



## New Jersey Student Learning Standards *Mathematics*

### Algebra 1 Instructional Support Tool for Teachers: **Functions**

This instructional support tool is designed to assist educators in interpreting and implementing the New Jersey Student Learning Standards for Mathematics. It contains explanations or examples of each Algebra 1 course standard to answer questions about the standard's meaning and how it applies to student knowledge and performance. To ensure that descriptions are helpful and meaningful to teachers, this document also identifies grades 6 to 8 prerequisite standards upon which each Algebra 1 standard builds. It includes the following: sample items aligned to each identified prerequisite; identification of course level connections; instructional considerations and common misconceptions; sample academic vocabulary and associated standards for mathematical practice. **Examples are samples only** and should not be considered an exhaustive list.

This instructional support tool is considered a living document. The New Jersey Department of Education believe that teachers and other educators will find ways to improve the document as they use it. Please send feedback to [mathematics@doe.state.nj.us](mailto:mathematics@doe.state.nj.us) so that the Department may use your input when updating this guide.

Please consult the [New Jersey Student Learning Standards for Mathematics](#) for more information.

## Functions Standards Overview

Functions describe situations where one quantity determines another. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, “I’ll give you a state, you give me the capital city”; by an algebraic expression like  $f(x) = a + bx$ ; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can shed light on the function’s properties.

### Interpreting Functions (F-IF)

<b>Understand the concept of a function and use function notations:</b>	<a href="#">F-IF.A.1</a>	<a href="#">F-IF.A.2</a>	<a href="#">F-IF.A.3</a>
<b>Interpret functions that arise in applications in terms of the context:</b>	<a href="#">F-IF.B.4</a>	<a href="#">F-IF.B.5</a>	<a href="#">F-IF.B.6</a>
<b>Interpret linear models:</b>	<a href="#">F-IF.C.7a-b</a>	<a href="#">F-IF.C.8a</a>	<a href="#">F-IF.C.9</a>

### Linear, Quadratic, and Exponential Models (F-LE)

<b>Construct and compare linear, quadratic, and exponential models and solve problems:</b>	<a href="#">F-LE.A.1</a>	<a href="#">F-LE.A.2</a>	<a href="#">F-LE.A.3</a>
<b>Interpret expressions for functions in terms of the situation they model:</b>	<a href="#">F-LE.B.5</a>		

### Building Functions (F-BF)

<b>Build a function that models a relationship between two quantities:</b>	<a href="#">F-BF.A.1</a>
<b>Build new functions from existing functions:</b>	<a href="#">F-BF.B.3</a>

### Mathematical Practices

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.A.1.

**Understand** that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ . (Content Emphases: Major)

### Explanations and/or Examples

Given the function  $(m)$ , students explain that input values are guaranteed to produce unique output values and use the function rule to generate a table or graph. They identify  $m$  as an element of the domain, the input, and  $(m)$  as an element in the range, the output. The domain of a function given by an algebraic expression, unless otherwise specified, is the largest possible domain. Students recognize that the graph of the function,  $f$ , is the graph of the equation  $y = (m)$  and that  $(m, (m))$  is a point on the graph of  $f$ .

**Example 1:** A pack of pencils cost \$0.75. If  $n$  number of packs are purchased, then the total purchase price is represented by the function  $t(n) = 0.75n$ .

- Explain why  $t$  is a function.
- What is a reasonable domain and range for the function?

### Ex. 1 Solution:

- $t$  is a function because any input (element of the domain) yields exactly one unique output (element of the range).
- A reasonable domain is any whole number. A reasonable range is any multiple of .75 that is greater than or equal to zero.

### Prerequisite Examples:

Grade 8

#### 8.F.A.1:

Determine if the graph represents a function. Justify your reasoning. Determine which of the tables represents a function. Explain why.

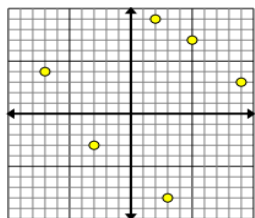


Table A

x	f(x)
0	1
1	2
2	2
3	4

Table B

x	f(x)
0	0
1	2
1	3
4	5

### Building Blocks (grades 6 to 8 prerequisites):

Grade 8

#### 8.F.A.1

Understand that **a function is a rule that assigns to each input exactly one output**. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

#### 8.F.A.2

Compare properties (e.g. rate of change, intercepts, domain and range) of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

#### 8.F.A.3

Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line; **give examples of functions that are not linear**. *For example, the function  $A = s^2$  giving the area of a square as a function of its side length is not linear because its graph contains the points  $(1,1)$ ,  $(2,4)$  and  $(3,9)$ , which are not on a straight line.*

## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.A.1.

**Understand** that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ . (Content Emphases: Major)

### Prerequisite Examples:

Grade 8

#### 8.F.A.2:

Compare the following functions to determine which has the greater rate of change:

Function 1:  $f(x) = 2x + 4$

Function 2

$x$	$f(x)$
-1	6
0	-3
2	3

#### 8.F.A.3:

Determine which of the functions listed below are linear and which are not linear. Explain your reasoning for each.

- $y = -2x^2 + 3$
- $y = 2x$
- $A = \pi r^2$
- $y = 0.25 + 0.5(x - 2)$

### Grade/Course Level Connections:

n/a

### Instructional Considerations:

- Reinforce that “domain” refers to the set of all possible input values.
- Carefully develop the appropriate language associated with functions. For example,  $f(2)$  means the output or value of the function  $f$  when the input is 2.
- Build upon the idea of a function as an input-output machine. Provide applied contexts in which to explore functions. For example, examine the amount of money earned when given the number of hours worked on a job. Contrast this with a situation in which a single fee is paid by the “carload” of people, regardless of whether 1, 2 or more people are in the car.
- Examine graphs of functions and non-functions. Students should explain *why* the vertical line test works given their understanding of each element of the domain ( $x$ ) being assigned to exactly one element of the range.

### Common Misconceptions:

- Students may believe that all relationships having an input and an output are functions, and therefore, misuse the function terminology.
- Students may believe that the notation  $f(x)$  means to multiply some value  $f$  by another value  $x$ . The notation alone can be confusing and needs careful development. For example,  $f(2)$  means the output value of the function  $f$  when the input value is 2.

### Academic Vocabulary

domain, range, function, independent variable, dependent variable, input, output

## Algebra 1 Instructional Support Tool - FUNCTIONS

### Standard F-IF.A.1.

**Understand** that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ . (*Content Emphases: Major*)

### Standards for Mathematical Practice:

- (2) Reason abstractly and quantitatively
- (6) Attend to precision

## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.A.2.

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (*Content Emphases: Major*)

#### Explanations and/or Examples

**Example 1:** Evaluate  $f(2)$  for the function  $f(m) = 5(m - 3) + 17$ .

**Ex. 1 Solution:**  $f(2) = 12$

**Example 2:** Evaluate  $f(2)$  for the function  $f(m) = 1200(1 + .04)^m$ .

**Ex. 2 Solution:**  $f(2) = 1297.92$

*Interpret statements that use function notation in terms of a context.*

**Example 3:** You placed a yam in the oven and, after 45 minutes, you take it out. Let  $f$  be the function that assigns to each minute after you placed the yam in the oven, its temperature in degrees Fahrenheit. Write a sentence for each of the following to explain what it means in everyday language.

- a)  $f(0) = 65$
- b)  $f(5) < f(10)$
- c)  $f(40) = f(45)$
- d)  $f(45) > f(60)$

#### Ex. 3 Solution

[Task 625 by IM](#) by [CC BY-NC-SA 4.0](#)

**Example 4:** The function  $g(x) = 50(0.85)^x$  represents the amount of a drug in milligrams,  $g(x)$ , which remains in the bloodstream after  $x$  hours. Evaluate and interpret each of the following:

- a)  $g(0)$
- b)  $g(2) = k \cdot g(1)$ . What is the value of  $k$ ?

#### **Ex. 4 Solution:**

- a)  $f(0) = 50$
- b)  $k = .85$

#### Building Blocks (Grades 6 to 8 prerequisites):

*Grade 6*

##### **6.EE.A.2c**

Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas  $V = s^3$  and  $A = 6s^2$  to find the volume and surface area of a cube with sides of length  $s = 1/2$ .*

#### Prerequisite Examples:

*Grade 6*

##### **6.EE.A.2c**

Evaluate the expression  $3x + 2y$  when  $x$  is equal to 4 and  $y$  is equal to 2.4.

Evaluate  $5(n + 3) - 7n$ , when  $n = 1/2$ .

## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.A.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. (*Content Emphases: Major*)

#### Grade/Course Level Connections:

F-IF.A.1

#### Instructional Considerations:

- Ensure that students recognize that once a relation has been determined to be a function, then function notation can be used.
- Decode function notation and explain how the output of a function is matched to its input (e.g., the function  $f(x) = 3^x + 4$  raises 3 to the power of the input, then adds four to produce the output).
- Introduce other variables to represent a function (e.g.  $h(x)$ ,  $k(y)$ ,  $r(t)$  ...)
- Carefully develop the appropriate language associated with functions. For example,  $f(2)$  means the output or value of the function  $f$  when the input is 2.

#### Common Misconceptions:

- Students may believe that the notation  $f(x)$  means to multiply some value  $f$  times another value  $x$ .
- Students may believe that all relationships having an input and an output are functions, and therefore, misuse the function terminology.

#### Academic Vocabulary

function notation, evaluate, function, domain, range

#### Standards for Mathematical Practice:

- (6) Attend to precision
- (7) Look for and make use of structure

## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.A.3

**Recognize** that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by  $f(0) = f(1) = 1$ ,  $f(n+1) = f(n) + f(n-1)$  for  $n \geq 1$ . (Content Emphases: Major)*

### Explanations and/or Examples

A set of numbers arranged in a definite order according to some definite rule is called a sequence. The individual elements in a sequence are called terms. A sequence is also a function and can be written algebraically using function notation.

A sequence is a function, whose input is a subset of the integers and whose output is the terms of the sequence. The most common subset for the domain of a sequence is the Natural numbers  $\{1, 2, 3, \dots\}$ ; however, there are instances where it is necessary to include  $\{0\}$  or possibly negative integers. Whereas, some sequences can be expressed explicitly, there are those that are a function of the previous terms. In which case, it is necessary to provide the first few terms to establish the relationship.

**Example 1:** A theater has 60 seats in the first row, 68 seats in the second row, and 76 seats in the third row. The number of seats in the remaining rows continue in this pattern.

- c) Write the number of seats as a sequence.
- d) What is the domain of the sequence? Explain what the domain represents in context.

### Ex. 1 Solution:

- c) 60, 68, 76, 84, ...
- d) The whole numbers are the domain of the sequence. We start with the first row (is there a theater with zero rows?) and we do not have 1.5 or 2.1 rows so those types of numbers are not part of my domain. It should be a point in which my domain has a limiting number of rows. (I do not know a theater with a 1000 rows).

### Building Blocks (Grades 6 to 8 prerequisites):

n/a

### Prerequisite Examples:

n/a

**Example 2:** In a video game called Snake, a player moves a snake through a square region in the plane, trying to eat the white pellets that appear. If we imagine the playing field as a 32-by-32 grid of pixels, then the snake starts as a 4-by-1 rectangle of pixels, and grows in length as it eats the pellets:

- After the first pellet, it grows in length by one pixel.
- After the second pellet, it further grows in length by two pixels.
- After the third pellet, it further grows in length by three pixels.
- And so on, with the n-th pellet increasing its length by n pixels.

Let  $L(n)$  denote the length of the snake after eating n pellets. For example,  $L(3)=10$ .

How long is the snake after eating 4 pellets? After 5 pellets? After 6 pellets?

### Ex. 2 Solution

[Task 1695 by IM](#) by [CC BY-NC-SA 4.0](#)



## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.A.3

**Recognize** that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by  $f(0) = f(1) = 1$ ,  $f(n+1) = f(n) + f(n-1)$  for  $n \geq 1$ .* (Content Emphases: Major)

### Grade/Course Level Connections:

F-IF.A.1, F-IF.A.2, and F-BF.A.1a

### Instructional Considerations:

- Help students to understand that the word “domain” implies the set of all possible input values and that the integers are a set of numbers made up of  $\{\dots-2, -1, 0, 1, 2, \dots\}$ .
- Students should know that sequences can be defined recursively, or using previous terms to define future terms. For instance, The Fibonacci sequence is a list of numbers where each term is the sum of the two before it. As such, we end up with 1, 1, 2, 3, 5, 8, 13, and

so on. We can define this recursively as  $f(n+1) = f(n) + f(n-1)$ . Once the first two terms are defined, ( $f(0) = f(1) = 1$ ), the sequence can just keep on going.

### Common Misconceptions:

- Students may believe that function notation only shows up on the left side of the equation, however it can be both sides of the equation.
- Students may believe that all relationships having an input and an output are functions, and therefore, misuse the function terminology.

### Academic Vocabulary:

arithmetic sequence, geometric sequence, term, common difference, common ratio, domain, range, recursive, function

### Standards for Mathematical Practice:

- (7) Look for and make use of structure
- (8) Look for and express regularity in repeated reasoning

## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.B.4.

For a function that models a relationship between two quantities, **interpret key features of graphs and tables** in terms of the quantities, and **sketch graphs** showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (Content Emphases: Major)*

### Explanations and/or Examples

Students interpret the key features of the different functions types. When given a table or graph of a function that models a real-life situation, students explain the meaning of the characteristics of the table or graph in the context of the problem.

Key features of an exponential function include y-intercept and increasing/decreasing intervals.

Key features of a linear function include slope and intercepts.

**Example 1:** The local newspaper charges for advertisements in their community section. A customer has called to ask about the charges. The newspaper gives the first 50 words for free and then charges a fee per word. Use the table below to describe how the newspaper charges for the ads. Include all important information.

Advertisement word count vs. cost

# of words	Cost (\$)
50	0.00
60	0.50
70	1.00
80	1.50
90	2.00
100	2.50

#### Ex. 1 Solution:

Table shows that the first 50 words is free. After 50 words, the newspaper charges \$0.50 for every additional 10 words or \$0.05 for every additional word. So the relationship is linear. The slope is 0.05 and the x-intercept is 50.

**Example 2:** Jack planted a mysterious bean just outside his kitchen window. Jack kept a table (shown below) of the plant's growth. He measured the height at 8:00am each day.

- What was the initial height of Jack's plant?
- How is the height changing each day?

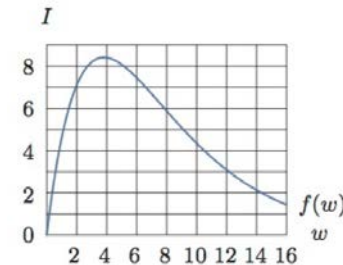
Plant Height over time

Day	Height (cm)
0	2.56
1	6.4
2	16
3	40
4	100

#### Ex. 2 Solution

- The initial height is 2.56 cm.
- Each day, the plant is 2.5 times taller than previous day.

**Example 3:** An epidemic of influenza spreads through a city. The figure below is the graph of  $I=f(w)$ , where  $I$  is the number of individuals (in thousands) infected  $w$  weeks after the epidemic begins.



- Estimate  $f(2)$  and explain its meaning in terms of the epidemic.
- Approximately how many people were infected at the height of the epidemic? When did that occur? Write your answer in the form  $f(a)=b$ .
- For approximately which  $w$  is  $f(w) = 4.5$ ; explain what the estimates mean in terms of the epidemic.
- An equation for the function used to plot the image above is  $f(w)=6w(1.3)^{-w}$ . Use the graph to estimate the solution of the inequality  $6w(1.3)^{-w} \geq 6$ . Explain what the solution means in terms of the epidemic.

#### Ex. 3 Solution

Task 637 by IM by CC BY-NC-SA 4.0

## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.B.4.

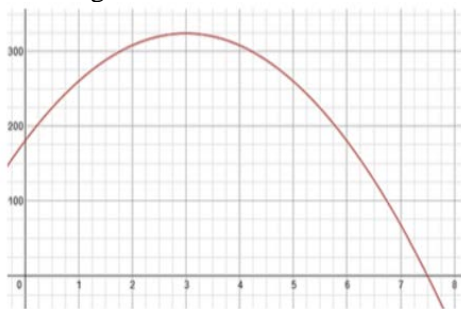
For a function that models a relationship between two quantities, **interpret key features of graphs and tables** in terms of the quantities, and **sketch graphs** showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* (Content Emphases: Major)

### Explanations and/or Examples

Given a function, identify key features in graphs and tables including: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.

Key features of a quadratic function include intervals of increase/decrease, intervals where the function is positive/negative, relative maximum/minimum, symmetry, and intercepts.

**Example 4:** The graph represents the height (in feet) of a rocket as a function of the time (in seconds) since it was launched. Use the graph to answer the following:



- What is the appropriate domain for  $t$  in this context? Why?
- What is the height of the rocket two seconds after it was launched?
- What is the maximum value of the function and what does it mean in context?
- When is the rocket 100 feet above the ground?
- When is the rocket 250 feet above the ground?
- Why are there two answers to part e but only one practical answer for part d?
- What are the intercepts of this function? What do they mean in the context of this problem?
- What are the intervals of increase and decrease on the practical domain? What do they mean in the context of the problem?

