STATE OF NEW JERSEY
DEPARTMENT OF EDUCATION

New Jersey Student Learning Standards
Mathematics

Algebra 1

Instructional Support Tool for Teachers: Number & Quantity

This instructional support tool is designed to assist educators in interpreting and implementing the New Jersey Student Learning Standards for Mathematics. It contains explanations or examples of each Algebra 1 course standard to answer questions about the standard's meaning and how it applies to student knowledge and performance. To ensure that descriptions are helpful and meaningful to teachers, this document also identifies grades 6 to 8 prerequisite standards upon which each Algebra 1 standard builds. It includes the following: sample items aligned to each identified prerequisite; identification of course level connections; instructional considerations and common misconceptions; sample academic vocabulary and associated standards for mathematical practice. **Examples are samples only** and should not be considered an exhaustive list.

This instructional support tool is considered a living document. The New Jersey Department of Education believe that teachers and other educators will find ways to improve the document as they use it. Please send feedback to mathematics@doe.state.nj.us so that the Department may use your input when updating this guide.

Please consult the New Jersey Student Learning Standards for Mathematics for more information.

Number and Quantity Standards Overview

In real-world problems, the answers are usually not numbers but quantities - numbers with units - which involves measurement. In high school courses such as Algebra 1, students encounter a wide variety of measurement units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year.

Quantities (N-Q)

Reason quantitatively and use units to solve problems.

N-Q.A.1 N-Q.A.2 N-Q.A.3

The Real Number System (N-RN)

Use properties of rational and irrational Numbers

N-RN.B.3

Mathematical Practices

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Standard N-Q.A.1

Use units as a way **to understand problems** and to guide the solution of multi-step problems; **Choose and interpret units** consistently in formulas; **Choose and interpret the scale** and the origin in graphs and data displays. *(Content Emphases: Supporting)*

Explanations and/or Examples:

Example 1: As Felicia gets on the freeway to drive to her cousin's house, she notices that she is a little low on gas. There is a gas station at the exit she normally takes, and she wonders if she will have to get gas before then. She normally sets her cruise control at the speed limit of 70mph and the freeway portion of the drive takes about an hour and 15 minutes. Her car gets about 30 miles per gallon on the freeway, and gas costs $3.50 per gallon.

a) Describe an estimate that Felicia might do in her head while driving to decide how many gallons of gas she needs to make it to the gas station at the other end.

b) Assuming she makes it, how much does Felicia spend per mile on the freeway?

Ex. 1 Solution:

Example 2:

Consider the formula R $= \frac{V}{C*t}$, where

- R represents rate of infection
- V represents the number of viruses in a cell
- C represents the number of cells, and
- t represents time and is represented in hours.

Select an appropriate measurement unit for rate of infection

a) $\frac{\text{viruses}}{\text{cells * hours}}$

b) $\frac{\text{viruses * hours}}{\text{cells}}$

c) $\frac{\text{cells * viruses}}{\text{hours}}$

d) $\frac{\text{cells * hours}}{\text{viruses}}$

Ex. 2 Solution: a

Building Blocks (Grades 6 to 8 prerequisites):

Grade 6

6.RP.A.3d

Use **ratio and rate reasoning** to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

d. Use ratio reasoning to **convert measurement units; manipulate and transform units appropriately** when multiplying or dividing quantities.

Standard N-Q.A.1.
Use units as a way **to understand problems** and to guide the solution of multi-step problems; **Choose and interpret units** consistently in formulas; **Choose and interpret the scale** and the origin in graphs and data displays. *(Content Emphases: Supporting)*

Explanations and/or Examples:

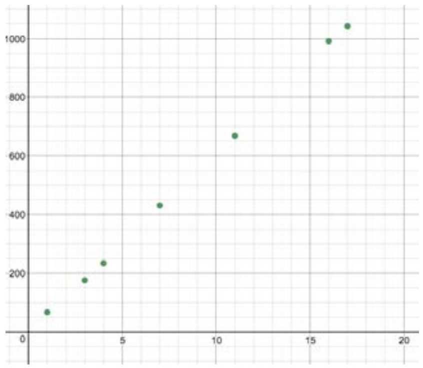
Example 3: Mr. Smith decided to travel to Orlando, Florida for Spring Break. His children recorded the total number of miles traveled each hour on the road trip. After arriving in Orlando, Mr. Smith's children had misplaced some of their data.

a) Using the data they had found, choose an appropriate scale for each axes in creating a scatter plot.
b) Interpret the meaning of the origin on the graph.

Mileage over time

Length of Trip (hours)	Total Distance Traveled (miles)
1	67
3	176
4	234
7	431
11	668
16	991
17	1042

Ex. 3 Solution:



a)
b) Any point on the graph represents the total distance traveled for an amount of time traveled. The origin would represent a situation in which zero miles are traveled for a trip whose length is zero hours.

