8)** {end new}
2. Compare properties (e.g. rate of change, intercepts, domain and range) of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
3. Interpret the equation as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

##### B. Use functions to model relationships between quantities

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

#### Geometry 8.G

##### A. Understand congruence and similarity using physical models, transparencies, or geometry software

1. Verify experimentally the properties of rotations, reflections, and translations:
   1. Lines are transformed to lines, and line segments to line segments of the same length.
   2. Angles are transformed to angles of the same measure.
   3. Parallel lines are transformed to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

##### B. Understand and apply the Pythagorean Theorem

1. Explain a proof of the Pythagorean Theorem and its converse.
2. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
3. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

##### C. Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres

1. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. {begin new} **(Students may solve real-world mathematical problems involving the physical properties of the principle gasses that cause climate change molecules.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}

#### Statistics and Probability 8.SP

##### A. Investigate patterns of association in bivariate data

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. {begin new} **(Students may Investigate patterns of association in bivariate data involving the amount of a greenhouse gas in the atmosphere and its effect on temperature.)**highlighted as an opportunity for inclusion in an interdisciplinary climate change unit. {end new}
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit (e.g. line of best fit) by judging the closeness of the data points to the line. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height. {begin new} **(Students may use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept involving the physical properties of the principle gasses that cause climate change.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}
4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.?

### High School—Number and Quantity

#### The Real Number System N.RN

##### A. Extend the properties of exponents to rational exponents

{N.RN.A.3 is new.}

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define to be the cube root of 5 because we want to hold, somust equal 5.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.
3. {begin new} **Simplify radicals, including algebraic radicals (e.g. , simplify ).** {end new}

{begin deletion}

##### [B. Use properties of rational and irrational numbers

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.]

{end deletion}

#### Quantitiesmodeling standard. N.Q

##### A. Reason quantitatively and use units to solve problems

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. {begin new} **(Students may reason quantitatively and use units to explain how variations in the flow of energy into and out of the Earth’s systems result in climate change. (Clarification Statement: changes in climate is limited to changes in surface temperatures, precipitation patterns, glacial ice volumes, sea levels, and biosphere distribution.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}
2. Define appropriate quantities for the purpose of descriptive modeling. {begin new} **(Students may reason quantitatively to define a descriptive model of how variations in the flow of energy into and out of Earth’s systems result in climate change. (Clarification Statement: changes in climate is limited to changes in surface temperatures, precipitation patterns, glacial ice volumes, sea levels, and biosphere distribution.).** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. {begin new} **(Students may reason quantitatively and use units to explain how variations in the flow of energy into and out of the Earth’s systems result in climate change.** )highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}

#### The Complex Number System N.CN

##### A. Perform arithmetic operations with complex numbers

1. Know there is a complex number *i* such that , and every complex number has the form   
    with *a* and *b* real.
2. Use the relation and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. () Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

##### B. Represent complex numbers and their operations on the complex plane

1. () Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
2. () Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, because has modulus 2 and argument 120°.
3. () Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

##### C. Use complex numbers in polynomial identities and equations

1. Solve quadratic equations with real coefficients that have complex solutions.
2. () Extend polynomial identities to the complex numbers. For example, rewrite as .
3. () Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

#### Vector and Matrix Quantities N.VM

##### A. Represent and model with vector quantities

1. () Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ).
2. () Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
3. () Solve problems involving velocity and other quantities that can be represented by vectors.

##### B. Perform operations on vectors

{The v's and w's because they are matrices; they are not new.}

1. () Add and subtract vectors.
   1. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
   2. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
   3. Understand vector subtraction as, where is the additive inverse of *w*, with the same magnitude as *w* and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
2. () Multiply a vector by a scalar.
   1. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as .
   2. Compute the magnitude of a scalar multiple using . Compute the direction ofknowing that when |*c* |***v*** ≠0, the direction of *c****v***is either along ***v***(for *c* > 0) or against ***v***(for *c* < 0).

##### C. Perform operations on matrices and use matrices in applications

1. () Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
2. () Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
3. () Add, subtract, and multiply matrices of appropriate dimensions.
4. () Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
5. () Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
6. () Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
7. () Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.

