



NEW JERSEY STUDENT LEARNING STANDARDS

Mathematics

Algebra 1:

Instructional Support Tool for Teachers: **Algebra**

This instructional support tool is designed to assist educators in interpreting and implementing the New Jersey Student Learning Standards for Mathematics. It contains explanations or examples of each Algebra 1 course standard to answer questions about the standard's meaning and how it applies to student knowledge and performance. To ensure that descriptions are helpful and meaningful to teachers, this document also identifies grades 6 to 8 prerequisite standards upon which each Algebra 1 standard builds. It includes the following: sample items aligned to each identified prerequisite; identification of course level connections; instructional considerations and common misconceptions; sample academic vocabulary and associated standards for mathematical practice.

Examples are samples only and should not be considered an exhaustive list.

This instructional support tool is considered a living document. The New Jersey Department of Education believe that teachers and other educators will find ways to improve the document as they use it. Please send feedback to mathematics@doe.state.nj.us so that the Department may use your input when updating this guide.

Please consult the [New Jersey Student Learning Standards for Mathematics](#) for more information.

Algebra Standards Overview

Broadly, Algebra, as a branch of mathematics, reflects the use of mathematical statements to describe relationships between quantities that vary. Notions of expressions and equations are foundational to the study of Algebra. Critical understandings include recognizing equations as statements of equality between two expressions and recognizing expressions as records reflecting computation with numbers and symbols that represent numbers.

ARITHMETIC WITH POLYNOMIALS AND RATIONAL FUNCTIONS (A-APR)

Perform arithmetic operations on polynomials: [A-APR.A.1](#)

Understand the relationship between zeros and factors of polynomials: [A-APR.B.3](#)

CREATING EQUATIONS (A-CED)

Create equations that describe numbers or relationships: [A-CED.A.1](#) [A-CED.A.2](#) [A-CED.A.3](#) [A-CED.A.4](#)

REASONING WITH EQUATIONS AND INEQUALITIES (A-REI)

Understand solving equations as a process of reasoning and explain the reasoning: [A-REI.A.1](#)

Solve equations and inequalities in one variable: [A-REI.B.3](#) [A-REI.B.4](#)

Solve systems of equations: [A-REI.C.5](#) [A-REI.C.6](#)

Represent and solve equations and inequalities graphically: [A-REI.D.10](#) [A-REI.D.11](#) [A-REI.D.12](#)

SEEING STRUCTURE IN EXPRESSIONS (A-SSE)

Interpret the structure of expressions: [A-SSE.A.1](#) [A-SSE.A.2](#)

Write expressions in equivalent forms to solve problems: [A-SSE.B.3a-b](#) [A-SSE.B.3c](#)

Mathematical Practices

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Algebra 1 Instructional Support Tool - Algebra

Standard A-APR.A.1

Understand that polynomials form a system **analogous to the integers**, namely, they are closed under the operations of addition, subtraction, and multiplication; **add, subtract, and multiply** polynomials. *(Content Emphases: Major)*

Explanations and/or Examples

Example 1: Simplify each of the following:

- $(3x - 4y)(5x - 2y)$
- $(x^2 + 5x - 9) + 2(4x - 3)$
- $(4x - 5)(2x^2 + 3x - 6)$
- $4b^3(9b^2 - 3b - 6)$
- $(4x^4 + 3) - (2x^4 + x^3 - 1)$

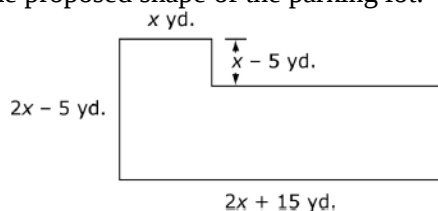
Ex. 1 Solution: a. $15x^2 - 26xy + 8y^2$ b. $9x^2 - x - 9$
c. $8x^3 + 2x^2 - 39x + 30$ d. $36b^5 - 12b^4 - 24b^3$ e. $2x^4 - x^3 + 4$

Example 2: Write the expression below as a constant times a power of x. Use your answer to decide whether the expression gets larger or smaller as the value of x increases

$$\frac{(2x^3)^2(3x^4)}{(x^2)^3}$$

Ex. 2 Solution: $12x^4$; The value of the expression gets larger as the value of x increases.

Example 3: A town council plans to build a public parking lot. The outline below represents the proposed shape of the parking lot.



Write an expression for the area, in square yards, of this proposed parking lot. Explain the reasoning you used to find the expression.

Ex. 3 Solution: The missing vertical dimension is
 $2x - 5 - (x - 5) = 2x - x - 5 + 5 = x$

$$\begin{aligned} \text{Area} &= x(x - 5) + x(2x + 15) \\ &= x^2 - 5x + 2x^2 + 15x \\ &= 3x^2 + 10x \text{ square yards} \end{aligned}$$

Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

8.EE.A.1

Know and apply the properties of integer exponents to **generate equivalent numerical expressions**. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$

Grade 7

7.EE.A.1

Apply properties of operations as strategies to **add, subtract, factor, and expand linear expressions** with rational coefficients

Grade 6

6.EE.A.4

Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.

Prerequisite Examples:

Grade 8

8.EE.A.1

Simplify: $\frac{(3^2)^4}{(3^2)(3^3)}$

Grade 7

7.EE.A.1

Factor: $-5m - 10$

Grade 6

6.EE.A.4

Determine whether the following expressions are equivalent. Explain your answer.

$$4(m + 2) \qquad 3m + 8 + m \qquad 2 + 2m + m + 6 + m$$

Algebra 1 Instructional Support Tool - Algebra

Standard A-APR.A.1

Understand that polynomials form a system **analogous to the integers**, namely, they are closed under the operations of addition, subtraction, and multiplication; **add, subtract, and multiply** polynomials. *(Content Emphases: Major)*

Grade/Course Level Connections:

n/a

Instructional Considerations:

- Avoid misleading mnemonics, such as FOIL, which is relevant only when multiplying two binomials. Reinforce use of the distributive property.
- Make connections between arithmetic of integers and arithmetic of polynomials. In order to understand this standard, students need to work toward both understanding and fluency with polynomial arithmetic.
 - For example, compare 213×47 with the product of $2b^2 + 1b + 3$ and $4b + 7$

$$\begin{array}{r} 2b^2 + 1b + 3 \\ \times \quad 4b + 7 \\ \hline \end{array}$$

$$\begin{array}{r} 14b^2 + 7b + 21 \\ 8b^3 + 4b^2 + 12b \\ \hline \end{array}$$

$$8b^3 + 18b^2 + 19b + 21$$

$$\begin{array}{r} 213 \\ \times \quad 47 \\ \hline \end{array}$$

$$\begin{array}{r} 1491 \\ 8520 \\ \hline \end{array}$$

$$10011$$

- Stress correct vocabulary, such as integer, monomial, polynomial, factor, and term.
- Emphasize the concept of closure. In order to understand that polynomials are closed under addition, subtraction and multiplication, students can compare these ideas with the analogous claims for integers: The sum, difference or product of any two integers is an integer, but the quotient of two integers is not always an integer.
- Emphasize how the distributive property is used. This is important in polynomial multiplication.

Common Misconceptions:

- Students may believe the following about the distributive property:
 - $3(2x^2 - 5) = 6x^2 - 5$ *rather than* $6x^2 - 15$ (the second term is ignored)
 - $2(r - 5)^2 = (2r - 10)^2$ *rather than* $2r^2 - 20r + 50$
- Students should be discouraged from using the term FOIL and instead the emphasis should be on the distributive property.
- Students may believe that the FOIL method is the tool that can be used to multiply all polynomials.
- Students may believe that the degree of the variable changes when adding or subtracting like terms. For example, $6x + 2x = 8x^2$ rather than $8x$.
- Students may believe that in multiplying polynomials, only like terms are multiplied together. For example, $(s + 5)(s - 6) = s^2 - 30$ rather than $s^2 - s - 30$.
- Students may believe that there is no connection between the arithmetic of integers and the arithmetic of polynomials.

Academic Vocabulary

monomial, binomial, trinomial, polynomial, closure, exponent, term, properties of integer exponents, equivalent forms

Standards for Mathematical Practice:

- (2) Reason abstractly and quantitatively
- (7) Look for and make use of structure

Algebra 1 Instructional Support Tool - Algebra

Standard A-APR.B.3

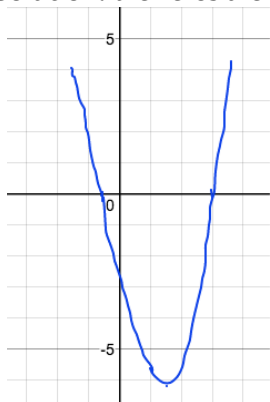
Identify zeros of polynomials **when suitable factorizations are available**, and **use the zeros to construct** a rough graph of the function defined by the polynomial. *[Algebra 1: limit to quadratic and cubic functions in which linear and quadratic factors are available] *(Content Emphases: Supporting)*

Explanations and/or Examples:

This standard calls for a sketch of the graph after zeros are identified. Sketching indicates that the graph should be done by hand rather than generated by a graphing calculator.

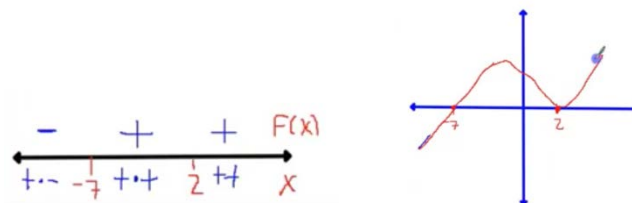
Example 1: Given the function $f(x) = 2x^2 - 5x - 3$, list the zeroes of the function and sketch the graph

Ex. 1 Solution: the zeros are $x = -1/2$ and $x = 3$



Example 2: Sketch the graph of the function $g(x) = (x - 2)^2(x + 7)$

Ex 2. Solution: zeros are $x = 2$ and $x = -7$



Building Blocks (Grades 6 to 8 prerequisites):

Grade 7

7.EE.A.1

Apply properties of operations as strategies to add, subtract, **factor**, and expand **linear expressions** with rational coefficients

Prerequisite Examples:

Grade 7

7.EE.A.1

Write an equivalent expression for: $-3a + 12$

Grade/Course Level Connections:

A-SSE.A.2 and A.SSE.B.3a

Algebra 1 Instructional Support Tool - Algebra

Standard A-APR.B.3

Identify zeros of polynomials **when suitable factorizations are available**, and **use the zeros to construct** a rough graph of the function defined by the polynomial. *[Algebra 1: limit to quadratic and cubic functions in which linear and quadratic factors are available] (*Content Emphases: Supporting*)

Instructional Considerations:

- Students sketch a graph, as opposed to plotting the graph, to develop ideas about the shape of the graph without plotting the points.
- A general exploration of polynomial (2nd and 3rd degree) functions – graphically, numerically, symbolically, and verbally – is important.
- Include identifying the multiplicity of the zeros of factored polynomials and examining how the multiplicity of zeros provides a clue as to how the graph will behave when it approaches and leaves the x-intercept.
- By using technology to explore the graphs of many polynomial functions, and describing the shape, end behavior, and number of zeros, students can begin to make the following informal observations:
 - Graphs of polynomial functions are continuous
 - An n th degree polynomial has at most n roots and at most $n-1$ “changes of direction” (i.e. from increasing to decreasing or vice versa)
 - Quadratics (an even-degree polynomial) have the same end-behavior in both the positive and negative directions: both heading to positive infinity, or both heading to negative infinity, depending on the sign of the leading coefficient
- Cubics (an odd-degree polynomial) have opposite end-behavior in the positive versus the negative directions, depending upon the sign of the leading coefficient.