Prerequisite Examples:

Grade 8

6.RP.A.3d:
The lot that Dana is buying for her new one-story house is 35 yards by 50 yards. Dana's house plans show that her house will cover 1,600 square feet of land. What percent of Dana's lot will not be covered by the house? Explain your reasoning. (Task 118 by IM by CC BY-NC-SA 4.0)

Algebra 1 Instructional Support Tool - Number & Quantity

Standard N-Q.A.1.
Use units as a way **to understand problems** and to guide the solution of multi-step problems; **Choose and interpret units** consistently in formulas; **Choose and interpret the scale** and the origin in graphs and data displays. *(Content Emphases: Supporting)*

Grade/Course Level Connections:

N-Q.A.2

Instructional Considerations:

- While proficiency in using dimensional analysis is necessary, the instructional focus should be on tasks given in context.
- Include word problems where quantities are given in different units, which must be converted to make sense of the problem. For example, a problem might have an object moving 12 feet per second and another at 5 miles per hour. To compare speeds, students convert 12 feet per second to miles per hour.
    - $\frac{12 \text{ ft}}{1 \text{ sec}} \bullet \frac{1 \text{ mile}}{5280 \text{ ft}} \bullet \frac{60 \text{sec}}{1 \text{ min}} \bullet \frac{60 \text{min}}{1 \text{ hr}} = \frac{43200 \text{ mile}}{5280 \text{ hr}} \approx 8.18$
- Some contextual problems may require an understanding of derived measurements and capability in unit analysis.
    - *A derived measurement is one that is developed from another measurement or measurements. Density is a good example. There is no way of measuring density directly, so one measures both the weight and volume of an object or substance. The density is then derived from the weight divided by the volume.*
- Keeping track of derived units during computations, making reasonable estimates and drawing conclusions about precision of the answers support the problem-solving process.
- Students spend time identifying various misleading graphs *before* choosing units and scales in order to create a correct representation of a situation.

Common Misconceptions:

- Students may believe that units are not important during computations. They need to be able to keep track of all derived units during computations, as well as determine if the answer is in the appropriate units.
- Students may assume that the given units are appropriate to the solution. Students may not realize the importance of the units' conversions. Students may not use units to evaluate the appropriateness of solutions.
- Students may believe that ratios expressed with different units cannot be equivalent to one.
- Students may believe that the information presented in a graph is correct, even if there are misrepresentations in terms of scales, etc. and therefore their interpretations would be incorrect.
- Students may believe that the labels and scales on a given graph or on one they must construct are not important. Students should determine if the units and scales chosen represent the problem correctly. They may not understand that the scales and units cannot be assumed by someone reading the graph.

Academic Vocabulary:

exponential function, unit analysis, dimensional analysis, ratio, rate

Standards for Mathematical Practice:

(1) Make sense of problems and persevere in solving them

(6) Attend to precision

Standard N-Q.A.2.
Define appropriate quantities for the purpose of **descriptive** modeling. *(Content Emphases: Supporting)*

Explanations and/or Examples:

Descriptive modeling is a mathematical process that describes real-world events and the relationships between factors responsible for them
Students list and describe the factors (quantitative variables) that could be important in describing a situation.

Example 1: A small company wants to give raises to their 5 employees. They have $10,000 available to distribute. Imagine you are in charge of deciding how the raises should be determined.

a) What are some variables you should consider?
b) Describe mathematically different methods to distribute the raises.
c) What information do you need to compute the raises for each employee?
d) Make up the information you need to compute specific raises for 2 different methods and apply them to the situation. Compute the specific dollar amount each employee receives as a raise.
e) Choose one of you methods that you think is most fair and construct an argument that supports your decision.

Ex. 1 Solution:

Building Blocks (Grades grades 6 to 8 prerequisites):

n/a

Prerequisite Examples:

n/a

Grade/Course Level Connections:

N-Q.A.1

Instructional Considerations:

- Teachers may ask students to list all of the factors important for a describing a situation or context. Then have students identify the factors from that list that are quantitative in nature.
- Consideration of units for each factor is important. For example, if you were modeling traffic flow, you might come up with cars per minute (passing a certain point) to measure flow. You might also consider the number of cars per mile per lane to measure how closely packed the traffic is.

Common Misconceptions:

n/a

Academic Vocabulary:

unit analysis, quantitative, factors, descriptive, models

Standards for Mathematical Practice:

(1) Make sense of problems and persevere in solving them
(4) Model with mathematics

Standard N-Q.A.3.

Choose a **level of accuracy** appropriate to **limitations** on **measurement** when reporting quantities. *(Content Emphases: Supporting)*

Explanations and/or Examples:

Error and tolerance vary according to measurement tool, measure and context.