### Ex. 4 Solution:

- $0 \leq t \leq 7.5$  Time cannot be negative, so the smallest value is 0. The rocket hits the ground at  $t=7.5$  seconds, There will be no change in height after that.
- Approximately 310 feet
- The maximum value is approximately 325 feet and represents the maximum height reached by the rocket.
- The rocket is 100 feet above the ground at approximately 6.75 seconds
- At approximately .75 seconds and approximately 5.1 seconds, the rocket is 250 feet above the ground.
- There are two times at which the rocket has a height of 250 feet - once as it ascends and again as it descends. There is only one time at which the rocket has a height of 100 feet. The initial height is 175 feet, so the rocket is never at 100 feet as it ascends. Its height is 100 feet only on the descent.
- The y-intercept is 175 feet and represents the initial height of the rocket. The x-intercept is 7.5 seconds and represents the time elapsed before the rocket hits the ground.
- The intervals of increase is  $0 \leq t \leq 3$  and represents the time over which the rocket is ascending to its maximum height. The interval of decrease,  $3 \leq t \leq 7.5$ , represents the interval over which the rocket descends until it hits the ground.

## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.B.4

For a function that models a relationship between two quantities, **interpret key features of graphs and tables** in terms of the quantities, and **sketch graphs** showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* (Content Emphases: Major)

### Explanations and/or Examples

**Example 5:** The table below shows a diver's height above the water (in meters),  $t$  seconds after the diver leaves the springboard.

Height above water over time

$t$ (seconds)	Height (meters)
0	3
.25	5.1875
.5	6.75
.75	7.6875
1	8
1.25	7.6875
1.5	6.75
1.75	5.1875
2	3
2.25	0.1875
2.5	-3.25

- How high above the water is the springboard? Justify your answer.
- When does the diver hit the water? Justify your answer.
- At what time on the diver's descent toward the water is the diver again at the same height as the springboard? Justify your answer.
- When does the diver reach the peak of the dive? Justify your answer.

### Ex. 5 Solution:

- 3 meters.  $t = 0$  seconds indicates the time that the diver will leave the springboard. The corresponding height is 3 meters, so the initial height of the diver above the water is the same as the height of the springboard above the water - 3 meters
- The diver hits the water when the height of the diver above the water is zero. This occurs between  $t = 2.25$  seconds and  $t = 2.5$  seconds.
- 2 seconds: The height of the springboard is 3 meters. At  $t = 2$  seconds, the height of the diver is also 3 meters.
- 1 second. The maximum height would represent the peak of the dive. The maximum height of 8 meters occurs at  $t = 1$  second.

### Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

#### 8.F.B.5

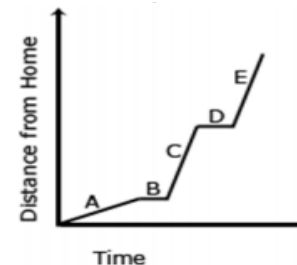
**Describe qualitatively** the functional relationship between two quantities by **analyzing a graph** (e.g., where the function is increasing or decreasing, linear or non-linear). **Sketch a graph that exhibits the qualitative features of a function that has been described verbally.**

### Prerequisite Examples:

Grade 8

#### 8.F.B.5

The graph shows a student's trip to school. This student walks to his friend's house and, together, they ride a bus to school. The bus stops once before arriving at school. Describe how each part A-E of the graph relates to the story.



## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.B.4

For a function that models a relationship between two quantities, **interpret key features of graphs and tables** in terms of the quantities, and **sketch graphs** showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. (Content Emphases: Major)*

### Grade/Course Level Connections:

F-IF.A.1, N-Q.A.1 and F-IF.B.5

### Instructional Considerations

- The expectation at the Algebra I level is to focus on linear and exponential functions. Later in the year, focus on quadratic functions and compare them with linear and exponential functions.
- Students should be given graphs to interpret, as well as required to produce graphs given an expression or table for the function. Students should have experience producing graphs both by hand and using technology.
- Students should flexibly move from examining a graph and describing its characteristics (e.g., intercepts, relative maximums, etc.) to using a set of given characteristics to sketch the graph of a function.
- Students have multiple opportunities to examine a table of related quantities and identify features in the table, such as intervals on which the function increases, decreases, or exhibits periodic behavior.
- Students are provided with many examples of quadratic, functional relationships. Use a variety of real-world contexts, so that students can not only describe what they see in a table, equation, or graph, but also can relate the features to the real-life meanings.

### Academic Vocabulary

intercepts; increasing, decreasing, positive, negative, intervals, relative maximums and minimums, symmetries, end behavior, periodicity

### Standards for Mathematical Practice:

- (2) Reason abstractly and quantitatively
- (4) Model with mathematics

### Common Misconceptions

- Students may believe that the slope of a linear function is merely a number used to sketch the graph of the line. In reality, slopes have real-world meaning, and the idea of a rate of change is fundamental to understanding major concepts from geometry to calculus.
- Students may believe that a straight line connects points when sketching quadratics and resist drawing a smooth curve.
- For projectile motion, students may believe that the coefficient of the  $x^2$  (or  $t^2$ ) term is independent of the units of measure. ( $-9.8 \text{ m/s}^2$ ,  $-16 \text{ ft/s}^2$ ).
- For projectile motion problems, students may not make the distinction between height above the ground and distance from point of release.

## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.B.5

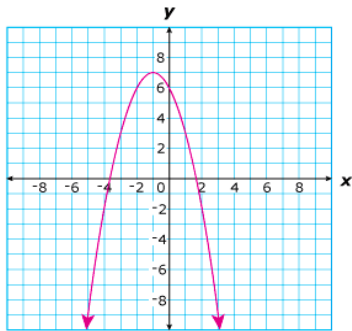
**Relate** the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.* (Content Emphases: Major)

### Examples and/or Explanations

Students relate the domain of a function to its graph and, where applicable, the quantitative relationship it describes.

#### Example 1:

Which could be the domain of the function shown in the graph below so that the range is  $-2 \leq y \leq 7$ ?



- a)  $-4 \leq x \leq 2$
- b)  $-2 \leq x \leq 4$
- c)  $-5 \leq x \leq 3$
- d)  $0 \leq x \leq 2$

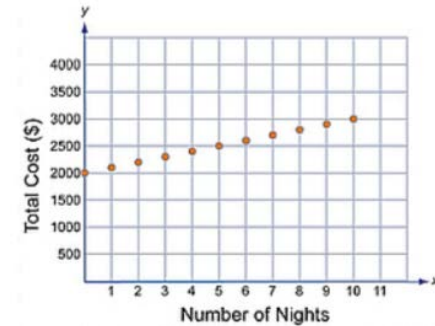
#### Ex. 1 Solution: a

**Example 2:** A company has developed a new video game console. After completing cost analysis and demand forecasts, the company has determined that the profit function for the new console is  $f(g) = -250g^2 + 70,000g - 4,570,000$  where  $g$  is the number of consoles sold. What is the domain of the profit function?

- a) all integers
- b) all rational numbers
- c) all integers greater than or equal to 0
- d) all rational numbers greater than or equal to 0

#### Ex 2 Solution: c

**Example 3:** An all-inclusive resort in Los Cabos, Mexico provides everything for their customers during their stay including food, lodging, and transportation. Use the graph below to describe the domain of the total cost function.



#### Ex. 3 Solution:

The domain for the total cost function is  $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ .

**Example 4:** A hotel has 10 stories above ground and 2 levels in its parking garage below ground. What is an appropriate domain for a function,  $T(n)$ , that gives the average number of times an elevator in the hotel stops at the  $n^{\text{th}}$  floor each day?

**Ex. 4 Sample Solution:** Students should recognize that  $n=0$  is not in the domain. Buildings don't have a 0 floor, so the domain is strictly  $\{-2, -1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ .

## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.B.5

**Relate** the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.* (Content Emphases: Major)

#### Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

##### 8.F.A.1

Understand that **a function is a rule that assigns to each input exactly one output**. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

#### Prerequisite Examples

Grade 8

##### 8.F.A.1

Given the table below, determine an ordered pair that will make the relation not a function.

Table: Relation or Function

x	y
-5	-1
-6	9
5	2
3	6
9	6

#### Grade/Course Level Connections:

F-IF.A.1 and F-IF.B.4

#### Instructional Considerations

- Ensure that students recognize *appropriate* domains of functions in real-world settings. For example, when determining a weekly salary based on hours worked, the hours (input) could be a rational number, such as 25.5. However, if a function relates the number of cans of soda sold in a machine to the money generated, the domain must consist of whole numbers.
- Provide students with *many* examples of quadratic, functional relationships. Use a variety of real-world contexts, so that students can describe what they see in a graph while relating the features to the real-life meanings.
- Students should examine natural restrictions on the domain because of the context (e.g. time cannot be negative) as well as restrictions on

the domain because of modeling constraints.