Common Misconceptions:

- Students may believe that they cannot test points when the equation is in factored form
- Students may confuse areas of the graph where the value of the function is negative with areas of the graph where the function is decreasing

Academic Vocabulary

zeros, polynomial, quadratic function, cubic function, degree (of a polynomial), leading coefficient, roots, x-intercept, point of inflection

Standards for Mathematical Practice:

- (2) Reason abstractly and quantitatively
- (5) Use appropriate tools strategically

Algebra 1 Instructional Support Tool - Algebra

Standard A-CED.A.1

Create equations and inequalities in one variable and **use them to solve problems**. Include equations arising from linear functions and quadratic functions, and simple rational and exponential functions. *(Content Emphases: Major)*

Explanations and/or Examples

Example 1: Chase and his brother like to play basketball. About a month ago they decided to keep track of how many games they have each won. As of today, Chase has won 18 out of the 30 games against his brother.

- How many games would Chase have to win in a row in order to have a 75% winning record?
- How many games would Chase have to win in a row in order to have a 90% winning record?
- Is Chase able to reach a 100% winning record? Explain.
- Suppose that after reaching a winning record of 90% in part (b), Chase had a losing streak. How many games in a row would Chase have to lose in order to drop down to a winning record below 55% again?

Ex. 1 Solution:

[Task 702 by IM](#) by [CC BY-NC-SA 4.0](#)

Create **exponential** equations in one variable.

Example 2: In a professional tennis tournament, the money a player wins depends on their finishing place in the standings. The first place finisher wins half of \$1,500,000 in total prize money. The second place finisher wins half of what is left; then the third place finisher wins half of that, and so on. Write an equation to calculate the actual prize money (P), in dollars, won by the player finishing in 10^{th} place.

Ex. 2 Solution: $P = 1500000(.50)^{10}$

Create **quadratic** equations in one variable and use them to solve problems.

Example 3: Cheerleaders are launching t-shirts into the stands at a football game. They are launching them from a height of 3 feet off the ground and an initial velocity of 36 feet per second. Write an equation that could be used to determine the next time (since launching) that a t-shirt is 3 feet from the ground. (Use $16t^2$ for the effect of gravity on the height of the t-shirt)

Ex. 3 Solution: $16t^2 + 36t + 3 = 3$, or other equivalent form of this equation.

Create quadratic equations in one variable **from quadratic functions**.

Example 4: An object is launched at 14.7 meters per second (m/s) from a 49-meter tall platform. The equation for the object's height s at time t seconds after launch is $s(t) = -4.9t^2 + 14.7t + 49$, where s is in meters. At what time does the object strike the ground?

Ex. 4 Solution:

$$\begin{aligned} 0 &= -4.9t^2 + 14.7t + 49 \\ 0 &= t^2 - 3t - 10 \\ 0 &= (t + 2)(t - 5) \end{aligned}$$

The solutions for t are $t = 5$ seconds and $t = -2$ seconds. $t = -2$ seconds does not make sense in the context of the problem, therefore the object strikes the ground 5 seconds after it was launched.

Example 5: At $t=0$, a car driving on a straight road at a constant speed passes a telephone pole. From then on, the car's distance from the telephone pole is given by $C(t) = 30t$, where t is in seconds and C is in meters.

Also at $t=0$, a motorcycle pulls out onto the road, driving in the same direction. Initially the motorcycle is 90 m ahead of the car. From then on, the motorcycle's distance from the telephone pole is given by $M(t) = 90 + 2.5t^2$ where t is in seconds and M is in meters.

- At what time t does the car catch up to the motorcycle?
- How far are the car and the motorcycle from the telephone pole when this happens?

Ex. 5 Solution:

a. The car and motorcycle are the same distance from the pole when $C(t) = M(t)$

$$\begin{aligned} 30t &= 90 + 2.5t^2 \\ 0 &= 2.5t^2 - 30t + 90 \\ 0 &= t^2 - 12t + 36 \\ 0 &= (t - 6)(t - 6) \quad t = 6 \text{ seconds.} \end{aligned}$$

b. At $t = 6$, the car's distance from the pole is $C(6) = 30(6) = 180$ meters. The motorcycle is also 180 meters from the telephone at this time ($t = 6$).

Algebra 1 Instructional Support Tool - Algebra

Standard A-CED.A.1

Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions and quadratic functions, and simple rational and exponential functions. *(Content Emphases: Major)*

Building Blocks (Grades 6 to 8 prerequisites):

Grade 7

7.EE.B.4

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

- Solve word problems leading to **equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers**. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
- Solve word problems leading to **inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers**. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Grade/Course Level Connections:

A-REI.B.3, A-REI.A.1 and A-REI.B.4

Instructional Considerations:

- In providing examples of real-world problems that can be modeled by writing an equation or inequality, begin with simple equations and inequalities and build up to more complex equations
- Provide examples of real-world problems in a variety of contexts that can be modeled by writing a quadratic equation.
- Encourage students to use variables other than x and y where sensible. For example, when modeling the relationship between height and time, h and t could be used as variables instead of x and y .
- Encourage students to use variables with subscripts where sensible. Students should recognize that the same letter (or symbol) with different subscripts (e.g., x_1 and x_2) should be viewed as different variables.

Prerequisite Examples:

Grade 7

7.EE.B.4a

NJ Taxi Company charges a \$3 initial fee and \$2 for each mile traveled. How far would you be able to travel if you had a total of \$50 and planned on giving the taxicab driver a \$5 tip? Write an equation to represent the total distance that can be traveled.

7.EE.B.4b

As a saleswoman, Laura earns \$600/week salary and 5% commission on her total sales. How much would Laura have to sell to earn at least \$1000 in a week? Write an inequality for the total sales Laura would need.

Common Misconceptions:

- Students may believe that when creating models from verbal descriptions, all values or quantities must be used.
- Students may believe that equations of quadratic and other functions are abstract and exist only “in a math book,” without seeing the usefulness of these functions as modeling real-world phenomena.
- Students may believe that only “ x ” or “ y ” can represent variables.
- Students may believe that when equations contain variables with subscripts, those subscripts can be combined by like terms.
- Students often do not understand what the variable represents in context. For example, if the height, h in feet, of a piece of lava t seconds after it is ejected from a volcano is modeled by $h(t) = -16t^2 + 64t + 936$, the student might be asked to find the time it takes for the piece of lava to hit the ground. The student may have difficulty understanding that when the lava hits the ground the height is zero ($h = 0$) and that they must solve for t .

Algebra 1 Instructional Support Tool - Algebra

Standard A-CED.A.1

Create equations and inequalities in one variable and **use them to solve problems**. Include equations arising from linear functions and quadratic functions, and simple rational and exponential functions. *(Content Emphases: Major)*

Academic Vocabulary

equation, inequality, solution, linear equation, quadratic equation, exponential equation, function, factor, zeros

Standards for Mathematical Practice:

- (1) Make sense of problems and persevere in solving them
- (4) Model with mathematics

Algebra 1 Instructional Support Tool - Algebra

Standard A-CED.A.2

Create equations in two or more variables to represent relationships between quantities; **Graph** equations on coordinate axes with labels and scales. *(Content Emphases: Major)*

Explanations and/or Examples

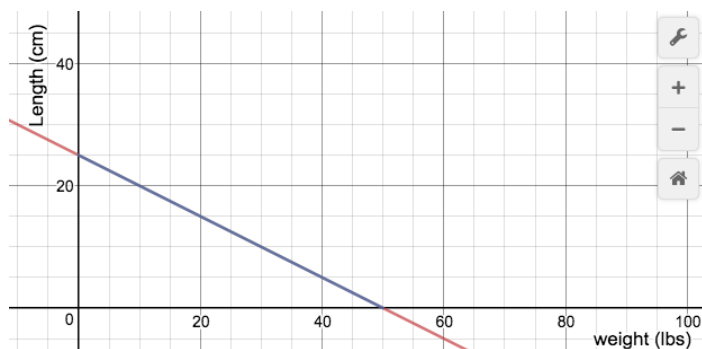
In graphing equations on coordinate axes, students select intervals for the scale that are appropriate for the context and display adequate information about the relationship. Students also interpret the context as part of choosing appropriate minimum and maximum values for a graph.

Example 1: A spring with an initial length of 25 cm will compress 0.5 cm for each pound applied.

- Write an equation to model the relationship between the amount of weight applied and the length of the spring.
- Graph the relationship between pounds and length.
- What does the graph reveal about limitations on weight?

Ex. 1 Solution:

- Let L = length of the spring in centimeters and w = weight applied to the spring in pounds; $L = 25 - .5w$



- The graph reveals that the weight must be less than or equal to 50. For values greater than 50, the length of the spring is negative, which is physically impossible

Example 2: The cheerleaders are launching T-shirts into the stands at a football game. They are launching the T-shirts from a height of 3 feet off the ground and at an initial velocity of 36 feet per second. What equation represents a T-shirt's height, h for a time, t ? (Use $16t^2$ for the effect of gravity on the height of the T-Shirt.)