Example 1: The label on a 16.9 ounce bottle of a sports drink indicates that one serving of 8 ounces contains 50 calories.

a) Based on this information, about how many calories are in the full bottle?
b) The label also says that the full bottle contains 120 calories. Does this agree with your estimate from part (a)? How can you explain the discrepancy (if there is a discrepancy)?
c) The label on a 20 ounce bottle of the same sports drink says the bottle contains 130 calories. Is this consistent with the information on the 16.9 ounce bottle?

Ex.1 Solution

Building Blocks (Grades 6 to 8 prerequisites)

Grade 8

8.EE.A.4

Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and **choose units of appropriate size for measurements of very large or very small quantities** (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Prerequisite Examples:

Grade 8

8.EE.A.4: George told his teacher that he spent over 21,000 seconds working on his homework. Express this amount using scientific notation. What would be a more appropriate unit of time for George to use? Explain and convert to your new units. (Solution)

Grade/Course Level Connections:

n/a

Instructional Considerations:

- Students may be learning about accuracy, as well as precision, in science. Engage students in measurement activities to reinforce the distinction between the two concepts.
- Ensure that students understand that final solutions to real world problems are not always the computed value. For example, when computing the price of an item is rounded to the hundredths place (the nearest cent).

Common Misconceptions:

- Students may believe that precision and accuracy are the same idea.
- Students should be able to correctly identify the degree of precision needed in a solution and that this should not be far greater than the actual accuracy of the measurements.
- Students may believe that it is not important to interpret units in the context of the problem.

Academic Vocabulary:

accuracy, level of accuracy, precision, rounding

Standards for Mathematical Practice:

(5) Use appropriate tools strategically
(6) Attend to precision

Standard N-RN.B.3.
Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. *(Content Emphases: Additional)*

Explanations and/or Examples

Since every difference can be rewritten as a sum and every quotient as a product, this includes differences and quotients as well.

Example 1: Explain why the number 2π must be irrational given that π is irrational.

Ex. 1 Solution: If 2π were rational, it could be written as a ratio of two integers – a fraction. If I were to divide 2π by 2, I would get π. Dividing this fractional form of 2π by 2 would give me that same fraction with its denominator multiplied by 2. The result, which would represent π, would still be a fraction. Since it is given and known that π is irrational, and by definition irrational numbers cannot be written as the ratio of two integers, 2π could not have been rational. Therefore 2π – which is the product of a rational and irrational number – must be irrational.

Building Blocks (Grades 6 to 8 prerequisites):

Grade 8

8.NS.A.1

Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Prerequisite Examples:

Grade 8

8.NS.A.1

Decide whether each of the numbers is rational or irrational. If it is rational, explain how you know.

a) $0.33\overline{3}$
b) $\sqrt{4}$
c) $\sqrt{2} = 1.414213\ldots$
d) 1.414213
e) $\pi = 3.141592$
f) 11
g) $\frac{1}{7} = 0.\overline{142857}$
h) $12.34565656\overline{56}$

Grade/Course Level Connections:

n/a

Standard N-RN.B.3
Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. *(Content Emphases: Additional)*

Instructional Considerations:

- Students should explore concrete examples that illustrate that for any two rational numbers written in form a/b and c/d, where b and d are natural numbers and a and c are integers, the following are true.
  $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$ and $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$
- Use indirect algebraic proof to generalize the statement that the sum of a rational and irrational number is irrational.
- Given that x is an irrational number, assume that the sum of x and a rational number, a/b, is also rational and that sum is represented as c/d.
  $x+\frac{a}{b}=\frac{c}{d}$
  $x=\frac{c}{d}-\frac{a}{b}$
  $x=\frac{cb-ad}{bd}$, which represents a rational number (a ratio of two integers). Since this last statement contradicts a given fact that x is an irrational number, the assumption is wrong and a sum of a rational number and an irrational number has to be irrational.
- Illustrate through student exploration of multiple cases that when adding a rational number and an irrational number the result is always an irrational number.
- Illustrate through student exploration of multiple cases that when multiplying a rational number (other than zero) and an irrational number the result is always an irrational number.
- Students need to see that the results of the operations performed between numbers from a particular number set do not always belong to the same set. For example, the sum of two irrational numbers $2+\sqrt{3}$ and $2-\sqrt{3}$ is 4, which is a rational number.

Common Misconceptions:

- Students may believe that both terminating decimals are rational while all non-terminating decimals are irrational.
- Students may believe that irrational numbers are part of the complex numbers.
- By using false extensions of the properties of rational numbers, students may assume that the sum of any two irrational numbers is also irrational. This statement is not always true, (e.g. $2+\sqrt{3}$ and $2-\sqrt{3}$ is 4, a rational number) and therefore cannot be considered a property.

Academic Vocabulary:

rational numbers, irrational numbers, terminating/non-terminating decimals, repeating/non-repeating decimals, ratio, integers

Standards for Mathematical Practice:

(2) Reason abstractly and quantitatively
(3) Construct viable arguments and critique the reasoning of others