#### Common Misconceptions

- Students may believe that restrictions on the domain only depend upon whether a quantity must be positive given the context.
- Students may also believe that it is reasonable to input any x-value into a function. Students should examine multiple situations in which there are various limitations to the domains.

#### Academic Vocabulary:

function, domain, interval, relationship, restriction, constraint

#### Standards for Mathematical Practice:

- (2) Reason abstractly and quantitatively
- (4) Model with Mathematics



## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.B.6

**Calculate and interpret the average rate of change** of a function (presented symbolically or as a table) over a specified interval. **Estimate** the rate of change from a graph. (Content Emphases: Major)

### Examples and/or Explanations

The average rate of change of a function  $y=f(x)$  over an interval  $[a,b]$  is

$$\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$$

In addition to finding average rates of change from functions given symbolically, graphically, or in a table, students may collect data from experiments or simulations (ex. falling ball, velocity of a car, etc.) and find average rates of change for the function modeling the situation.

#### Example 1:

The plug is pulled in a small hot tub. The table gives the volume of water in the tub from the moment the plug is pulled, until it is empty.

Table: Volume over time

Time (s)	Volume (L)
0	1600
10	1344
20	1111
30	900
40	711
50	544
60	400
70	278
80	178
90	100
100	44
110	11
120	0

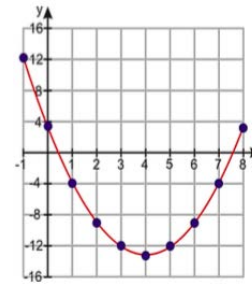
What is the average rate of change between:

- 60 seconds and 100 seconds?
- 0 seconds and 120 seconds?
- 70 seconds and 110 seconds?

#### Ex. 1 Solutions:

- 8.9 per second
- 13.3 per second
- 6.67 per second

**Example 2:** For the quadratic represented below, find the average rate of change over the interval  $[1, 4]$ .



**Ex. 2 Solution:**  $-8/3$

**Example 3:** Ian tosses a bone up in the air for his dog, Spot. The height,  $h$ , in feet, that Spot is above the ground at the time  $t$  seconds after she jumps for the bone can be represented by the function  $h(t) = -16t^2 + 3t$ . What is Spot's average rate of ascent, in feet per second, from the time she jumps into the air to the time she catches the bone at  $t=3$  seconds?

**Ex. 3 Solution:** The average rate of ascent is 2.52 ft/sec



## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.B.6

**Calculate and interpret the average rate of change** of a function (presented symbolically or as a table) over a specified interval. **Estimate** the rate of change from a graph. *(Content Emphases: Major)*

#### Building Blocks (Grades 6 to 8 prerequisites)

Grade 8

##### 8.F.B.4

Construct a function to model a linear relationship between two quantities. **Determine the rate of change and initial value** of the function **from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph.** Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

#### Prerequisite Examples

Grade 8

##### 8.F.B.4

A car is traveling down a long, steep hill. The elevation,  $E$ , above sea level (in feet) of the car when it is  $d$  miles from the top of the hill is shown in the table below. Determine the rate of change and the initial value of this function and explain what they mean in the context of the moving car.

Elevation over time

Miles (d)	Elevation (E)
0	7500
1	7250
2	7000
3	6750
4	6500
5	6250

#### Grade/Course Level Connections:

F-IF.A.2

#### Instructional Considerations

- Begin by focusing on linear and exponential functions whose domain is a subset of the integers. Later, focus on quadratic functions and compare them with linear and exponential functions. In Algebra 2, students will extend this standard to address other types of functions.
- Given a table of values, such as height of a plant over time, students can estimate the rate of plant growth. Also, if the relationship is expressed as a linear equation, students explain the meaning of the slope of the line. If the relationship is illustrated graphically, students may select points on the graph to determine the rate of change over a given interval.
- For non-linear graphs, students should determine the rate of change for various intervals, confirming that rate of change varies for non-linear functions.

#### Academic Vocabulary:

rate of change, interval, slope, change in  $y$ , change in  $x$ , delta  $y$ , delta  $x$

#### Standards for Mathematical Practice:

- (2) Reason abstractly and quantitatively
- (5) Use appropriate tools strategically

#### Common Misconceptions

- Students may believe that the slope of a linear function is merely a number used to sketch the graph of the line. In reality, slopes have real-world meaning and the idea of rate of change is fundamental to understanding major concepts from geometry to calculus.
- Students may believe that varying rates of change for non-linear functions are not reflected graphically. Sketching a line that connects the points used to determine rate of change for various intervals may be helpful.

## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.C.7a-b.

**Graph functions** expressed symbolically and **show key features** of the graph, by hand in simple cases and using technology in complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima and minima

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. *(Content Emphases: Supporting)*

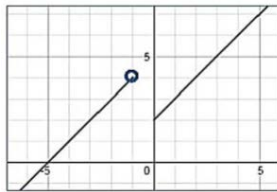
### Explanations and/or Examples

Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, end behavior, and asymptotes. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions

**Example 1:** Sketch the graph and identify the key features of the function described below.

$$f(x) = \begin{cases} x + 2 & \text{for } x \geq 0 \\ x + 5 & \text{for } x < -1 \end{cases}$$

**Ex. 1 Solution:**



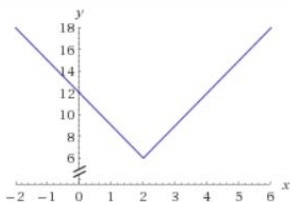
The function:

- has an x-intercept at -5 and a positive slope ( $m = 1$ ) when  $x < -1$ .
- has a y-intercept at  $y = 2$  and a positive slope ( $m = 1$ ) when  $x \geq 0$ .
- is undefined on the interval  $-1 < x < 0$

**Example 2:** An absolute value function in the form  $f(x) = a|x + b| + c$  is graphed in the xy-coordinate plane, where  $a, b,$  and  $c$  are constants.

Graph and describe the key features of function  $f(x) = |6 - 3x| + 6$ .

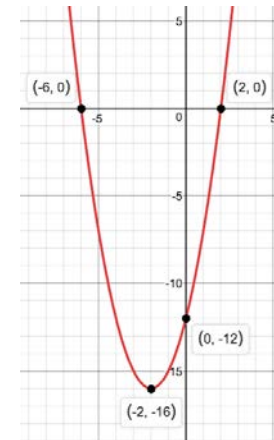
**Ex. 2 Solution:**



The key features of  $f(x) = |6 - 3x| + 6$  are a vertex at (2,6) and slopes of -1.5 and 1.5.

Students use technology such as graphing calculators, software programs, spreadsheets, or computer algebra systems to graph more complicated cases of quadratic functions. Key characteristics include maximum/minimum and intercepts.

**Example 3:** Describe key features of the graph of the quadratic function  $y = x^2 + 4x - 12$ .



**Ex. 3 Solution:**

- graph crosses x-axis at (-6,0) and (2, 0)
- graph crosses y-axis at (0, -12)
- graph minimum is the vertex at (-2, -16)

## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.C.7a-b.

**Graph functions** expressed symbolically and **show key features** of the graph, by hand in simple cases and using technology in complicated cases.

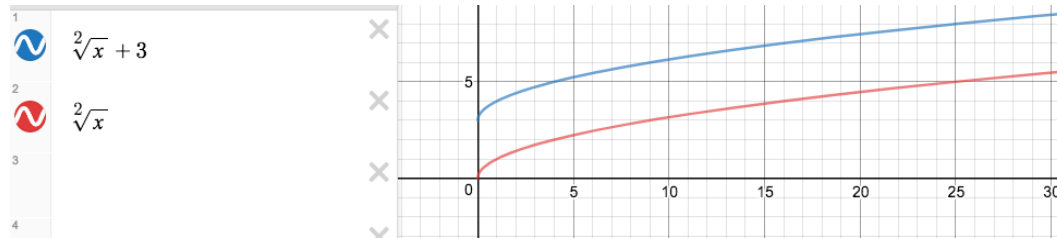
a. Graph linear and quadratic functions and show intercepts, maxima and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. *(Content Emphases: Supporting)*

### Explanations and/or Examples

**Example 4:** Graph, using technology,  $f(x) = \sqrt{x}$  and  $f(x) = \sqrt{x} + 3$ . Describe the key features of each graph.

### Ex. 4 Solution:



- Both graphs are always increasing and positive.
- Neither graph has an absolute maximum.
- The (absolute) minimum for  $f(x) = \sqrt{x}$  is zero and for  $f(x) = \sqrt{x} + 3$ , it is positive 3. These are also the y-intercepts.
- Zero is also the x-intercept for  $f(x) = \sqrt{x}$ .
- The end behavior for both is that an approach to positive infinity as  $x$  approaches positive infinity and an approach to zero as  $x$  approaches zero.