Ex. 2 Solution: $h = -16t^2 + 36t + 3$

Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

8.F.B.4

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.A.3

Interpret the equation $y = mx + b$ as **defining a linear function, whose graph is a straight line**; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points $(1,1)$, $(2,4)$ and $(3,9)$, which are not on a straight line.*

8.EE.B.5

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

Grade 7

7.RP.A.2c

Recognize and represent proportional relationships between quantities

- Represent proportional relationships by equations.** *For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.*

Algebra 1 Instructional Support Tool - Algebra

Standard A-CED.A.2

Create equations in two or more variables to represent relationships between quantities; **Graph** equations on coordinate axes with labels and scales. *(Content Emphases: Major)*

Prerequisite Examples:

Grade 8

8.F.B.4

Write an equation that models the linear relationship in the table below.

Table 1 *x*-*y* values

<i>x</i>	<i>y</i>
-2	8
0	2
1	-1

Grade 8

8.EE.B.5

Lena paid \$18.96 for 3 pounds of coffee.

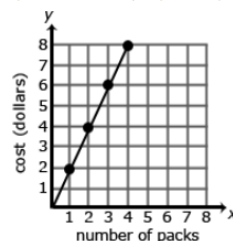
- Draw a graph in the coordinate plane of the relationship between the number of pounds of coffee and the total cost.
- In this situation, what is the meaning of the slope of the line you drew in part (c)

[Task 129 by IM by CC BY-NC-SA 4.0](#)

Grade 7

7.RP.A.2c

The graph below represents the cost of packs of gum as a unit rate of \$2 for every pack of gum. Represent the relationship using an equation.



8.F.A.3

Determine if the functions shown below represent a linear function

- $y = 0.25 + 0.5(x - 2)$
- $A = \pi r^2$
- Table 2. *x*-*y* values

<i>x</i>	<i>y</i>
1	12
2	7
3	4
4	3
5	4
6	7

Grade/Course Level Connections:

A-CED.A.1 and N-Q.A.1

Instructional Considerations:

- Build on the work previously done with creating equations in *one* variable.
- In providing examples of real-world problems that can be modeled by writing an equation or inequality, begin with simple equations and inequalities and build up to more complex equations in two variables.
- Discuss the importance of using appropriate labels and scales on the axes when representing equations with graphs.
- Provide examples of real-world problems that can be solved by writing an equation, and have students explore the graphs of the equations on a graphing calculator to determine which parts of the graph are relevant to the problem context.
- Linear equations can be written in a multitude of ways; $y = mx + b$ and $ax + by = c$ are commonly used forms (given that x and y are the two variables). Students should be flexible in using multiple forms and recognizing from the context, which is appropriate to use in creating the equation.

Algebra 1 Instructional Support Tool - Algebra

Standard A-CED.A.2

Create equations in two or more variables to represent relationships between quantities; **Graph** equations on coordinate axes with labels and scales. *(Content Emphases: Major)*

Instructional Considerations (cont'd):

- Quadratic equations can be written in a multitude of ways; $y = ax^2 + bx + c$ and $y = (x+m)(x+n)$ are commonly used forms (given that x and y are the two variables). Students should be flexible in using multiple forms and recognizing from the context, which is appropriate to use in creating the equation.
- Exponential equations can be written in different ways; $y = ab^x$ and $y = a(1 \pm r)^x$ are the most common forms (given that x and y are variables). Students should be flexible in using all forms and recognizing from the context which is appropriate to use in creating the equation.

Common Misconceptions:

- Students may believe that the default scales shown in a technology tool are the best scales.
- Students may believe that when creating models from verbal descriptions, all values or quantities must be used.

Academic Vocabulary

relationship, quantity, variable, graph, coordinate plane, axes, scale, labels, units

Standards for Mathematical Practice:

- (1) Make sense of problems and persevere in solving them
- (4) Model with mathematics

Algebra 1 Instructional Support Tool - Algebra

Standard A-CED.A.3

Represent constraints by equations or inequalities, and **by systems of equations and/or inequalities**, and **interpret solutions** as viable or nonviable options in a **modeling context**. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. (Content Emphases: Major)

Explanations and/or Examples

Write equations and inequalities from a modeling context. Write systems of equations and systems of inequalities from a modeling context.

Example 1: Michelle loves to go to the movies. She goes both at night and during the day. The cost of a matinee is \$6. The cost of an evening show is \$8. Michelle went to see a total of 5 movies and spent \$36. How many of each type of movie did she attend?

Ex. 1 Solution: $m + e = 5$
 $6m + 8e = 36$;
 $m = 2, e = 3$

Represent constraints by a system of equations or inequalities.

Example 2: The only coins that Alexis has are dimes and quarters. Her coins have a total value of \$5.80. She has a total of 40 coins.

Which of the following systems of equations can be used to find the number of dimes, d , and the number of quarters, q , Alexis has? Explain your choice.

- a) $\begin{cases} d + q = 5.80 \\ 40d + 40q = 5.80 \end{cases}$
- b) $\begin{cases} d + q = 40 \\ .25d + .10q = 5.80 \end{cases}$
- c) $\begin{cases} d + q = 5.80 \\ .10d + .25q = 40 \end{cases}$
- d) $\begin{cases} d + q = 40 \\ .10d + .25q = 5.80 \end{cases}$

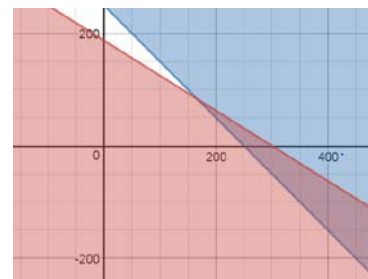
Ex. 2 Solution:

[Task 220 by IM](#) by [CC BY-NC-SA 4.0](#)

Example 3: A club is selling hats and jackets as a fundraiser. Their budget is \$1500 and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs \$5 and each jacket costs \$8.

- a. Write a system of inequalities to represent the situation.
- b. Solve the system and determine three viable solutions.

Ex. 3 Solution: a. h = number of hats, j = number of jackets
 $h + j \geq 250$; $5h + 8j \leq 1500$



b.

NOTE: teachers should call attention to the fact that in this scenario, the club cannot buy more jackets than hats.

Interpret solutions as viable or nonviable options in a modeling context.

Example 4: A club is selling hats and jackets as a fundraiser. Their budget is \$1500 and they want to order at least 250 items. They must buy at least as many hats as they buy jackets. Each hat costs \$5 and each jacket costs \$8.

- a. If the club buys 150 hats and 100 jackets, will the conditions be satisfied?
- b. What is the maximum number of jackets they can buy and still meet the conditions?

Ex. 4 Solution:

- a. No, if the club buys 150 hats and 100 jackets, they will spend \$1550.
- b. The maximum number of jackets the club can buy is 83.

Algebra 1 Instructional Support Tool - Algebra

Standard A-CED.A.3

Represent constraints by equations or inequalities, and **by systems of equations and/or inequalities**, and **interpret solutions** as viable or nonviable options in a **modeling context**. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* (Content Emphases: Major)

Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

8.F.B.4

Construct a function to model a linear relationship between two quantities. **Determine the rate of change and initial value** of the function **from a description of a relationship** or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Grade 7

7.EE.B.4a-b

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

- Solve word problems leading to equations** of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.
- Solve word problems leading to inequalities** of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Prerequisite Examples:

Grade 8

8.F.B.4

You work for a video streaming company that has two plans available.

Plan 1: A flat rate of \$7 per month plus \$2.50 per video viewed

Plan 2: \$4 per video viewed

- What type of function models this situation? Explain how you know.
- Define variables that make sense in the context and write an equation representing a function that describes each plan.
- How much would 3 videos in a month cost for each plan? 5 videos?
- Compare the two plans and explain what advice you would give to a customer trying to decide which plan is best for them, based on their viewing habits.

Grade 7

7.EE.B.4a

The sum of three consecutive even numbers is 48. What is the smallest of these numbers? Write and solve an equation.

7.EE.B.4b

Jonathan wants to save up enough money so that he can buy a new sports equipment set that includes a football, baseball, soccer ball, and basketball. This complete boxed set costs \$50. Jonathan has \$15 he saved from his birthday. In order to make more money, he plans to wash neighbors' windows. He plans to charge \$3 for each window he washes, and any extra money he makes beyond \$50 he can use to buy the additional accessories that go with the sports boxed set.

Write and solve an inequality that represents the number of windows Jonathan can wash in order to save at least the minimum amount he needs to buy the boxed set. Graph the solutions on the number line. What is a realistic number of windows for Jonathan to wash? How would that be reflected in the graph?

[Task by IM](#) by [CC BY-NC-SA 4.0](#)

Algebra 1 Instructional Support Tool - Algebra

Standard A-CED.A.3

Represent constraints by equations or inequalities, and **by systems of equations and/or inequalities**, and **interpret solutions** as viable or nonviable options in a **modeling context**. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* (Content Emphases: Major)

Building Blocks (Grades 6 to 8 prerequisites):

Grade 6

6.EE.A.2a

Write, read, and evaluate **expressions** in which letters stand for numbers.

- Write expressions that record operations with numbers and with letters standing for numbers.

6.EE.B.8

Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Grade/Course Level Connections:

A-CED.A.1, A-CED.A.2, and A-REI.D.12

Instructional Considerations:

- Explore examples illustrating modeling contexts that contain multiple constraints.
- Constraints are restrictions (i.e., limitations, boundaries) placed upon variables used in equations that model real-world situations.
- Equations and/or inequalities may represent constraints in a given context.
 - e.g. if I have to have at least two gallons of gas, x , for every gallon of oil, y , then $x \geq 2y$ is an inequality constraint.
 - e.g. if I have to have exactly three gallons of gas, x , for every gallon of oil, y , then $x = 3y$ is a constraint equation.
- It is possible that certain solutions which make an equation true mathematically, may not make any sense in the context of a real-world word problem. Constraints then become necessary to allow the mathematical model to realistically represent the situation.
- Applications such as linear programming can help students to recognize how businesses use constraints to maximize profit while minimizing the use of resources. These situations often involve the use of systems of two variable inequalities.

Prerequisite Examples:

Grade 6

6.EE.A.2a

Write an expression for each:

- 3 times the sum of a number and 5
- Twice the difference between a number and 5

6.EE.B.8

Jonas spent more than \$50 at an amusement park. Write an inequality to represent the amount of money Jonas spent. What are some possible amounts of money Jonas could have spent? Represent the situation on a number line.

Common Misconceptions:

- Students may believe that a constraint must be an inequality.
- Students may believe that systems of equations or inequalities are abstract and exist only “in a math book,” without seeing their usefulness as models of real-world situations.
- Students may believe that only ‘ x ’ or ‘ y ’ can represent variables or that ‘ x ’ and ‘ y ’ *should* be the variables used in modeling situations.