### High School—Algebra

#### Arithmetic with Polynomials and Rational Expressions A.APR

##### A. Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials

##### B. Understand the relationship between zeros and factors of polynomials

1. Know and apply the Remainder Theorem: For a polynomial and a number *a*, the remainder on division by is , so if and only if is a factor of .
2. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

##### C. Use polynomial identities to solve problems

{A.APR.C.4 was changed to a plus ()) standard.}

1. **()** Prove polynomial identities and use them to describe numerical relationships. For example, the difference of two squares; the sum and difference of two cubes; the polynomial identity can be used to generate Pythagorean triples.
2. () Know and apply the Binomial Theorem for the expansion of in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal’s Triangle. {begin new} **(Clarification: The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.)** {end new}

##### D. Rewrite rational expressions

1. Rewrite simple rational expressions in different forms; write in the form , where , and are polynomials with the degree of less than the degree of , using inspection, long division, or, for the more complicated examples, a computer algebra system.
2. () Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions

#### Creating Equationsmodeling standard. A.CED

##### A. Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. {begin new} **(Students may examine the economic impact of climate change and investigate the claim that the economic impact of climate change modeled by a quadratic function rather than a linear function. A new paper,** [**Few and Not So Far Between: A Meta-analysis of Climate Damage Estimates**](https://link.springer.com/article/10.1007%2Fs10640-017-0166-z) **is a good source for background information.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. {begin new} **(Students may examine the economic impact of climate change and represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.)**highlighted as an opportunity for inclusion in an interdisciplinary climate change unit. {end new}
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law *V* = *IR* to highlight resistance *R*. {begin new} **(Students may examine the economic impact of climate change and rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}

#### Reasoning with Equations and Inequalities A.REI

##### A. Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

##### B. Solve equations and inequalities in one variable

1. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
2. Solve quadratic equations in one variable.
   1. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form that has the same solutions. Derive the quadratic formula from this form.
   2. Solve quadratic equations by inspection (e.g., for ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as for real numbers and.

##### C. Solve systems of equations

{A.REI.C.5 was changed to a plus standard.}

1. **()** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
2. Solve systems of linear equations {begin deletion} [exactly and approximately (e.g., with graphs)]{end deletion} {begin new} **algebraically (include using the elimination method) and graphically** {end new}, focusing on pairs of linear equations in two variables.
3. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line   
    and the circle .
4. () Represent a system of linear equations as a single matrix equation in a vector variable.
5. () Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).

##### D. Represent and solve equations and inequalities graphically

1. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
2. Explain why the *x*-coordinates of the points where the graphs of the equations and intersect are the solutions of the equation; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. 
3. Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

#### Seeing Structure in Expressions A.SSE

##### A. Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context. 
   1. Interpret parts of an expression, such as terms, factors, and coefficients.
   2. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret as the product of *P* and a factor not depending on *P*.
2. Use the structure of an expression to identify ways to rewrite it. For example, see   
   as , thus recognizing it as a difference of squares that can be factored as   
   .

##### B. Write expressions in equivalent forms to solve problems

{A.SSE.B.4 was changed to a plus standard.}

1. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. 
   1. Factor a quadratic expression to reveal the zeros of the function it defines.
   2. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
   3. Use the properties of exponents to transform expressions for exponential functions. For example, the expression can be rewritten as to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
2. **()** Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments*.* 

### High School—Functions

#### Building Functions F.BF

##### A. Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities. 
   1. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   2. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
   3. () Compose functions. For example, if is the temperature in the atmosphere as a function of height, and is the height of a weather balloon as a function of time, then is the temperature at the location of the weather balloon as a function of time.
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. 

##### B. Build new functions from existing functions

1. Identify the effect on the graph of replacing *f*(*x*) by, , , andfor specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
2. Find inverse functions.
   1. Solve an equation of the form for a simple function *f* that has an inverse and write an expression for the inverse. For example, or for .
   2. () Verify by composition that one function is the inverse of another.
   3. () Read values of an inverse function from a graph or a table, given that the function has an inverse.
   4. () Produce an invertible function from a non-invertible function by restricting the domain.
3. () Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.

#### Interpreting Functions F.IF

##### A. Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation .
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. {begin new} **(Students can use the function to determine the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline), *x*, where *f*(*x*) is the number of molecules of carbon dioxide.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.{end new}
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by , for .