### Building Blocks (Grades 6 to 8 prerequisites)

#### Grade 8

##### 8.F.A.3

Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line; **give examples of functions that are not linear**. For example, the function  $A = s^2$  giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.

##### 8.EE.B.5

**Graph proportional relationships**, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

#### Grade 7

##### 7.RP.A.2a

Recognize and represent proportional relationships between quantities.

Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or **graphing on a coordinate plane** and observing whether the graph is a straight line through the origin.

## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.C.7a-b.

**Graph functions** expressed symbolically and **show key features** of the graph, by hand in simple cases and using technology in complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. *(Content Emphases: Supporting)*

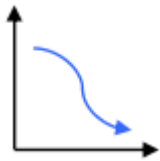
### Prerequisite Examples

Grade 8

#### 8.F.A.3:

Determine which of the following are nonlinear functions.

$$y = 9 - 2x$$



x	y
0	50
5	100
10	150
15	200
20	250
25	300

$$y = 3x^2 - 1$$



x	y
1	15
2	35
3	20
4	40
5	45
6	25

#### 8.EE.B.5:

Mariko has a job mowing lawns that pays \$7 per hour.

- Draw a graph that represents  $y$ , the amount of money Mariko would make for working  $x$  hours.
- Determine the slope of your graph. How does it compare to the Mariko's rate of pay (unit rate)?

Grade 7

#### 7.RP.A.2a:

A student is making trail mix. Create a graph to determine if the quantities of nuts and fruit are proportional for each serving size listed in the table. If the quantities are proportional, what is the constant of proportionality or unit rate that defines the relationship? Explain how you determined the constant of proportionality and how it relates to both the table and graph.

Trail mix quantities

Serving Size	Cups of Nuts (x)	Cups of Fruit (y)
1	1	2
2	2	4
3	3	6
4	4	8

### Grade/Course Level Connections:

F-IF.A.1

### Instructional Considerations

- Graphing utilities on a calculator and/or computer can be used to demonstrate the changes in behavior of a function as various parameters are varied.
- Explore various families of functions and help students make connections in terms of general features. For example, just as a function  $y = (x + 3)^2 - 5$  represents a translation of the parent function  $y = x^2$  by 3 units to the left and 5 units down, the same is true for  $y = |x + 3| - 5$  as a translation of parent function  $y = |x|$ .

### Common Misconceptions

- Students may believe that each family of function (e.g. quadratic and square root) is independent of the others.
- Students may believe that as with  $y = x^2$  and  $y = (x + 3)^2 - 5$ , the '+ 3' indicates movement in the positive x-direction.

## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.C.7a-b.

**Graph functions** expressed symbolically and **show key features** of the graph, by hand in simple cases and using technology in complicated cases.

a. Graph linear and quadratic functions and show intercepts, maxima and minima.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (*Content Emphases: Supporting*)

### Academic Vocabulary:

quadratic function, parabola, vertex, zero, maximum, minimum, vertex form, square root function, cube root function, piecewise-defined function, intercepts

### Standards for Mathematical Practice:

(4) Model with mathematics

(7) Look for and make use of structure

## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.C.8a.

**Write a function** defined by an expression in different but equivalent forms **to reveal and explain different properties** of the function.

a. Use the process of **factoring and completing the square** in a quadratic function to **show zeros**, extreme values, and **symmetry** of the graph, and **interpret** these **in terms of a context**. (*Content Emphases: Supporting*)

### Explanations and/or Examples

Students factor quadratics in which the coefficient of the quadratic term is an integer that may or may not be the GCF of the expression.

Students use the factors to reveal and explain properties of the function, interpreting them in context. Factoring just to factor does not fully address this standard.

**Example 1:** The quadratic expression  $-5x^2 + 10x + 15$  represents the height of a diver jumping into a pool off a platform, with  $x$  representing the time since leaving the platform. Use the process of factoring to determine key properties of the function defined by the expression and interpret them in the context of the problem.

### Ex. 1 Solution:

The function defined by the factored form of the expression is  $f(x) = -5(x - 3)(x + 1)$ . Key properties of the function are:

- The zeros of the quadratic function defined by this expression would be  $x = 3, -1$ . Since  $x$  represents time, a zero of  $-1$  has no meaning in context. The zero at  $3$  means that  $3$  seconds after leaving the platform, the diver entered the pool.
- The line of symmetry would be  $x = 1$  (midway between the zeros).
- The maximum of the function would be on the line of symmetry.

### Building Blocks (Grades 6 to 8 prerequisites)

Grade 7

#### 7.EE.A.1

Apply properties of operations as strategies to add, subtract, **factor**, and expand **linear expressions** with rational coefficients.

### Prerequisite Examples

Grade 7

#### 7.EE.A.1

Factor:  $-10y + 2$

### Grade/Course Level Connections:

F-IF.C.7a

### Instructional Considerations

- Focus on factoring as a strategy to show zeros, extreme values, and symmetry of the graph.
- Students discover that the factored form of a quadratic can be used to determine the zeros which, in turn, can be used to identify maxima/minima.
- Students use various representations of the same function to emphasize different characteristics of that function. For example, the  $y$ -intercept of the function  $y = x^2 - 4x - 12$  is easy to recognize as  $(0, -12)$ . However, rewriting the function as  $y = (x - 6)(x + 2)$  reveals zeros at  $(6, 0)$  and  $(-2, 0)$ . Furthermore, completing the square allows the equation to be written as  $y = (x - 2)^2 - 16$ , which shows the vertex (and minimum point) of the parabola at  $(2, -16)$ .

### Common Misconceptions

- Students may believe that skills such as factoring a trinomial or completing the square are isolated within a unit on polynomials/quadratics. They will come to understand the usefulness of these skills in the context of examining characteristics of functions.
- Students may also believe that the process of rewriting equations into various forms is simply an Algebra symbol manipulation exercise, rather than serving a purpose of allowing different features of the function to be exhibited.

## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.C.8a.

**Write a function** defined by an expression in different but equivalent forms **to reveal and explain different properties** of the function.

- a. Use the process of **factoring and completing the square** in a quadratic function to **show zeros**, extreme values, and **symmetry** of the graph, and **interpret** these **in terms of a context**. (*Content Emphases: Supporting*)

### Academic Vocabulary:

perfect square binomial, complete the square, trinomial, factor

### Standards for Mathematical Practice:

- (4) Model with mathematics
- (7) Look for and make use of structure

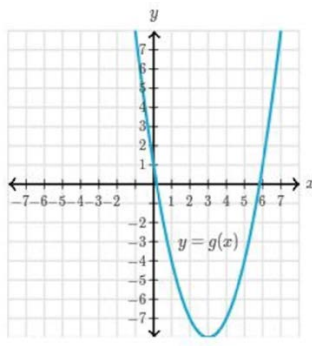
## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.C.9

**Compare properties** of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (Content Emphases: Supporting)

### Explanations and/or Examples

**Example 1:** How do the intercepts and vertex for the following functions compare?  $f(x) = x^2 - 2x + 12$



**Ex. 1 Solution:**  $f(x) = x^2 - 2x + 12$  has a greater y-intercept at (0,12) than  $g(x)$  whose y-intercept is negative. The vertex of the graph of  $f(x)$  is a minimum at (1, 11) while the vertex of  $g(x)$  is a minimum at approximately (3, -8). The graph of  $g(x)$  has two positive x-intercepts. The graph of  $f(x)$  does not intersect the x-axis.

**Example 2:** Examine the functions below. Which function decreases faster?  
 $f(r) = -\frac{5}{9}r + 4$

Table Function h

r	h(r)
0	6
1	5.25
2	4.5
3	3.75

**Ex. 2 Solution:**  $h(r)$  decreases at a faster rate than  $f(r)$ .

### Building Blocks (Grades 6 to 8 prerequisites)

#### Grade 8

#### 8.EE.B.5

Graph proportional relationships, interpreting the unit rate as the slope of the graph. **Compare two different proportional relationships represented in different ways.** *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

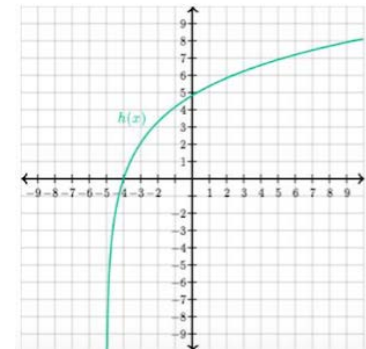
**Example 3:** Which of the following function has no x- intercept(s)?