Algebra 1 Instructional Support Tool - Algebra

Standard A-CED.A.3

Represent constraints by equations or inequalities, and **by systems of equations and/or inequalities**, and **interpret solutions** as viable or nonviable options in a **modeling context**. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* (Content Emphases: Major)

Academic Vocabulary

constraints, system of inequalities, boundaries, limitations, solution set, system of linear equations

Standards for Mathematical Practice:

- (4) Model with mathematics
- (7) Look for and make use of structure

Algebra 1 Instructional Support Tool - Algebra

Standard A-CED.A.4

Rearrange formulas to highlight a quantity of interest, **using the same reasoning** as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R .* (Content Emphases: Major)

Explanations and/or Examples

Example 1:

Rearrange the formula to solve for F . Explain your reasoning.

$$C = \frac{5}{9}(F - 32)$$

Ex. 1 Solution:

I can multiply both sides of the equation by $\frac{9}{5}$. Multiplying by the multiplicative inverse of $\frac{5}{9}$ leaves the product of 1 and $(F - 32)$ on the right side of the equation. The product of 1 and $(F - 32)$ is simply $F - 32$. Next using the addition property of equality, add 32 to both sides of the equation. The result is $\frac{9}{5}C + 32 = F$ or $F = \frac{9}{5}C + 32$

Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

8.EE.C.7b

Solve linear equations in one variable.

- b. **Solve linear equations with rational number coefficients**, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Prerequisite Examples:

Grade 8

8.EE.C.7b :

Solve the following equations

- $-3(x + 1) - 5 = 3x - 2$
- $\frac{1}{4} - \frac{2}{3}y = \frac{3}{4} - \frac{1}{3}y$

Grade/Course Level Connections:

A-REI.B.3 and A-CED.A.2

Instructional Considerations:

- Solving equations for a specified parameter (e.g. $A = .5h(B + b)$ when solving for b) is similar to solving an equation with one variable. Provide students with opportunities to abstract from particular numbers and to apply the same kind of manipulations to formulas as they do to equations.
- Draw students' attention to equations containing variables with subscripts.
 - The same variables with different subscripts should be viewed as different variables that cannot be combined as like terms (e.g. x_1 and x_2).
 - A variable with a variable subscript, such as a_n , should be treated as a single variable - the n^{th} term, where variables a and n have different meaning.
- Explore examples illustrating when it is useful to rewrite a formula by solving for one of the variables in the formula. For example, the formula for the area of a trapezoid ($A = .5h(b_1 + b_2)$) can be solved for h if the area and lengths of the bases are known but the height needs to be calculated. This strategy of selecting a different representation has many applications in science and business when using formulas.
- Give students formulas, such as area and volume (or from science or business), and have students solve the equations for each of the different variables in the formula.

Algebra 1 Instructional Support Tool - Algebra

Standard A-CED.A.4

Rearrange formulas to highlight a quantity of interest, **using the same reasoning** as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R .* (Content Emphases: Major)

Common Misconceptions:

- Students may believe that when equations contain variables with subscripts, those subscripts can be combined by like terms. Students should realize that the same variables with different subscripts (e.g., x_1 and x_2) should be viewed as different variables.
- Students may believe that only “x” or “y” can represent variables.
- Students may believe there is no connection between given formulas (for example, geometric formulas) and real-world applications.

Academic Vocabulary

formula, additive inverse, multiplicative inverse, properties of equality (addition, subtraction, multiplication, and division)

Standards for Mathematical Practice:

- (2) Reason abstractly and quantitatively
- (7) Look for and make use of structure

Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.A.1

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. **Construct a viable argument** to justify a solution method. *(Content Emphases: Major)*

Explanations and/or Examples

Example 1:

Assuming the equation $5(x + 3) - 3x = 55$ has a solution, construct a convincing argument that justifies each step in the solution process.

Ex. 1 Solution:

Solution, Justification and Explanation

Solution Process	Justification	Explanation
$5(x + 3) - 3x = 55$	given	given
$5x + 15 - 3x = 55$	Distributive property	Multiply 5 by x and 5 by 3 in order to eliminate the parenthesis.
$2x + 15 = 55$	Distributive property (combine like terms)	Add 5x to -3x (or 5x minus 3x) to get 2x in order to combine the like terms on the left side of the equation
$2x + 15 - 15 = 55 - 15$ $2x = 40$	Subtraction property of equality	Subtract 15 from both sides of the equation in order to get all constants on one side of the equal sign
$\frac{2x}{2} = \frac{40}{2}$	Division property of equality	Divide both sides of the equation by 2 so that the coefficient of x is 1

Prerequisite Examples:

Grade 8

8.EE.C.7a:

For each linear equation, indicate whether the equation has ‘no solution’, ‘one solution’, or ‘infinitely many solutions’.

- $7x + 21 = 21$
- $12x + 15 = 12x - 15$
- $-5x - 25 = 5x + 25$

Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

8.EE.C.7a

Solve linear equations in one variable.

- Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by **successively transforming the given equation into simpler forms**, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

Grade 7

7.EE.B.4a

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers.** Solve equations of these forms **fluently**. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.A.1

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. **Construct a viable argument** to justify a solution method. *(Content Emphases: Major)*

Prerequisite Examples cont'd:

Grade 7

7.EE.B.4a:

Sue takes a taxi to the train station from her home. The taxi fare was \$1.75 per mile. She gave the driver \$5 tip and paid a total of \$45.25. Write an equation that represents the distance, in miles, between the train station and Sue's house. Solve your equation to find the distance between Sue's house and the train station.

Grade/Course Level Connections:

A-CED.A.1, A-CED.A.4, A-REI.B.3 and A-REI.B.4a

Instructional Considerations:

- Students justify each step of solving an equation, understanding that transforming $2x - 5 = 7$ into $2x = 12$ is possible because $5 = 5$. Therefore, adding the same quantity to both sides of the equation makes the resulting equation true as well. Each step of solving an equation can be defended, much like providing evidence for steps of a geometric proof.
- Connect the idea of adding two equations together as a means of justifying steps of solving a simple equation to the process of solving a system of equations. A system consisting of two linear equations such as $2x + 3y = 8$ and $x - 3y = 1$ can be solved by adding the equations together. This can be justified by exactly the same reason that solving the equation $2x - 4 = 5$ can begin by adding the equation $4 = 4$.
- Provide examples of how the same equation might be solved in a variety of ways. as long as equivalent quantities are added or subtracted to both sides of the equation, the order of steps taken will not matter.

$\begin{array}{r} 3n + 2 = n - 10 \\ -2 \quad -2 \\ \hline 3n = n - 12 \\ -n \quad -n \\ \hline 2n = -12 \\ 2 \quad 2 \\ \hline n = -6 \end{array}$	$\begin{array}{r} 3n + 2 = n - 10 \\ +10 \quad +10 \\ \hline 3n + 12 = n \\ -3n \quad -3n \\ \hline 12 = -2n \\ -2 \quad -2 \\ \hline -6 = n \end{array}$	$\begin{array}{r} 3n + 2 = n - 10 \\ -n \quad -n \\ \hline 2n = -10 \\ -2 \quad -2 \\ \hline 2n = -12 \\ 2 \quad 2 \\ \hline n = -6 \end{array}$
or		

- Students should be able to state and apply properties when working with equations (and inequalities) that include more than one variable, fractions and decimal.

Common Misconceptions:

- Students may believe that fractions are a special case. They may believe that it is not necessary to multiply both sides of an equation by the multiplicative inverse of a fraction. For example, given $\frac{1}{2}x = 10$, students may believe it is only necessary to multiply the left side of the equation by the multiplicative inverse of $\frac{1}{2}$.
- Students may believe that all equations have a solution.
- Students may believe that adding the same term to both sides of an equation or multiplying both sides by a non-zero constant produces an equation with different solutions.

Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.A.1

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. **Construct a viable argument** to justify a solution method. *(Content Emphases: Major)*

Academic Vocabulary

additive inverse, multiplicative inverse, properties of equality (addition, subtraction, multiplication, and division)

Standards for Mathematical Practice:

- (1) Make sense of problems and persevere in solving them
- (3) Construct viable arguments and critique the reasoning others

Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.B.3

Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (Content Emphases: Major)

Explanations and/or Examples

Solve equations with coefficients represented by letters

Example 1:

- Solve $-cz + 6z = tz + 83$ for z
- Solve $d(-3 + x) = kx + 9$ for x

Ex. 1 Solution:

- $z = \frac{83}{6-c-t}$
- $x = \frac{3d+9}{d-k}$

Example 2: Solve each inequality

- $4 - \frac{2}{3}x > 2 - x$
- $-2(x + 2) > 4 - x$
- $\frac{2}{3}h + 9 < 18$
- $53w + 13 < 56w + 16$
- $12y + 7 < -11$ or $5y - 8 > 40$
- $x + 8 \geq 9$ and $\frac{x}{7} < 1$

Ex. 2 Solution:

- $x > -6$
- $-x > 8$
- $h < \frac{27}{2}$
- $-1 < w$
- $y < -\frac{3}{2}$ or $y > \frac{48}{5}$
- $x \geq 1$ and $x < 7$

Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

8.EE.C.7b

Solve linear equations in one variable.

- Solve linear equations with **rational number coefficients**, including equations whose solutions require **expanding expressions using the distributive property and collecting like terms**.

Grade 7

7.EE.B.4a

Use variables to represent quantities in a real-world or mathematical problem, and **construct simple equations and inequalities to solve problems** by reasoning about the quantities.

- Solve word problems leading to **equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers**. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

Prerequisite Examples:

Grade 8

8.EE.C.7b

Solve $3x - 8 = 4x - 8$

Solve $3(x + 1) - 5 = 3x - 2$

Grade 7

7.EE.B.4a

Solve $-3(x + 7) = 4$

Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.B.3

Solve linear equations and inequalities in one variable, including **equations with coefficients represented by letters**. (Content Emphases: Major)

Building Blocks (Grades 6 to 8 prerequisites):

Grade 7

7.EE.B.4b

Use variables to represent quantities in a real-world or mathematical problem, and **construct simple equations and inequalities to solve problems** by reasoning about the quantities.

b. Solve word problems leading to **inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers**. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Grade 6

6.EE.B.7

Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.

Prerequisite Examples:

Grade 7

7.EE.B.4b

Jonathan wants to save up enough money so that he can buy a new sports equipment set that includes a football, baseball, soccer ball, and basketball. This complete boxed set costs \$50. Jonathan has \$15 he saved from his birthday. In order to make more money, he plans to wash neighbors' windows. He plans to charge \$3 for each window he washes, and any extra money he makes beyond \$50 he can use to buy the additional accessories that go with the sports box set.