##### B. Interpret functions that arise in applications in terms of the context

1. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. 
2. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. {begin new} **(Students may graph the functionto represent the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline), in which case the positive integers would be an appropriate domain for the function.)** {end new}highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
3. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. {begin new} **(Students may use the graph of the function , which represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline), x, students will calculate and interpret the rate of change *f*(*x*).** {end new} highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.

##### C. Analyze functions using different representations

{The second half of HS.F.IF.7e was moved to f. HS.IF.7.f was changed to a plus standard.}

1. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. 
   1. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   2. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   3. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
   4. () Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
   5. Graph exponential and logarithmic functions, showing intercepts and end behavior {begin deletion}[, and] {end deletion}**.**
   6. {begin new} **()Graph** {end new} trigonometric functions, showing period, midline, and amplitude.
2. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
3. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
4. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as , ,  
    , , and classify them as representing exponential growth or decay.
5. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

#### Linear and Exponential Models F.LE

##### A. Construct and compare linear and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
   1. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
   2. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
   3. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.

##### B. Interpret expressions for functions in terms of the situation they model

1. Interpret the parameters in a linear or exponential function in terms of a context. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.

#### Trigonometric Functions F.TF

##### A. Extend the domain of trigonometric functions using the unit circle

{HS.F.TF.A.1. and HS.F.TF.A.2. were changed to plus standards.}

1. **()** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. **()**Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. () Use special triangles to determine geometrically the values of sine, cosine, tangent for , , and , and use the unit circle to express the values of sine, cosines, and tangent for , , and in terms of their values for *x*, where *x* is any real number.
4. () Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

##### B. Model periodic phenomena with trigonometric functions

{HS.F.TF.B.5. was changed to a plus standard.}

1. **()** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. 
2. ()Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
3. () Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. 

##### C. Prove and apply trigonometric identities

{HS.F.TF.C.8. was changed to a plus standard.}

1. **()** Prove the Pythagorean identity and use it to find , or given , , or and the quadrant of the angle.
2. () Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

### High School—Geometry

#### Circles G.C

##### A. Understand and apply theorems about circles

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
4. () Construct a tangent line from a point outside a given circle to the circle.

##### B. Find arc lengths and areas of sectors of circles

1. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

#### Congruence G.CO

##### A. Experiment with transformations in the plane

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

##### B. Understand Congruence in terms of rigid motions

1. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
2. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
3. Explain how the criteria for triangle congruence.

##### C. Prove geometric theorems

1. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.
2. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
3. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

##### D. Make geometric constructions

1. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
2. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

#### Geometric Measurement and Dimension G.GMD

##### A. Explain volume formulas and use them to solve problems

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri’s principle, and informal limit arguments.
2. () Give an informal argument using Cavalieri’s principle for the formulas for the volume of a sphere and other solid figures.
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.

##### B. Visualize relationships between two-dimensional and three-dimensional objects

1. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

#### Expressing Geometric Properties with Equations G.GPE

##### A. Translate between the geometric description and the equation for a conic section

{HS.G.GPE.A.1. and HS.G.GPE.A.2. were changed to plus standards.}

1. **()** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2. **()** Derive the equation of a parabola given a focus and directrix.
3. () Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

##### B. Use coordinates to prove simple geometric theorems algebraically

{HS.G.GPE.B.6. was changed to a plus standard.}

1. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).
2. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
3. **()** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
4. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.

#### Modeling with Geometry★ G.MG

##### A. Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). {begin new} **(Students may use geometric shapes, their measures, and their properties to describe a technological design solution that reduces the impact of human activities on climate change and other natural systems.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit. {end new}
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). {begin new} **(Students may apply concepts of density to analyze how well a technical solution is in reducing the amount of carbon in the atmosphere.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit. {end new}
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). {begin new} **(Students may apply geometric methods to analyze how well a technical solution is in reducing the amount of carbon in the atmosphere.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit. {end new}

#### Similarity, Right Triangles, and Trigonometry G.SRT

##### A. Understand similarity in terms of similarity transformations

1. Verify experimentally the properties of dilations given by a center and a scale factor:
   1. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
   2. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

##### B. Prove theorems involving similarity

1. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
2. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

##### C. Define trigonometric ratios and solve problems involving right triangles

1. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
2. Explain and use the relationship between the sine and cosine of complementary angles.
3. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. 