- $f$  is a function defined on all real numbers. Its formula is given by  $f(x) = x(x + 4)$
- $g$  is a function defined on all integers between -3 and 3 (inclusive). Its values are given in the table below.

Table Function g

x	g(x)
-3	0
-2	2
-1	3
0	-1
1	-2
2	-3
3	-7

- $h$  is a function defined on all real numbers greater than -5. Its graph is given below.



**Ex. 3 Solution:** All of the functions have at least one x-intercept



## Algebra 1 Instructional Support Tool - Functions

### Standard F-IF.C.9.

**Compare properties** of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (Content Emphases: Supporting)

### Prerequisite Examples

Grade 8

#### 8.EE.B.5:

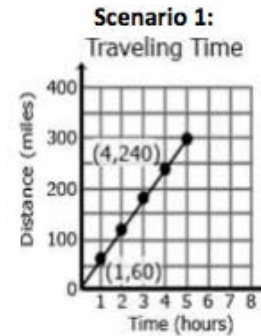
Compare the scenarios to determine which represents a greater speed. Explain your choice including a description of each scenario. Be sure to include the rates in your explanation.

Scenario 2:

$$y = 55x$$

x is time in hours

y is distance in miles



### Grade/Course Level Connections:

F-IF.B.4 and F-IF.C.8a

### Instructional Considerations

- Use different representations of different functions from the same function family to compare properties. For example, the y-intercept of the function  $y = x^2 - 4x - 12$  is easy to recognize from the equation as  $(0, -12)$ . In comparing that function to the graph of  $y = x^2 - 2x + 12$ , the y-intercept is easily identifiable from the graph as  $(0,12)$ . Students can discuss these points comparatively in different ways. For example, the y-intercepts are equal distances from the x-axis.

### Common Misconceptions

- Students may have trouble making connections between graphs and other representations especially in multiple situations in which there are various limitations to the domains.
- Students may believe that each family of functions (e.g., quadratic, square root, etc.) is independent of the others, so they may not recognize commonalities among all functions and their graphs.
- NOTE: Student misconceptions are generally related to determining properties for each function which would be reflected in guidance for other standards. See F-IF.C.6 through F-IF.C.8a*

### Academic Vocabulary:

properties of functions

### Standards for Mathematical Practice:

- (6) Attend to precision
- (7) Look for and make use of structure

## Algebra 1 Instructional Support Tool - Functions

### Standard F-LE.A.1

**Distinguish between situations** that can be modeled with linear functions and with exponential functions.

- Prove** that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- Recognize situations** in which one quantity changes at a constant rate per unit interval relative to another.
- Recognize situations** in which a quantity grows or decays by a constant percent rate per unit interval relative to another. *(Content Emphases: Supporting)*

### Explanations and/or Examples

Given a situation in context, determine whether a described quantity changes at a constant rate or at a constant percent rate per unit interval. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and compare linear and exponential functions.

**Example 1:** The following tables show the values of linear functions, exponential functions and function that are neither linear nor exponential. Indicate which function type corresponds to each table. Justify your choice.

Table A

x	y
1	6
2	9
3	12
4	15

Table B

x	y
1	7
2	14
3	28
4	56

Table C

x	y
1	6
2	9
3	14
4	21

Table D

x	y
1	56
2	28
3	14
4	7

### Ex. 1 Solution

[Task 238 by IM](#) by [CC BY-NC-SA 4.0](#)

**Examples 2:** For each of the scenarios below, decide whether the situation can be modeled by a linear function, an exponential function, or neither. For those with a linear or exponential model, justify your reasoning.

- From 1910 until 2010 the growth rate of the United States has been steady at about 1.5% per year. The population in 1910 was about 92,000,000.
- The circumference of a circle as a function of the radius.
- According to an old legend, an Indian King played a game of chess with a traveling sage on a beautiful, hand-made chessboard. The sage requested, as reward for winning the game, one grain of rice for the first square, two grains for the second, four grains for the third, and so on for the whole chess board.
- How many grains of rice would the sage win for the  $n^{th}$  square?
- The volume of a cube as a function of its side length.

### Ex. 2 Solution

[Task 1910 by IM](#) by [CC BY-NC-SA 4.0](#)

## Algebra 1 Instructional Support Tool - Functions

### Standard F-LE.A.1

**Distinguish between situations** that can be modeled with linear functions and with exponential functions.

- Prove** that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- Recognize situations** in which one quantity changes at a constant rate per unit interval relative to another.
- Recognize situations** in which a quantity grows or decays by a constant percent rate per unit interval relative to another. (*Content Emphases: Supporting*)

### Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

#### 8.F.A.3

**Interpret the equation  $y = mx + b$  as defining a linear function**, whose graph is a straight line; **give examples of functions that are not linear**.

*For example, the function  $A = s^2$  giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

### Prerequisite Examples

Grade 8

#### 8.F.A.3

A graph contains the following points: (1, 1), (2, 5) and (3, 11). Would the equation for the graph formed by these points above define a linear or nonlinear function?

### Grade/Course Level Connections:

F-BF.A.1a

### Instructional Considerations:

- Compare tabular representations of a variety of functions to show that linear functions have a common first difference (i.e., equal differences over equal intervals), while exponential functions do not (instead function values grow by equal factors over equal  $x$ -intervals).
- Emphasize common *first* difference, as opposed to just ‘common differences’ in preparation for analysis of quadratic functions which have common second differences.

- Students should model and compare linear and exponential functions using graphing calculators or programs, spreadsheets, or computer algebra systems.
- Students may investigate functions and graphs reflecting compound interest and simple interest scenarios.

### Common Misconceptions:

- Students may believe that all functions have a common first difference (or a common factor) and need to explore to realize that, for example, a quadratic function will have common second differences.

### Academic Vocabulary:

exponential function, exponential growth, exponential decay, linear function, constant rate of change, common difference, common factor

### Standards for Mathematical Practice:

- (3) Construct viable arguments and critique the reasoning of others
- (4) Model with Mathematics

## Algebra 1 Instructional Support Tool - Functions

### Standard F-LE.A.2

**Construct** linear and exponential **functions**, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a tab. *(Content Emphases: Supporting)*)

### Explanations and/or Examples

A set of numbers arranged in a definite order according to some definite rule is called a sequence. The individual elements in a sequence are called terms.

Students recognize that arithmetic sequences are linear functions and geometric sequences are exponential functions. As such, consecutive terms in an arithmetic sequence have a common difference (e.g., 1, 3, 5, 7...is generated by adding 2 to the previous term; thus, the common difference is 2). Consecutive terms in a geometric sequence have the same ratio (e.g., 1, 3, 9, 27, 81 is generated by multiplying the previous term by a factor of 3; therefore, the common ratio is 3:1).

**Example 1:** A theater has 60 seats in the first row, 68 seats in the second row, and 76 seats in the third row. The number of seats in the remaining rows continue in this pattern. Is the sequence arithmetic or geometric? Explain how you know.

**Ex. 1 Solution:** The sequence is arithmetic because there is a constant difference of 8 seats per row.

**Example 2:** Every day Brian takes 20 mg of a drug that helps with his allergies. His doctor tells him that each hour the amount of drug in his blood stream decreases by 15%.

- Construct an exponential function of the form  $f(t) = ab^t$ , for constants  $a$  and  $b$ , that gives the quantity of the drug, in milligrams, that remains in his bloodstream  $t$  hours after he takes the medication.
- How much of the drug remains one day after taking it?
- Do you expect the percentage of the dose that leaves the blood stream in the first half hour to be more than or less than 15%? What percentage is it?
- How much of the drug remains one minute after taking it?

### Ex. 2 Solution

[Task 2125 by IM](#) by [CC BY-NC-SA 4.0](#)

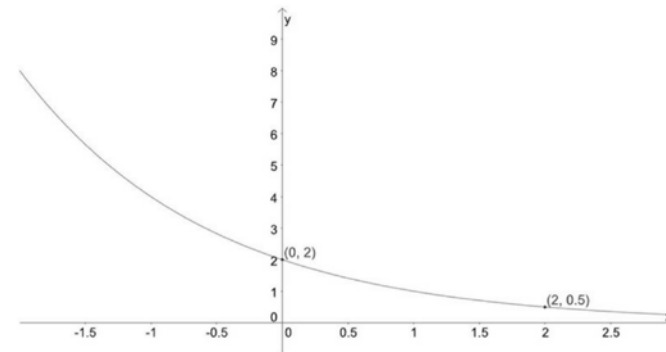
**Example 3:** After a record setting winter storm, there are 10 inches of snow on the ground! Now that the sun is finally out, the snow is melting. At 7 am there were 10 inches and at 12 pm there were 6 inches of snow.

- Construct a linear function rule to model the amount of snow.
- Construct an exponential function rule to model the amount of snow.
- Which model best describes the amount of snow? Provide reasoning for your choice.