Write and solve an inequality that represents the number of windows Jonathan can wash in order to save at least the minimum amount he needs to buy the boxed set. [Task by IM](#) by [CC BY-NC-SA 4.0](#)

Grade 6

6.EE.B.7

Sierra walks her dog Pepper twice a day. Her evening walk is two and a half times as far as her morning walk. At the end of the week she tells her mom, "I walked Pepper for 30 miles this week!" Write an equation that could be used to find the length of her morning walk. Use your equation to find the length of her morning walk. [Task by IM](#) by [CC BY-NC-SA 4.0](#)

Grade/Course Level Connections:

A-CED.A.1, A-CED.A.4 and A-REI.A.1

Instructional Considerations:

- Two reasons for discussing the topic of inequalities along with the topic of equations:
 - There are analogies between solving equations and solving inequalities that help students understand both.
 - The applications that lead to equations almost always lead in the same or similar way to inequalities.
- Students must be aware of what it means to check an inequality's solution. The substitution of the endpoints of the solution set into the original inequality should give equality, regardless of the presence or absence of an equal sign in the original sentence. The substitution of any value from the rest of the solution set should give a correct inequality.

Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.B.3

Solve linear equations and inequalities in one variable, including **equations with coefficients represented by letters**. *(Content Emphases: Major)*

Common Misconceptions:

- Students may believe there are no similarities between solving equations and solving inequalities that help them better understand both types of problems.
- Students may believe there is only one way to solve problems. Students need to be reminded of the most common solving techniques, such as converting fractions from one form to another, removing parentheses in the sentences, or multiplying both sides of an equation or inequality by the common denominator of the fractions.

Academic Vocabulary:

distributive property, additive inverse, multiplicative inverse, properties of inequality (addition, subtraction, multiplication, and division), literal equation

Standards for Mathematical Practice:

- (2) Reason abstractly and quantitatively
- (4) Model with mathematics

Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.B.4

Solve quadratic equations in one variable.

4a. Use the method of **completing the square** to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. **Derive** the quadratic formula from this form.

4b. **Solve** quadratic equations **by inspection** (e.g., for $x^2 = 49$), **taking square roots**, **completing the square**, **the quadratic formula** and **factoring**, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives **complex solutions** and **write them as $a \pm bi$** for real numbers a and b .

(Content Emphases: Major)

Explanations and/or Examples

Complete the square to transform a quadratic equation into the form $(x - p)^2 = q$

Example 1: Use the method of completing the square to transform the equation $x^2 + 6x + 10 = 0$.

Ex. 1 Solution: $x^2 + 6x = -10$
 $x^2 + 6x + 9 = -10 + 9$
 $(x + 3)^2 = -1$

Example 2: Choose and use an efficient method to find the solutions of $x^2 - 8x + 16 = 7$. Why was the method you selected efficient in this case?

Ex. 2 Solution: $(x - 4)^2 = 7$

$$(x - 4) = \pm \sqrt{7}$$

$x = 4 \pm \sqrt{7}$. Factoring (and then taking the square root) is efficient because the trinomial on the left of the equal sign is a perfect square trinomial.

Example 3: Are the solutions of $2x^2 + 5 = 2x$ real or complex? How many solutions does it have? Find all solutions to the equation.

Ex. 3 Solution: $b^2 - 4ac = (-2)^2 - 4(2)(5) = 4 - 40 = -36$. The discriminant is negative so there are two complex solutions $(2 + 6i)/4$ and $(2 - 6i)/4$ (or other equivalent forms of these solutions).

Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

8.EE.A.2

Use **square root** and **cube root** symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

Grade 7

7.EE.A.1

Apply properties of operations as strategies to add, subtract, **factor**, and expand **linear expressions** with rational coefficients.

Grade/Course Level Connections:

A-SSE.B.3 and A-REI.A.1

Prerequisite Examples:

Grade 8

8.EE.A.2:

Solve $x^2 = \frac{4}{9}$

Grade 7

7.EE.A.1:

Factor the expression $6y - 15z$

Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.B.4

Solve quadratic equations in one variable.

4a. Use the method of **completing the square** to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. **Derive** the quadratic formula from this form.

4b. **Solve** quadratic equations **by inspection** (e.g., for $x^2 = 49$), **taking square roots, completing the square, the quadratic formula and factoring**, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives **complex solutions** and **write them as $a \pm bi$** for real numbers a and b .

(Content Emphases: Major)

Instructional Considerations:

- Demonstrate different approaches to solving a quadratic equation. For example, the same quadratic equation may be solved using the quadratic formula as well as by completing the square.
- Students may illustrate that the same quadratic equation, regardless of solution process, yields the same solution(s).
- Connect completing the square with balancing equations.
- Facilitate discourse and illustrations of various solution processes that enable students to see the value (e.g. efficiency) in each.
- Discourage students from using only one method to solve quadratic equations.

Common Misconceptions:

- Some students may think that rewriting equations into various forms (taking square roots, completing the square, using quadratic formula and factoring) are isolated techniques without purpose.
- Students may believe there is only one way to solve quadratic equations and may need help in strategically selecting an efficient way to solve a given problem.
- Students may believe that the graph of a quadratic function has no connection to the quadratic equation and no connection to real-world applications.
- Students may believe that every quadratic equation has two solutions.

Academic Vocabulary

quadratic, completing the square, quadratic formula, complex number, solution, square root, radical, zeros

Standards for Mathematical Practice:

- (2) Reason abstractly and quantitatively
- (7) Look for and make use of structure

Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.C.5

Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. (Content Emphases: Additional)

Explanations and/or Examples

Example 1: Lisa is working with the system of equations $x + 2y = 7$ and $2x - 5y = 5$. She multiplies the first equation by 2 and then subtracts the second equation to find $9y = 9$, telling her that $y = 1$. Lisa then finds that $x = 5$. Thinking about this procedure, Lisa wonders

“There are lots of ways I could go about solving this problem. I could add 5 times the first equation and twice the second or I could multiply the first equation by -2 and add the second. I seem to find that there is only one solution to the two equations but I wonder if I will get the same solution if I use a different method?”

- What is the answer to Lisa’s question? Explain
- Does the answer to (a) change if we have a system of two equations in two unknowns with no solutions? What if there are infinitely many solutions?

Ex. 1 Solution:

[Task 1903 by IM](#) by [CC BY-NC-SA 4.0](#)

Prerequisite Examples:

Grade 8

8.EE.C.8a

Consider the equation $5x - 2y = 3$. If possible, find a second linear equation to create a system of equations that has:

- Exactly 1 solution
- Exactly 2 solutions
- No solutions
- Infinitely many solutions

8.EE.C.8b

Find x and y using elimination and then using substitution.

$$3x + 4y = 7$$

$$-2x + 8y = 10$$

Grade/Course Level Connections:

A-CED.A.2, A-CED.A.3, A-REI.C.6, and A-REI.D.12

Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

8.EE.C.8

Analyze and solve pairs of simultaneous linear equations.

- Understand that **solutions** to a system of two linear equations in two variables **correspond to points of intersection of their graphs**, because **points of intersection satisfy both equations simultaneously**.
- Solve systems of two linear equations in two variables algebraically**, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
- Solve real-world and mathematical problems leading to two linear equations in two variables**. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Determine the number of solution(s) to the system of equations. Justify your reasoning.

$$2x - 3y = 8$$

$$6x - 9y = 24$$

8.EE.C.8c

Site A charges \$6 per month and \$1.25 for each movie downloaded. Site B charges \$2 for each movie and no monthly fee. Determine the number of movies that could be downloaded in one month that would make the costs for two sites the same.

Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.C.5

Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. (*Content Emphases: Additional*)

Instructional Considerations:

- The focus of this standard is to provide mathematics justification for the addition (elimination) and substitution methods of solving systems of equations that transform a given system of two equations into a simpler equivalent system that has the same solutions as the original.
- The Addition and Multiplication Properties of Equality allow finding solutions to certain systems of equations. In general, any linear combination, $m(Ax + By) + n(Cx + Dy) = mE + nF$, of two linear equations

$$Ax + By = E \text{ and}$$

$$Cx + Dy = F$$

intersecting in a single point contains that point. The multipliers m and n can be chosen so that the resulting combination has only an x -term or only a y -term in it.

- In the proof of a system of two equations in two variables, where one equation is replaced by the sum of that equation and a multiple of the other equation, produces a system that has the same solutions, let point (x_1, y_1) be a solution of both equations:

$$Ax_1 + By_1 = E \text{ (true)}$$

$$Cx_1 + Dy_1 = F \text{ (true)}$$

Replace the equation $Ax + By = E$ with ' $Ax + By + k(Cx + Dy)$ ' on its left side

Replace the equation $Ax + By = E$ with ' $E + kF$ ' on its right side.

The new equation is $Ax + By + k(Cx + Dy) = E + kF$.

Show that the ordered pair of numbers (x_1, y_1) is a solution of this equation by replacing (x_1, y_1) in the left side of this equation and verifying that the right side really equals $E + kF$:

$$Ax_1 + By_1 + k(Cx_1 + Dy_1) = E + kF \text{ (true)}$$

Common Misconceptions:

see A-REI.C.6

Academic Vocabulary

elimination method, solutions

Standards for Mathematical Practice:

- (4) Model with mathematics
- (7) Look for and make use of structure

Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.C.6

Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. *(Content Emphases: Additional)*

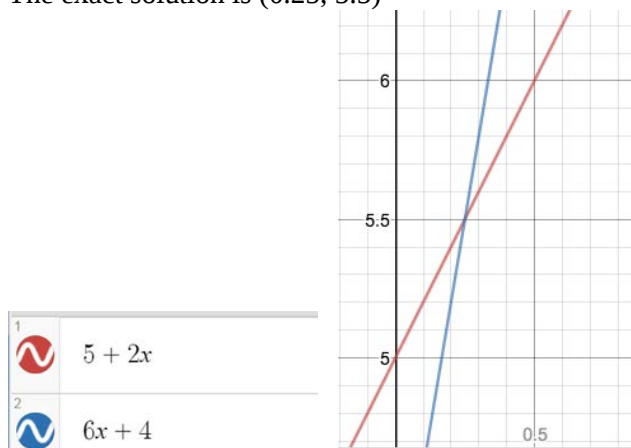
Explanations and/or Examples

Example 1: Solve the following system by graphing.

$$y_1 = 5 + 2x \quad y_2 = 6x + 4.$$

Give your answer to the nearest tenth.