##### D. Apply trigonometry to general triangles

1. () Derive the formula for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
2. () Prove the Laws of Sines and Cosines and use them to solve problems.
3. () Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

### High School—Statistics and Probability

#### Interpreting Categorical and Quantitative Data S.ID

##### A. Summarize, represent, and interpret data on a single count or measurement variable

1. Represent data with plots on the real number line (dot plots, histograms, and box plots). {begin new} **(Students my represent data, with plots on the real number line, as they analyze geoscience data and the results from global climate models to make an evidence-based-forecast of the current rate of global climate change.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit. {end new}
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

##### B. Summarize, represent, and interpret data on two categorical and quantitative variables

1. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
2. Represent data on two quantitative variables on a scatter plot and describe how the variables are related. {begin new} **(Students may represent data, with plots on the real number line, as they analyze geoscience data, and the results from global climate modeling, to make an evidence-based-forecast of the current rate of global climate change.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit. {end new}
   1. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
   2. Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology.
   3. Fit a linear function for a scatter plot that suggests a linear association.

##### C. Interpret linear models

1. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. {begin new} **(Students may represent data on two quantitative variables on a scatter plot and describe how the variables are related as they analyze geoscience data and the results from global climate models to make an evidence-based-forecast of the current rate of global climate change.)** highlighted as an opportunity for inclusion in an interdisciplinary climate change unit. {end new}
2. Compute (using technology) and interpret the correlation coefficient of a linear fit. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.
3. Distinguish between correlation and causation.highlighted as an opportunity for inclusion in an interdisciplinary climate change unit.

#### Making Inferences and Justifying Conclusions S.IC

##### A. Understand and evaluate random processes underlying statistical experiments

{All of the HS.S.IC.A. standards were changed to plus standards.}

1. **()** Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2. **()** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

##### B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies

{All of the HS.S.IC.B. standards were changed to plus standards.}

1. **()** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
2. **()** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
3. **()** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
4. **()** Evaluate reports based on data {begin new} **(e.g. interrogate study design, data sources, randomization, the way the data are analyzed and displayed, inferences drawn and methods used; identify and explain misleading uses of data; recognize when arguments based on data are flawed)**. highlighted as an opportunity for inclusion in an interdisciplinary climate change unit. {end new}

#### Conditional Probability and the Rules of Probability S.CP

##### A. Understand independence and conditional probability and use them to interpret data

{All of the HS.S.CP.A. standards were changed to plus standards.}

1. **()** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
2. **()** Understand that two events *A* and *B* are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. **()** Understand the conditional probability of *A* given *B* as , and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
4. **()** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
5. **()** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

##### B. Use the rules of probability to compute probabilities of compound events in a uniform probability model

{HS.S.CP.B.6. and HS.S.CP.B.7 were changed to plus standards.}

1. **()** Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model.
2. **()** Apply the Addition Rule, , and interpret the answer in terms of the model.
3. () Apply the general Multiplication Rule in a uniform probability model, , and interpret the answer in terms of the model.
4. () Use permutations and combinations to compute probabilities of compound events and solve problems.

#### Using Probability to Make Decisions S.MD

##### A. Calculate expected values and use them to solve problems

1. () Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2. () Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
3. () Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.
4. () Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?

##### B. (Use probability to evaluate outcomes of decisions

1. () Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
   1. () Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast food restaurant.
   2. () Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.
2. () Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
3. () Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

1. See Glossary, Table 1 [↑](#footnote-ref-1)
2. See Glossary, Table 1 [↑](#footnote-ref-2)
3. See standard 1.OA.C.6 for a list of mental strategies [↑](#footnote-ref-3)
4. See Glossary, Table 1 [↑](#footnote-ref-4)
5. See Glossary, Table 2 [↑](#footnote-ref-5)
6. Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8. [↑](#footnote-ref-6)
7. See Glossary, Table 2. [↑](#footnote-ref-7)
8. Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. [↑](#footnote-ref-8)
9. Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12 and 100. [↑](#footnote-ref-9)