### Ex. 3 Solution:

- $y = -0.8x + 10$
- $y = 10(0.9)^x$
- The exponential function best models the situation because the graph will eventually approach zero, whereas the linear function will have negative values.

**Example 4:** The graph of a function of the form  $f(x) = ab^x$  is shown below. Find the values of  $a$  and  $b$ .



### Ex. 4 Solution

[Task 349 by IM](#) by [CC BY-NC-SA 4.0](#)

## Algebra 1 Instructional Support Tool - Functions

### Standard F-LE.A.2

**Construct** linear and exponential **functions**, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). (*Content Emphases: Supporting*)

#### Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

##### 8.F.B.4

**Construct a function to model a linear relationship** between two quantities. **Determine the rate of change and initial value of the function from a description of a relationship** or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

#### Prerequisite Examples:

Grade 8

##### 8.F.B.4:

The table below shows the cost of renting a car. The company charges \$45 a day for the car as well as charging a one-time \$25 fee for the car's navigation system (GPS). Write an equation for the cost in dollars,  $c$ , as a function of the number of days,  $d$ .

Car rental cost

Days (d)	Cost (c) in dollars
1	70
2	115
3	160
4	205

#### Grade/Course Level Connections:

F-LE.A.1b, F-LE.A.1c, and F-BF.A.1a

#### Instructional Considerations:

- Apply linear and exponential functions to real-world situations. For example, a person earning \$10 per hour experiences a constant rate of change in salary given the number of hours worked, while the number of bacteria on a dish that doubles every hour will have equal factors over equal intervals.
- When handling functions, help students see that patterns emerge when comparing the  $x$  and  $y$  values to each other. Students should know that these patterns are not coincidences.
- Students should know that these patterns can be thought of as sequences, or a list of numbers. Sequences can be either arithmetic or geometric.
- Provide examples of arithmetic and geometric sequences in graphic, verbal, or tabular forms, and have students generate formulas and equations that describe the patterns.

#### Common Misconceptions:

- Students may believe that the end behavior of all functions depends on the context and not that exponential functions values will eventually exceed those of any other functions.

## Algebra 1 Instructional Support Tool - Functions

### Standard F-LE.A.2

**Construct** linear and exponential **functions**, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). *(Content Emphases: Supporting)*

### Academic Vocabulary:

exponential function, arithmetic sequence, geometric sequence, term, common difference, common ratio, linear function, slope, rate of change, y-intercept, interval, growth factor

### Standards for Mathematical Practice:

- (4) Model with Mathematics
- (7) Look for and make use of structure

## Algebra 1 Instructional Support Tool - Functions

### Standard F-LE.A.3

**Observe using graphs and tables** that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (*Content Emphases: Supporting*)

### Explanations and/or Examples

**Example 1:** Mr. Wiggins gives his daughter Celia two choices of payment for raking leaves:

Choice 1: Two dollars for *each* bag of leaves.

Choice 2: She will be paid for the number of bags of leaves she rakes as follows: two cents for one bag, four cents for two bags, eight cents for three bags, and so on with the amount doubling for each additional bag.

- If Celia rakes five bags of leaves, should she opt for payment method 1 or 2? What if she rakes ten bags of leaves?
- How many bags of leaves does Celia have to rake before method 2 pays more than method 1?

**Ex. 1 Solution:**

- A table of values giving the number of bags of leaves and the amount paid using methods 1 and 2 shows that method 1 pays more up to and including eleven bags.
- Analyzing a table of values shows that method 2 pays more as soon as Celia rakes at least twelve bags of leaves. We know that method 2 will always pay more, beyond the twelfth bag, because doubling an amount  $x$  gives a larger increase than adding 2 as soon as  $x$  is greater than 2:  
 $2x > x + 2$  whenever  $x > 2$

### Building Blocks (Grades 6 to 8 prerequisites):

n/a

### Prerequisite Examples:

n/a

### Grade/Course Level Connections:

F-LE.A.1c

### Instructional Considerations:

- Use a graphing calculator or computer program to compare tabular and graphic representations of exponential and quadratic/polynomial functions to show how the output of the exponential function eventually exceeds those of polynomial functions. A simple example would be to compare the graphs (and tables) of the functions  $f(x) = x^2$  and  $f(x) = 2^x$  to find that the  $y$  values are greater for the exponential function when  $x > 4$ .

**Example 2:** Contrast the growth of the  $f(x) = x^2$  and  $f(x) = 2^x$

**Ex. 2 Solution:** The exponential function grows faster than the quadratic function. The  $y$  values are greater for the exponential function when  $x > 4$ . Beyond  $x = 4$ , the exponential function  $f(x) = 2^x$  remains greater than the quadratic function  $f(x) = x^2$ .

### Common Misconceptions:

- Students may believe that the end behavior of all functions depends on the situation and not the fact that exponential function values will eventually get larger than those of any other polynomial functions.

## Algebra 1 Instructional Support Tool - Functions

### Standard F-LE.A.3

**Observe using graphs and tables** that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. *(Content Emphases: Supporting)*

### Academic Vocabulary:

exponential, quadratic, growth

### Standards for Mathematical Practice:

- (2) Reason abstractly and quantitatively
- (8) Look for and express regularity in repeated reasoning



## Algebra 1 Instructional Support Tool - Functions

### Standard F-LE.B.5

**Interpret the parameters** in a linear or exponential function **in terms of a context.** (*Content Emphases: Supporting*)

#### Explanations and/or Examples

Use real-world situations to help students understand how the parameters of linear and exponential functions depend on the context.

**Example 1:** A plumber who charges \$50 for a house call and \$85 per hour can be expressed as the function  $y = 85x + 50$ . If the rate were raised to \$90 per hour, how would the function change?

**Ex. 1 Solution:** The function would become  $y = 90x + 50$ .

**Example 2:** A function of the form  $f(n) = P(1 + r)^n$  is used to model the amount of money in a savings account that earns 8% interest, compounded annually, where  $n$  is the number of years since the initial deposit.

- What is the value of  $r$ ? Interpret what  $r$  means in terms of the savings account?
- What is the meaning of the constant  $P$  in terms of the savings account?
- Will  $n$  or  $f(n)$  ever take on the value 0? Why or why not?

**Ex. 2 Solution:**

- The value of  $r$  is 0.08 which represents the interest rate.
- $P$  represents the initial amount of money invested.
- $f(n)$  cannot be zero unless  $P$ , the amount invested is zero. The expression  $1+r$  is greater than 1. Therefore it is a rate of growth. The expression  $(1 + r)^n$  will always be greater than 1. As long as  $P$  is greater than 0, the value of the function will be greater than or equal to  $P$ , and less than infinity ( $P \leq f(n) < \infty$ ).

The number of years since the initial deposit,  $n$ , can be 0. That represents the beginning of the investment and  $f(0) = P$ .

#### Building Blocks (Grades 6 to 8 prerequisites):

n/a

#### Prerequisite Examples:

n/a

#### Grade/Course Level Connections:

A-SSE.A.1a, F-LE.A.2, and S-ID.C.7

#### Instructional Considerations

- Use real-world contexts to help students understand how the parameters of linear and exponential functions depend on the context.
  - For example, a plumber who charges \$50 for a house call and \$85 per hour would be expressed as the function  $y = 85x + 50$ , and if the rate were raised to \$90 per hour, the function would become  $y = 90x + 50$ .
  - On the other hand, an equation of  $y = 8,000(1.04)^x$  could model the rising population of a city with 8,000 residents when the annual growth rate is 4%. Students can examine what would happen to the population over 25 years if the rate were 6% instead of 4% or the effect on the equation and function of the city's population were 12,000 instead of 8,000.
- Graphs and tables can be used to examine the behaviors of functions as parameters are changed, including the comparison of two functions such as what would happen to a population if it grew by 500 people per year, versus rising an average of 8% per year over the course of 10 years.
- Students can be given different parameters of a function to manipulate and compare the results in order to draw conclusions about the effects of the changes.

## Algebra 1 Instructional Support Tool - Functions

### Standard F-LE.B.5

**Interpret the parameters** in a linear or exponential function **in terms of a context**. (*Content Emphases: Supporting*)

#### Common Misconceptions:

- Students may believe that parameters will not have a real-world interpretation.
- Students may find parameters, independent and dependent variables indistinguishable.