Ex. 1 Solution: One possible **approximate** solution is (.3, 6).
The exact solution is (0.25, 5.5)



Example 2: In 1983 the composition of pennies in the United States was changed due, in part, to the rising cost of copper. Pennies minted after 1983 weigh 2.50 grams while the earlier copper pennies weighed 3.11 grams

- A roll of pennies contains 50 coins. If a particular roll of pennies weighs 138.42 grams, how many of the old heavier pennies and how many of the new lighter pennies does this roll contain?

Ex. 2 Solution:

[Task 761 by IM](#) by [CC BY-NC-SA 4.0](#)

Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

8.EE.C.8

Analyze and solve pairs of simultaneous linear equations.

- Understand that **solutions** to a system of two linear equations in two variables **correspond to points of intersection of their graphs**, because **points of intersection satisfy both equations simultaneously**.
- Solve systems of two linear equations in two variables algebraically**, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
- Solve real-world and mathematical problems leading to two linear equations in two variables.** *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Prerequisite Examples:

Grade 8

8.EE.C.8a

Consider the equation $5x - 2y = 3$. If possible, find a second linear equation to create a system of equations that has:

- Exactly 1 solution
- Exactly 2 solutions
- No solutions
- Infinitely many solutions

8.EE.C.8b

Find x and y using elimination and then using substitution.

$$\begin{aligned} 3x + 4y &= 7 \\ -2x + 8y &= 10 \end{aligned}$$

8.EE.C.8c

Site A charges \$6 per month and \$1.25 for each movie downloaded. Site B charges \$2 for each movie and no monthly fee. Determine the number of movies that could be downloaded in one month that would make the costs for two sites the same.

Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.C.6

Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. *(Content Emphases: Additional)*

Grade/Course Level Connections:

A-CED.A.2, A-CED.A.3, and A-REI.C.5

Instructional Considerations:

- While using algebraic methods - elimination or substitution - to find exact solutions to systems of equations, stress the importance of having a **well-organized format** for developing the solutions.
- Solving a system of equations with n unknowns requires n equations (e.g. a system of 3 equations is required in order to solve equations in 3 variables).
- Systems of equations are classified into two groups, consistent or inconsistent, depending on whether or not solutions exist.
- Stress the benefit of making the appropriate selection of a method for solving systems (graphing vs. addition vs. substitution). This depends on the type of equations and combination of coefficients for corresponding variables, without giving a preference to either method.
- By making connections between algebraic and graphical solutions - and the **context** of the system of linear equations - students are able to make sense of their solutions.
- Students need opportunities to work with equations and **contexts** that include solutions in decimal and fraction form.

Common Misconceptions:

- Students may believe that the solutions to systems of equations are always integers and therefore may assume that a decimal/fractional answer is incorrect because they have not been sufficiently exposed to various types of solutions.
- Students may believe that when solving a system of equation by elimination, they must multiply both equations by the same number. Student should understand that when they are using the elimination method, they are creating multiples of the original equation to help them solve most efficiently.
- Many mistakes that students make are careless rather than conceptual.

Academic Vocabulary

linear system, solution, elimination method, substitution method, intersection, approximate

Standards for Mathematical Practice:

- (1) Make sense of problems and persevere in solving them
- (4) Model with mathematics

Algebra 1 Instructional Support Tool - Algebra

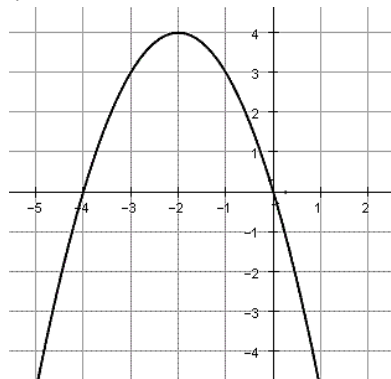
Standard A-REI.D.10

Understand that the **graph of an equation** in two variables is the set of **all** its solutions plotted in the coordinate plane, often forming a curve (which could be a line). *(Content Emphases: Major)*

Explanations and/or Examples

Example 1:

Given the graph of an equation shown below, provide at least three solutions to the equation.



Ex. 1 Solution: One possible set of solutions is $(-2, 4)$, $(0, 0)$, $(-4, 0)$

Prerequisite Examples:

Grade 8

8.EE.B.5

Construct a graph of a proportional relationship reflecting a slope that is less than 25.

8.EE.C.8a

Victor is half as old as Maria. The sum of their ages is 54. Let v = Victor's age. Let m = Maria's age.

$$\begin{cases} v + m = 54 \\ v = .5m \end{cases}$$

The graph represents the two equations that form this system. What does the intersection of the two equations represent? Justify your answer.

Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

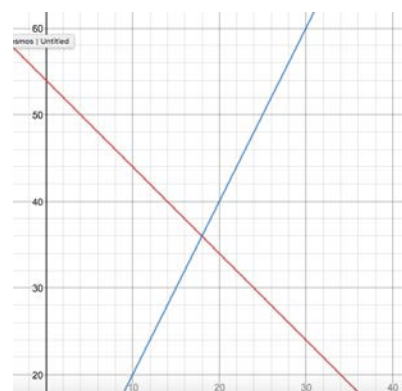
8.EE.B.5

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

8.EE.C.8a

Analyze and solve pairs of simultaneous linear equations.

- Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.**



Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.D.10

Understand that the **graph of an equation** in two variables is the set of **all** its solutions plotted in the coordinate plane, often forming a curve (which could be a line). *(Content Emphases: Major)*

Grade/Course Level Connections:

A-CED.A.2

Instructional Considerations:

- Beginning with simple, real-world examples, help students to recognize a graph as a set of solutions to an equation. For example, if the equation $y = 6x + 5$ represents the amount of money paid to a babysitter (i.e., \$5 for gas to drive to the job and \$6/hour to do the work), then every point on the line represents an amount of money paid, given the amount of time worked.

Common Misconceptions:

- Students may believe that only the points they create are solutions instead of recognizing that the graph represents all solutions.
- Students may believe that a table of values such as that shown on a graphing calculator represents the only solutions possible and should realize that the graph of the function represents all solutions.

Academic Vocabulary

solutions, solution set, graphical solution, line, represent

Standards for Mathematical Practice:

- (4) Model with mathematics
- (6) Attend to precision

Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.D.11

Explain why the **x-coordinates** of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the **solutions** of the equation $f(x) = g(x)$; **find the solutions approximately**, e.g., using technology to graph the functions, **make tables of values**, or **find successive approximations**. Include cases where $f(x)$ and/or $g(x)$ are **linear**, **polynomial**, rational, **absolute value**, **exponential**, and logarithmic functions. *(Content Emphases: Major)*

Explanations and/or Examples

Find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.

Example 1: Given $3x - 2 = (x - 3)^2 - 1$, use **successive approximations** to estimate solution(s).

Ex. 1 Solution: There is a solution that is greater than 1 and less than 2. There is a second solution that is greater than 7 and less than 8.

x	 $3x - 2$	 $(x - 3)^2 - 1$
-2	-8	24
-1	-5	15
0	-2	8
1	1	3
2	4	0
3	7	-1
4	10	0
5	13	3
6	16	8
7	19	15
8	22	24

x	 $3x - 2$	 $(x - 3)^2 - 1$
0	-2	8
.5	-0.5	5.25
1	1	3
1.5	2.5	1.25
2	4	0
2.5	5.5	-0.75
3	7	-1
6	16	8
6.5	17.5	11.25
7	19	15
7.5	20.5	19.25
8	22	24
8.5	23.5	29.25

Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

8.EE.C.8a-b

Analyze and solve pairs of simultaneous linear equations.

- Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.**
- Solve systems of two linear equations in two variables algebraically, and **estimate solutions by graphing the equations**. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.

Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.D.11

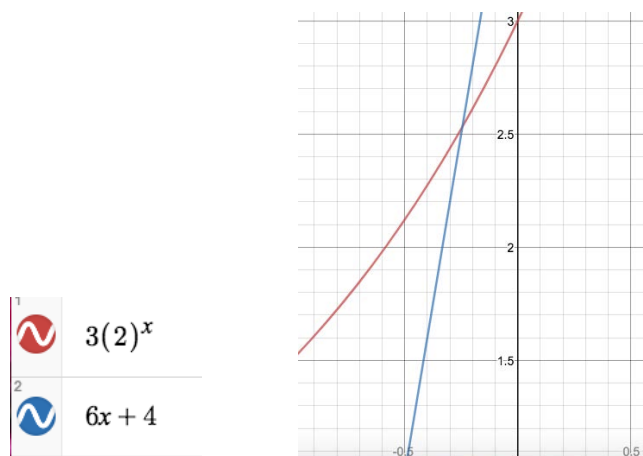
Explain why the **x-coordinates** of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the **solutions** of the equation $f(x) = g(x)$; **find the solutions approximately**, e.g., using technology to graph the functions, **make tables of values**, or **find successive approximations**. Include cases where $f(x)$ and/or $g(x)$ are **linear**, **polynomial**, rational, **absolute value**, **exponential**, and logarithmic functions. *(Content Emphases: Major)*

Explanations and/or Examples

Example 2:

Solve the following by graphing. $3(2^x) = 6x + 4$. Give your answer to the nearest tenth.

Ex. 2 Solution: $x = -0.2$



Prerequisite Examples

Grade 8

8.EE.C.8a

Two lines are graphed on the same coordinate plane. The lines only intersect at the point (3, 6). Which of these systems of linear equations could represent the two lines? Circle all that apply.

$$\begin{cases} x = 3 \\ y = 6 \\ y = 3x - 3 \\ y = x - 1 \\ y = x + 3 \\ y = 2x \end{cases}$$

$$\begin{cases} x = 6 + y \\ y = 3 + x \\ x = 3 + y \\ y = 6 + x \end{cases}$$

8.EE.C.8b

The equation of line s is $y = \frac{1}{3}x - 3$. The equation of line t is $y = -x + 5$. The equations of lines s and t form a system of equations. Graph line s and line t . What is the solution of the system of equations?

Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.D.11

Explain why the **x-coordinates** of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the **solutions** of the equation $f(x) = g(x)$; **find the solutions approximately**, e.g., using technology to graph the functions, **make tables of values**, or **find successive approximations**. Include cases where $f(x)$ and/or $g(x)$ are **linear, polynomial, rational, absolute value, exponential**, and logarithmic functions. *(Content Emphases: Major)*

Grade/Course Level Connections:

A-REI.C.5 and A-REI.C.6

Instructional Considerations:

- Students need to understand numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically.
- Students may use graphing calculators or programs to generate tables of values, graph, or solve for a variety of functions. Using technology, have students graph an equation and use the trace function to move the cursor along the curve. Discuss the meaning of the ordered pairs that appear at the bottom of the calculator, emphasizing that every point on the curve represents a solution to the equation.
- Begin with simple linear equations and how to solve them using the graphs and tables on a graphing calculator. Later, advance students to nonlinear situations so they can see that even complex equations that might involve quadratics or absolute value can be solved fairly easily using this same strategy. While a standard graphing calculator does not simply solve an equation for the user, it can be used as a tool to approximate solutions.
- Use the table function on a graphing calculator to solve equations. For example, to solve the equation $2x - 9 = x + 12$, students can examine the equations $y = 2x - 9$ and $y = x + 12$. They can then determine that they intersect when $x = 21$ by examining the table to find where the y-values are the same.
- Explore visual ways to solve an equation such as $2^x = x^2 - 7$ by graphing the functions $f(x) = 2^x$ and $g(x) = x^2 - 7$.

Common Misconceptions:

- Students may believe that solutions must be all data in a table of values or all graphical data.
- Students may believe that solutions must be a single number (e.g. $x = 3$).
- Students may believe that graphing linear and other functions is an isolated skill.
- Students may not realize that multiple graphs can be drawn in the same coordinate plane for the purpose of solving equations involving those functions (e.g. by graphing $y = 2x - 6$ and $y = x + 3$ in the same coordinate plane, the solution to $2x - 6 = x + 3$ can be identified).

Academic Vocabulary

solution, table of values, linear system, system of equations, exact solution, estimate, approximation, successive approximations

Standards for Mathematical Practice:

- (4) Model with mathematics
- (5) Use appropriate tools strategically

Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.D.12

Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and **graph** the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. *(Content Emphases: Major)*

Explanations and/or Examples

Example 1:

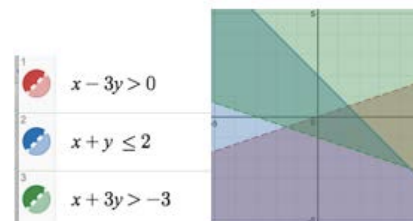
Graph the system of linear inequalities below and determine if (3, 2) is a solution to the system.

$$x - 3y > 0$$

$$x + y \leq 2$$

$$x + 3y > -3$$

Ex. 1 Solution:



(3, 2) is not solution. It is not within or on an acceptable boundary of the commonly shaded regions. Nor is it a solution to all three of the inequalities. Also, (3,2) is not a solution to $x + y \leq 2$.

Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

8.EE.B.5

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

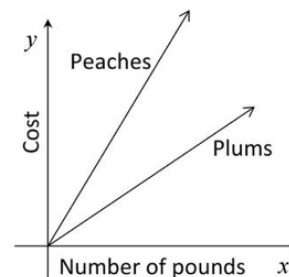
Prerequisite Examples

Grade 8

8.EE. B.5

The graphs below show the cost y of buying x pounds of fruit. One line models the cost of buying x pounds of peaches, and the other represents the cost of buying x pounds of plums.

- Bananas cost less per pound than peaches or plums. Draw a line on the graph that might represent the cost y of buying x pounds of bananas.



Algebra 1 Instructional Support Tool - Algebra

Standard A-REI.D.12

Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and **graph** the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. *(Content Emphases: Major)*

Building Blocks (Grades 6 to 8 prerequisites):

Grade 6

6.EE.B.8

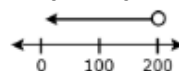
Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. **Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions;** represent solutions of such inequalities on number line diagrams.

Prerequisite Examples

Grade 6

6.EE.B.8

The Flores family spent less than \$200.00 last month on groceries. Does the following graph represent the amounts of money that the Flores family could have spent on groceries last month? Why or why not? Are there infinitely many solutions? Explain.



Grade/Course Level Connections:

A-CED.A.3 and A-REI.C.6

Instructional Considerations:

- Students should not shade above or below a line *based on the direction of the inequality symbol*. Students should test a point to determine whether or not that region contains the solutions to the inequality. This reinforces their understanding of representing solutions graphically and confirming solutions algebraically.
- Investigate real-world examples of two-variable inequalities in a variety of contexts.
- Students should think about boundary lines in terms of solutions. They must distinguish between when the boundary line is included in the solution (solid line) versus when the boundary line is not included in the solution (dashed line).

Common Misconceptions:

- Students may believe that they can determine whether solutions lie (and whether to shade the region) above or below a line *based on the direction of the inequality symbol*.
- Students may believe that two-variable inequalities have no application in the real world. Teachers can consider business related problems (e.g., linear programming applications) to engage students in discussions of how the inequalities are derived and how the feasible set includes all the points that satisfy the conditions stated in the inequalities.

Academic Vocabulary

system of inequalities, boundaries, conditions, solution

Standards for Mathematical Practice:

- (1) Make sense of problems and persevere in solving them
- (4) Model with mathematics

Algebra 1 Instructional Support Tool - Algebra

Standard A-SSE.A.1

Interpret expressions that represent a quantity in terms of its context (*Content Emphases: Major*)

a. **Interpret** parts of an expression, such as terms, factors, and coefficients.

b. **Interpret** complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)n$ as the product of P and a factor not depending on P .*

Explanations and/or Examples

Example 1:

A company uses two different-sized trucks to deliver sand. The first truck can transport x cubic yards, and the second y cubic yards. The first truck makes S trips to a job site, while the second makes T trips. What quantities do the following expressions represent in terms of the problem's context?

- a) $S + T$
- b) $x + y$
- c) $xS + yT$
- d) $(xS + yT)(S + T)$

Ex. 1 Solution

[Task 531 by IM](#) by [CC BY-NC-SA 4.0](#)

Example 2:

A candy shop sells a box of chocolates for \$30. It has \$29 worth of chocolates plus \$1 for the box. The box includes two kinds of candy: caramels and truffles. Lita knows how much the different types of candies cost per pound and how many pounds are in a box. She said,

If x is the number of pounds of caramels included in the box and y is the number of pounds of truffles in the box, then I can write the following equations based on what I know about one of these boxes: $x + y = 3$ and $8x + 12y + 1 = 30$

Assuming Lita used the information given and her other knowledge of the candies, use her equations to answer the following:

- a) How many pounds of candy are in the box?
- b) What is the price per pound of the caramels?
- c) What does the term $12y$ in the second equation represent?
- d) What does $8x + 12y + 1$ in the second equation represent?

Ex. 2 Solution

[Task 389 by IM](#) by [CC BY-NC-SA 4.0](#)

Example 3:

In a carefully controlled biology lab, a population of 100 bacteria reproduces via *binary fission*. That is, every hour, on the hour, each bacteria splits into two bacteria. In the next lab over, a population of *protists* reproduces hourly according to *multiple fission*. The function which gives the population of *protists* after t hours is $P(t) = 50 \cdot 3^t$. Interpret the significance of the numbers 50 and 3 in the context of the biological experiment.

Ex. 3 Solution

[Task 2116 by IM](#) by [CC BY-NC-SA 4.0](#)

Algebra 1 Instructional Support Tool - Algebra

Standard A-SSE.A.1

Interpret expressions that represent a quantity in terms of its context (*Content Emphases: Major*)

a. **Interpret** parts of an expression, such as terms, factors, and coefficients.

b. **Interpret** complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P .*

Building Blocks (Grades 6 to 8 prerequisites):

Grade 7

7.EE.A.2

Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05.”*

Grade 6

6.EE.A.2b

Write, read, and evaluate expressions in which letters stand for numbers.

- b. **Identify** parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.*

Grade/Course Level Connections:

F-LE.B.5

Instructional Considerations:

- Extending beyond simplifying an expression, this cluster addresses interpretation of the components in an algebraic expression. A student should recognize that in the expression $2x + 1$, “2” is the coefficient, “2” and “x” are factors, and “1” is a constant, as well as “2x” and “1” being terms of the binomial expression.
- Development and proper use of mathematical language is an important building block for future content.
- Using real-world context examples, the nature of algebraic expressions can be explored. For example, suppose the cost of cell phone service for a month is represented by the expression $0.40s + 12.95$. Students can analyze how the coefficient of 0.40 represents the cost of one minute (40¢), while the constant of 12.95 represents a fixed, monthly fee, and s stands for the number of cell phone minutes used in the month. Similar real-world examples, such as tax rates, can also be used to explore the meaning of expressions.

Prerequisite Examples

Grade 7

7.EE.A.2:

Jamie and Ted both get paid an equal hourly wage of \$9 per hour. This week, Ted made an additional \$27 dollars in overtime. Write an expression that represents the weekly wages of each person. Let J = the number of hours that Jamie worked this week and T = the number of hours Ted worked this week What is another way to write the expression?

Grade 6

6.EE.A.2b:

The expression $c + 0.07c$ can be used to find the total cost of an item with 7% sales tax. What does each term in the expression represent?

Common Misconceptions:

- Students may believe that the use of algebraic expressions is merely the abstract manipulation of symbols. Use examples of various real-world context that allow students to construct meaning of the parts of algebraic expressions is needed to counter this misconception.
- Students may believe that there is only one way to represent relationships and may require support in recognizing that two different expressions may represent the same relationship (for example, $P = 2(L + W)$ and $P = 2L + 2W$).
- Students may believe that expressions can be solved in the way that equations can be solved.

Algebra 1 Instructional Support Tool - Algebra

Standard A-SSE.A.1

Interpret expressions that represent a quantity in terms of its context (*Content Emphases: Major*)

a. **Interpret** parts of an expression, such as terms, factors, and coefficients.

b. **Interpret** complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P .*

Academic Vocabulary

expression, terms, factor, coefficient, sum, product, quotient, dependent, relationship

Standards for Mathematical Practice:

- (2) Reason abstractly and quantitatively
- (7) Look for and make use of structure

Algebra 1 Instructional Support Tool - Algebra

Standard A-SSE.A.2

Use the structure of an expression to **identify ways to rewrite** it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. (Content Emphases: Major)

Explanations and/or Examples

Example 1: Factor the quadratic expression $t^2 - 12t + 36$.

Ex. 1 Solution: $(t - 6)(t - 6)$

Example 2: Find a value for a , a value for k , and a value for n , so that $(3x + 2)(2x - 5)$ is equivalent to $ax^2 + kx + n$.