#### Academic Vocabulary:

parent function, parameter, even function, odd function, term, exponent, vertical shift, horizontal shift, stretch, compression

#### Standards for Mathematical Practice:

- (4) Model with mathematics
- (7) Look for and make use of structure

## Algebra 1 Instructional Support Tool - Functions

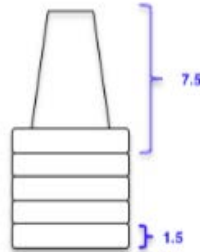
### Standard F-BF.A.1

**Write a function** that describes a relationship between two quantities.

a. **Determine** an explicit expression, a recursive process, or steps for calculation from a context. *(Content Emphases: Supporting)*

### Explanations and/or Examples

**Example 1:** The height of a stack of cups is a function of the number of cups in the stack. If a 7.5" tall cup with a 1.5" lip is stacked vertically, determine a function that would provide you with the height of the stack based on any number of cups.



**Ex. 1 Solution:**  $x$  = number of cups;  $f(x) = 6 + 1.5x$  or  $f(x) = 7.5 + 1.5(x - 1)$

**Example 2:** The price of a new computer decreases with age.

- Describe a recursive process for determining the value of the computer.
- Determine a function that represents the value of the computer for a given age.

### Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

#### 8.F.B.4

**Construct a function to model a linear relationship** between two quantities. **Determine the rate of change and initial value of the function from a description of a relationship** or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Computer value over time

Age (years)	Value (dollars)
1	\$1575
2	\$1180
3	\$885
4	\$665
5	\$500
6	\$375
7	\$280

**Ex. 2 Solution:**

- The value of the computer is 75% of the value of the computer for the previous year. The process would be to multiply the prior year's value by .75 to find the value of the computer for the year.
- $f(x) = 1575(.75)^{x-1}$  or  $2100(.75)^x$

### Prerequisite Examples

Grade 8

#### 8.F.B.4:

You have \$100 to spend on a barbeque where you want to serve chicken and steak. Chicken costs \$1.29 per pound and steak costs \$3.49 per pound.

- Find a function that relates the number of pounds of chicken to the number of pounds of steak that you can buy.
- Graph the function. What is the meaning of each intercept in this context? What is the meaning of the slope in this context?

## Algebra 1 Instructional Support Tool - Functions

### Standard F-BF.A.1

**Write a function** that describes a relationship between two quantities.

a. **Determine** an explicit expression, a recursive process, or steps for calculation from a context. *(Content Emphases: Supporting)*

### Grade/Course Level Connections:

F-IF.A.3

### Instructional Considerations:

- Provide a real-world example (e.g., a table showing how far a car has driven after a given number of minutes, traveling at a uniform speed), and examine the table by looking “down” the table to describe a recursive relationship, as well as “across” the table to determine an explicit formula to find the distance traveled if the number of minutes is known.
- Write out terms in a table in an expanded form to help students see what is happening. For example, if the y-values are 2, 4, 8, 16, they could be written as 2,  $2(2)$ ,  $2(2)(2)$ ,  $2(2)(2)(2)$ , etc., so that students recognize that 2 is being used multiple times as a factor.
- Focus on one representation and its related language – recursive or explicit – at a time so that students are not confusing the formats.

### Common Misconceptions:

- Students may believe that the recursive process and explicit expressions are the same.
- Students may believe that the best (or only) way to generalize a table of data is by using a recursive formula.

### Academic Vocabulary:

explicit expression, recursive process, rate of change, constant rate, constant percent rate, growth factor, decay factor

### Standards for Mathematical Practice:

- (1) Make sense of problems and persevere in solving them
- (4) Model with mathematics

## Algebra 1 Instructional Support Tool - Functions

### Standard F-BF.B.3.

**Identify the effect** on the graph of **replacing  $f(x)$**  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); **find the value of  $k$  given the graphs**. Experiment with cases and **illustrate an explanation of the effects on the graph using technology**. Include **recognizing even and odd functions** from their graphs and algebraic expressions for them. *(Content Emphases: Additional)*

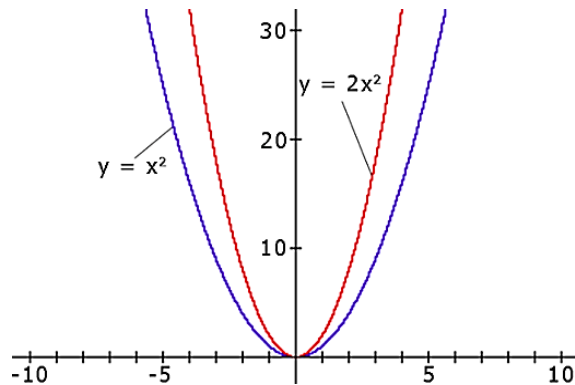
### Explanations and/or Examples

**Example 1:** For  $f(x) = 3x$ , give the expressions for  $g(x) = f(x) + 2$  and  $h(x) = f(x) - 1$  and describe the effect of adding 2 to  $f(x)$  and subtracting one from  $f(x)$ .

**Ex. 1 Solution:**  $g(x) = 3x + 2$ ;  $h(x) = 3x - 1$ ; The three lines are parallel.  $g(x)$  is a vertical shift of  $f(x)$  up 2 units and has a y-intercept of  $(0, 2)$ .  $h(x)$  is a vertical shift down 1 unit and has a y-intercept of  $(0, -1)$ .

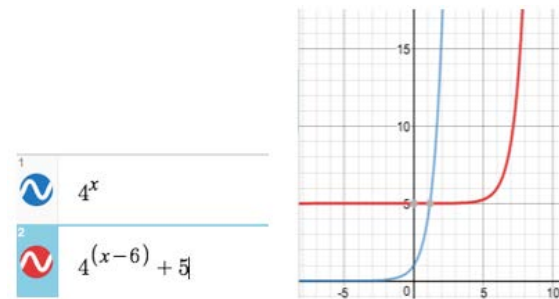
**Example 2:** Compare the shape and position of the graphs of  $f(x) = x^2$  and  $g(x) = 2x^2$ , and explain the differences in terms of the algebraic expressions for the functions.

**Ex. 2 Solution:**  $g(x)$  is  $2f(x)$ , therefore  $k$  is 2. Comparing  $f(x)$  to  $2f(x)$ , there is no vertical or horizontal translation. However,  $2f(x)$  is a compression of  $f(x)$  in the y-direction.



**Example 3:** Compare the shape and position of the graphs of  $f(x) = 4^x$  to  $g(x) = 4^{x-6} + 5$  and explain the differences in terms of the algebraic expressions for the functions.

**Ex. 3 Solution:** The shapes of the graphs are the same. In terms of position, 'x-6' in  $g(x)$  indicates a shift of 6 units to the right of  $f(x)$ . '+ 5' in  $g(x)$  indicates a vertical shift of 5 units up.



## Algebra 1 Instructional Support Tool - Functions

### Standard F-BF.B.3.

**Identify the effect** on the graph of **replacing  $f(x)$**  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); **find the value of  $k$  given the graphs**. Experiment with cases and **illustrate an explanation of the effects on the graph using technology**. Include **recognizing even and odd functions** from their graphs and algebraic expressions for them. *(Content Emphases: Additional)*

#### Building Blocks (Grades 6 to 8 prerequisites):

n/a

#### Prerequisite Examples:

n/a

#### Grade/Course Level Connections:

F-IF.C.7a-b

#### Instructional Considerations:

- Begin by focusing on vertical translations of graphs of linear and exponential functions. Extend to quadratic and absolute value functions. Relate the vertical translation of a linear function to its y-intercept.
- Use graphing calculators or software to distinguish between the graphs of  $y = x^2$  and  $y = 2x^2$ ,  $y = x^2 + 2$ ,  $y = (2x)^2$  and  $y = (x + 2)^2$ . This can be accomplished by allowing students to work with a single parent function and examine numerous parameter changes to make generalizations.
- Students note the common effect of each transformation *across function types*.
- Students use technology to graph, noting the value and sign of  $k$  while describing and illustrating an explanation of the effect on the graph. Using graphing utilities such as desmos allows students to easily observe and distinguish between the two graphs.
- Students note the effect of *multiple* transformations on a single graph.
- Distinguish between even and odd functions by providing several examples and helping students recognize that a function is even if  $f(-x) = f(x)$  and is odd if  $f(-x) = -f(x)$ . Visual approaches to identifying the graphs of even and odd functions should be used as well.

#### Common Misconceptions:

- Students may believe that the graph of  $y = (x - 4)^3$  is the graph of  $y = x^3$  shifted 4 units to the left (due to the subtraction symbol). Examples should be explored by hand and on a graphing calculator to overcome this, and other similar misconception.
- Students may believe that even and odd functions refer to the (largest) exponent of the expression or the exponent of the first term of the expression, rather than the sketch of the graph and the behavior of the function.

#### Academic Vocabulary:

parent function, parameter, even function, odd function, term, exponent, vertical shift, horizontal shift, stretch, compression

#### Standards for Mathematical Practice:

- (4) Model with mathematics
- (7) Look for and make use of structure