Ex. 2 Solution: Using the distributive property $(3x + 2)(2x) + (3x + 2)(-5)$
 $= 6x^2 + 4x - 15x - 10$
 $= 6x^2 - 11x - 10$

So, $a = 6$, $k = -11$, and $n = -10$

Example 3: Write two expressions that are equivalent forms of the expression $m^4 + 5m^2 + 4$

Ex. 3 Solution: $(m^2)^2 + 5(m^2) + 4$; $(m^2 + 4)(m^2 + 1)$

Building Blocks (Grades 6 to 8 prerequisites):

Grade 7

7.EE.A.1

Apply properties of operations as strategies to **add, subtract, factor, and expand linear expressions** with rational coefficients.

Grade 6

6.EE.A.3

Apply the properties of operations to generate equivalent expressions.

For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.

Prerequisite Examples

Grade 7

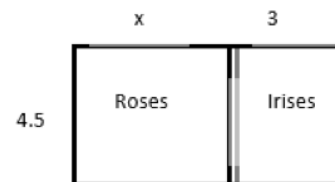
7.EE.A.1

Write an equivalent expression for: $-3(x + 4) - 2(-x + 2)$

Grade 6

6.EE.A.3

Given that the width of the flower bed shown is 4.5 units and the length can be represented by $x + 3$ units, the area of the flower bed can be expressed as $4.5(x + 3)$ square units. Write an equivalent expression that represents the area of the flower bed.



Grade/Course Level Connections:

n/a

Algebra 1 Instructional Support Tool - Algebra

Standard A-SSE.A.2

Use the structure of an expression to **identify ways to rewrite** it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. (Content Emphases: Major)

Instructional Considerations:

- Emphasize that when two expressions are equivalent, an equation relating the two is called an identity because it is true for all values of the variables.
- Highlight polynomial identities that are commonly used in solving problems, and work toward familiarity with special products such as those involving squares and cubes.

Common Misconceptions:

- Students may also believe that an expression cannot be factored because it does not fit a form that they recognize. Support students with *reorganizing* the terms whereby the structures become evident.
- Students may believe that factoring out and dividing by a monomial will result in the same expression. For example, given $12x - 6 = 9$ factoring out 6 results in $2x - 1 = 9$ instead of $6(2x - 1) = 9$.
- Students may believe that, in some instances, they can combine terms that are not like terms. For example, $6x + 2y$ is equivalent to $8xy$.
- Students may believe that the order in which terms are written when an expression contains a difference of terms doesn't matter. For example, $6z - 5y$ is the same as $5y - 6z$.
- Students may believe that the rules of exponents do not apply when using the distributive property. For example, $2t(5t - 4)$ is equivalent to $10t - 8t$ rather than $10t^2 - 8t$.

Academic Vocabulary

trinomial, perfect square trinomial, difference of squares, polynomial identity

Standards for Mathematical Practice:

- (2) Reason abstractly and quantitatively
- (7) Look for and make use of structure

Algebra 1 Instructional Support Tool - Algebra

Standard A-SSE.B.3a-b

Choose and produce an equivalent form of an expression to **reveal and explain properties** of the quantity represented by the expression. *(Content Emphases: Supporting)*

a. **Factor a quadratic expression to reveal the zeros** of the function it defines

b. **Complete the square** in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Explanations and/or Examples:

Factor a quadratic expression to reveal the zeros

Example 1:

The expression $-4x^2 + 8x + 12$ represents the height of a coconut thrown from a person in a tree to a basket on the ground. x is the number of seconds since the coconut was thrown.

- Rewrite the expression to reveal the linear factors
- Identify the zeros of the expression and interpret their meaning in the context of the problem

Ex.1 Solution

- $-4(x - 3)(x + 1)$
- $x = 3, -1$ are the zeros. Only $x = 3$ is meaningful in context and represents that the coconut hit the ground 3 seconds after it was thrown.

Building Blocks (Grades 6 to 8 prerequisites):

Grade 7

7.EE.A.1

Apply properties of operations as strategies to add, subtract, **factor**, and expand **linear expressions** with rational coefficients.

Grade 6

6.EE.A.3

Apply the properties of operations to **generate equivalent expressions**. For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.

Grade/Course Level Connections:

A-REI.B.4 and A-SSE.A.2

Example 2:

The quadratic expression $-x^2 + 24x + 255$ models the height, in centimeters, of a ball thrown vertically with x representing the number of seconds since the ball was thrown. Determine the vertex-form of the expression, determine the vertex from the rewritten form, and interpret the meaning of the vertex in context.

Ex. 2 Solution

$-(x - 12)^2 - 399$, the vertex is $(12, 399)$. The ball reaches its maximum height of 399 centimeters 12 seconds after it was thrown

Prerequisite Examples:

Grade 7

7.EE.A.1

Factor: $2m - \frac{2}{3}$

Grade 6

6.EE.A.3

Write an equivalent expression for $7(2x + 9)$

Algebra 1 Instructional Support Tool - Algebra

Standard A-SSE.B.3a-b

Choose and produce an equivalent form of an expression to **reveal and explain properties** of the quantity represented by the expression. (*Content Emphases: Supporting*)

a. **Factor a quadratic expression to reveal the zeros** of the function it defines

b. **Complete the square** in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Instructional Considerations

- It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.
- Teachers foster the idea that changing the forms of expressions, such as factoring or completing the square, are *not independent algorithms* that are learned for the sake of symbol manipulations. They are processes that are guided by goals (e.g., investigating properties of families of functions and solving contextual problems).
- These standards focus on linking expressions and functions, i.e., creating connections between multiple representations of functional relations – the dependence between a quadratic expression and a graph of the quadratic function it defines.
- Students use completing the square to rewrite a quadratic expression in the form $y = a(x - h)^2 + k$ to identify the vertex of the parabola (h, k) . They explain the meaning of the vertex in context and understand that they use this form when looking for the maximum or minimum value.
- Students, given a quadratic function, explain the meaning of the zeros of the function. That is if $f(x) = (x - c)(x - a)$ then $f(a) = 0$ and $f(c) = 0$.
- Students, given a quadratic expression, explain the meaning of the zeros graphically. That is, for an expression $(x - a)(x - c)$, a and c correspond to the x -intercepts (if a and c are real).
- Factoring methods that are typically introduced in elementary algebra and the method of completing the square reveals attributes of the graphs of quadratic functions, represented by quadratic equations.
 - The solutions of quadratic equations solved by factoring are the x – intercepts of the parabola or zeros of quadratic functions.
 - A pair of coordinates (h, k) from the general form $f(x) = a(x - h)^2 + k$ represents the vertex of the parabola, where h represents a horizontal shift and k represents a vertical shift of the parabola $y = x^2$ from its original position at the origin.
 - A vertex (h, k) is the minimum point of the graph of the quadratic function if $a > 0$ and is the maximum point of the graph of the quadratic function if $a < 0$. Understanding an algorithm of completing the square provides a solid foundation for deriving a quadratic formula.
- Translating among different forms of expressions, equations and graphs helps students to understand some key connections among arithmetic, algebra and geometry. Have students derive information about a function's equation, represented in standard, factored or general form, by investigating its graph.

Common Misconceptions:

- Some students may believe that factoring and completing the square are isolated techniques within a unit of quadratic equations. Teachers should help students to see the value of these skills in the context of solving higher degree equations and examining different families of functions.
- Students may think that the minimum (the vertex) of the graph of $y = (x + 5)^2$ is shifted to the right of the minimum (the vertex) of the graph $y = x^2$ due to the addition sign. Students should explore examples both analytically and graphically to overcome this misconception.
- Some students may believe that the minimum of the graph of a quadratic function always occur at the y -intercept. Students should explore examples both analytically and graphically to overcome this misconception.

Algebra 1 Instructional Support Tool - Algebra

Standard A-SSE.B.3a-b

Choose and produce an equivalent form of an expression to **reveal and explain properties** of the quantity represented by the expression. *(Content Emphases: Supporting)*

a. **Factor a quadratic expression to reveal the zeros** of the function it defines

b. **Complete the square** in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Academic Vocabulary

complete the square, factor, zeros, x-intercept, vertex-form, vertex, maximum, minimum, parabola

Standards for Mathematical Practice:

- (2) Reason abstractly and quantitatively
- (7) Look for and make use of structure

Algebra 1 Instructional Support Tool - Algebra

Standard A-SSE.B.3c

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. (Content Emphases: Supporting)
c. Use the properties of exponents to **transform expressions** for exponential functions. For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Explanations and/or Examples:

Example 1: After a container of ice cream has been sitting in a room for t minutes, its temperature in degrees Fahrenheit is $a - b2^{-t} + b$, where a and b are positive constants. Write this expression in a form that:

- c) Shows that the temperature is always less than $a + b$.
- d) Shows that the temperature is never less than a .

Ex.1 Solution

[Task 551 by IM](#) by [CC BY-NC-SA 4.0](#)

Example 2: Two physicists describe the amount of a radioactive substance, Q in grams, left after t years:

- c) $Q = 300 \cdot 0.9439^t$
- d) $Q = 252.290 \cdot 0.9439^{t-3}$

(i) Show that the expressions describing the radioactive substance are all equivalent (using appropriate rounding).

(ii) What aspect of the decay of the substance does each of the formulas highlight?

Ex.2 Solution

[Task 1305 by IM](#) by [CC BY-NC-SA 4.0](#)

Grade/Course Level Connections:

A-SSE.A.2

Instructional Considerations

- Students first choose a form of an expression. They then seek to produce it.
- Engage students in purposeful discourse about why a particular form of an exponential might be chosen over another in a given situation.
- Note the stated expectation that students use the properties of exponents. Thus rewriting $C(t) = 20,000(0.75)^t$ as $C(t) = 20,000(1 - 0.25)^t$ does not require use of the properties of exponents and does not reflect the transforming of expressions described in the standard. (See 7.EE.A.2).
- Note that in Algebra 1, students work with integer exponents only.

Common Misconceptions:

- Students may believe that the properties of exponents apply to numerical expressions but do not apply to algebraic expressions.

Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

8.EE.A.1

Know and **apply the properties of integer exponents** to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Prerequisite Examples:

Grade 8

8.EE.A.1: Simplify each expression. Select all that have a value between 0 and 1

- a) $8^7 \cdot 8^{-12}$
- b) $\frac{7^4}{7^3}$
- c) $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^9$
- d) $\frac{(-5)^6}{(-5)^{10}}$

Algebra 1 Instructional Support Tool - Algebra

Standard A-SSE.B.3c

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *(Content Emphases: Supporting)*
c. Use the properties of exponents to **transform expressions** for exponential functions. *For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Academic Vocabulary

exponential expression, exponential function

Standards for Mathematical Practice:

- (5) Use appropriate tools strategically
- (7) Look for and make use of structure